6.3 Curves of Best Fit

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

GOAL

Construct and interpret a curve of best fit for a given set of data.

LEARN ABOUT the Math

Sean and Parminder are studying the motion of a pendulum. Sean says that the pendulum will swing in the same way, no matter how long it is. Parminder is not so sure.

They did an experiment to see whether the length of a pendulum affects its period, which is how long it takes to go back and forth once.

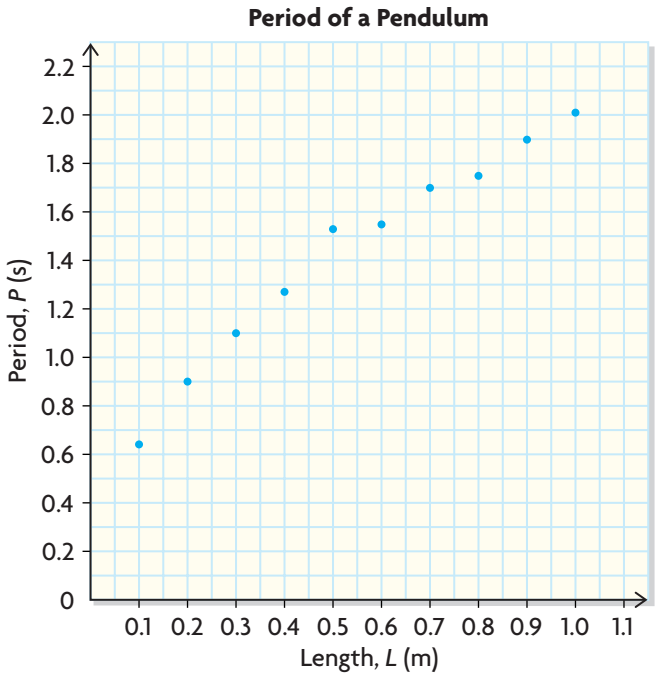
Length, *L*(m)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Period, *P*(s)	0.64	0.90	1.10	1.27	1.53	1.55	1.70	1.75	1.90	2.01

? What is the period when the length is 0.38 m?

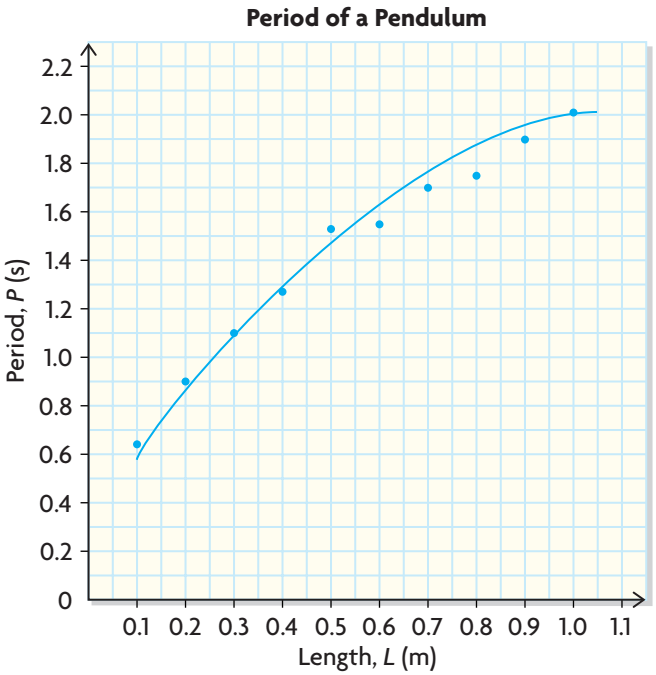
EXAMPLE 1 Solving a problem using a curve of best fit

Estimate the period of a pendulum that has a length of 0.38 m.

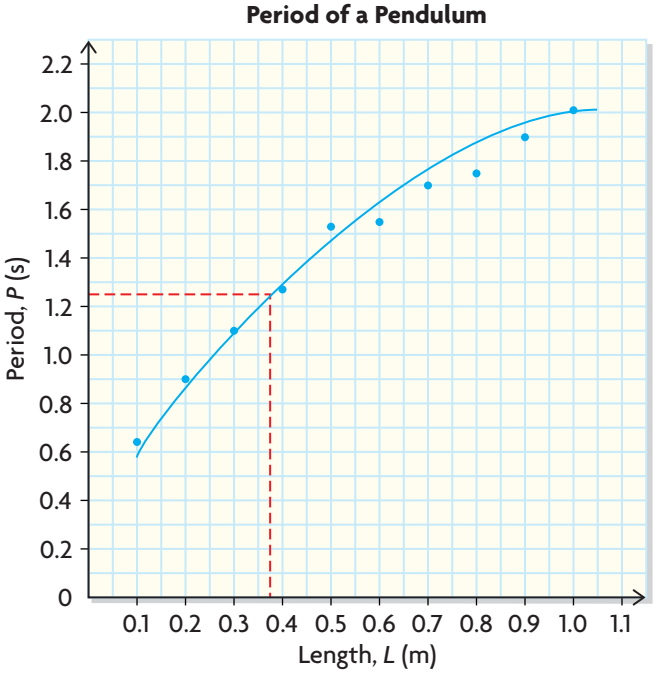
Walid's Solution



Since I want to know if the period depends on the length of a pendulum, I chose length as the independent variable and period as the dependent variable. Then, I created a scatter plot of the data.



The plotted points seem to follow a curve, so I used a smooth curve of best fit instead of a line of best fit. I made the curve solid because the variables are continuous.



I used the curve to estimate the period for a pendulum 0.38 m long.

My estimate for the period is 1.24 s when the length is 0.38 m.

Reflecting

A. How did Walid decide that a curve of best fit is a more appropriate representation than a line of best fit?

B. How did Walid use the curve of best fit to estimate the period when the length is 0.38 m?

C. If you were to draw a curve of best fit, would it be exactly the same as Walid's? If not, how would this change the estimate for the period when the length is 0.38 m?

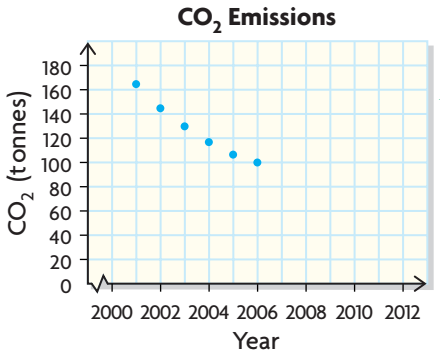
APPLY the Math

EXAMPLE 2 Using a curve of best fit to represent a trend

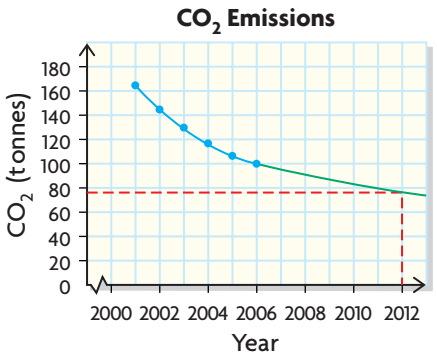
To help protect the environment, a steel factory is thinking about setting new standards for its carbon dioxide (CO_2) emissions. By 2012, the factory wants to emit less than 70 tonnes of CO_2 per year. If it does not change its practices, is the goal realistic?

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

Shannon's Solution



I plotted the data on a scatter plot.



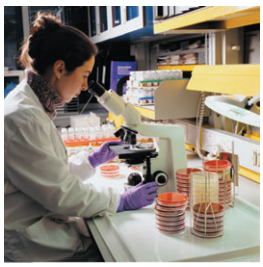
The emissions level for 2012 is estimated to be greater than 70 tonnes. This means the goal will probably not be met by 2012.

I sketched a curve of best fit. I could have used a line of best fit, but since the data seemed to follow a smooth curve, I thought using a curve of best fit would make my estimate more accurate. I used a solid curve because the variables are continuous.

I extended the curve out to the year 2012 assuming that this trend will continue.

I used the extrapolated graph to estimate the amount of CO_2 in 2012.

EXAMPLE 3 Using curves of best fit to reason about a trend



These tables show the population (in thousands) of two different bacterial colonies growing in separate Petri dishes.

Colony 1:

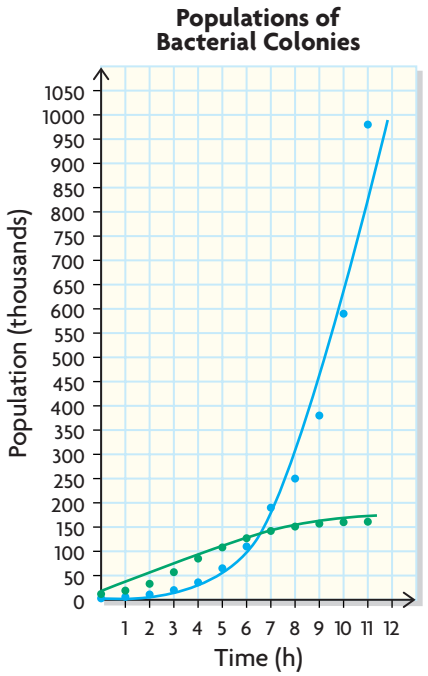
Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Population (thousands)	12	19	33	57	85	108	127	142	151	157	160	161

Colony 2:

Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Population (thousands)	3	6	11	20	36	65	110	190	250	380	590	980

Compare the growth patterns of the two colonies.

Alexis's Solution



I plotted both sets of data on a single scatter plot so that I could compare them more easily. I sketched each curve so that it was close to most of the plotted points.

Colony 2 keeps growing more and more quickly, but Colony 1 is growing more and more slowly.

Both curves are always going up, so the populations of both colonies are increasing.

Colony 1 grows rapidly at first. But later the curve almost levels off. The colony still grows, but very slowly.
Colony 2 starts off with a lower population than Colony 1 but, because it grows more quickly, at about 6.5 h it catches up and passes Colony 2.

Where the curves are steep, it means that the population is increasing rapidly. Where the curves are less steep, the population is growing slower.

In Summary

Key Idea

- Sometimes a curve represents the trend or pattern in a scatter plot better than a line.

Need to Know

- You can use a curve of best fit to extrapolate and interpolate values.
- Extending a curve involves more guesswork than extending a line, so you can't be as sure of your predictions.
- Sometimes it's not clear whether a curve or line of best fit can be drawn. This could be because there is no relationship between the variables, or it could mean that more data need to be collected.

CHECK Your Understanding

1. Computers use code numbers that are made up of only the digits 0 and 1. The data in the chart represent the number of possible code numbers of each length. (For example, the code numbers of length 1 are 0 and 1, the code numbers of length 2 are 00, 01, 10, and 11, etc.)

Length of Code Number	1	2	3	4	5	6
Number of Possible Code Numbers	2	4	8	16	32	64

a) Plot the data on a scatter plot.
b) Sketch a line of best fit.
c) Sketch a curve of best fit.
d) Which approximates the data better: the line in part b) or the curve in part c)?
e) Is it reasonable to use the line or curve of best fit to estimate the value of y when $x = 3.7$? Explain.

PRACTISING

2. K Weekly earnings for the movie *Dude, Where's My Math Book?* in the weeks following its opening are listed in the table.

Earnings ($ millions)	9.2	21.4	34.0	25.7	19.6	14.1	13.3	11.2	6.6	3.1
Weeks Since Opening	1	2	3	4	5	6	7	8	9	10

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Should you use a solid curve or a dashed curve? Explain.
c) Does it make sense to use the curve of best fit to estimate the earnings after 3.5 weeks? Explain.
d) Does it make sense to use the curve of best fit to estimate the earnings 20 weeks after opening? Explain.

3. A basketball is dropped from a height of 200 cm. The table shows how high it bounces on each bounce.

Maximum Height (cm)	200	120	72	44	26	16	10	6	4
Bounce Number	0	1	2	3	4	5	6	7	8

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Should you use a solid curve or a dashed curve? Explain.
c) Does it make sense to use the curve of best fit for interpolation? Explain.
d) Does it make sense to use the curve of best fit for extrapolation? Explain.

4. C In order to obtain a medical image of a patient's thyroid gland, a chemical is injected into the patient's bloodstream. The chemical's concentration in the blood gradually decreases with time.

Concentration (mg/L)	29.0	15.0	7.7	3.9	2.1	1.3	0.7	0.5	0.4
Time (h)	0	1	2	3	4	5	6	7	8

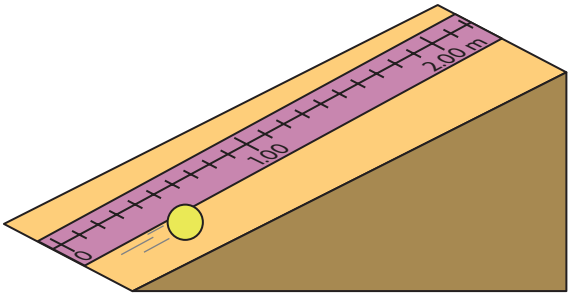
a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Did you use a solid curve or a dashed curve? Explain.
c) Describe the relationship between the variables.
d) Use your curve of best fit to estimate when the concentration of the chemical will be 6.1 mg/L.
e) Use your curve of best fit to estimate the concentration of the chemical after 12 h.

5. The ages and resting heart rates for some people are listed in the table.

Age (years)	21	24	26	29	31	35	39
Resting Heart Rate (beats per minute)	60	61	63	65	68	73	78

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Did you use a solid curve or a dashed curve? Explain why.
c) Describe the relationship between the variables.
d) Does it make sense to use the curve of best fit for interpolation? Explain.
e) Does it make sense to use the curve of best fit to estimate the resting heart rate for an 85-year-old person? Explain.

6. In his experiments to study the Earth's gravity, Galileo rolled objects
A on inclined planes. In one such experiment, a ball is rolled up a plane, and then, rolls back down. The data are in the following table.



Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Position (m)	0	1.13	1.50	1.88	2.00	1.88	1.50	1.13

a) Use a graph to estimate the position of the ball after 0.3 s.
b) Use a graph to estimate when the ball will return to the bottom of the inclined plane.

7. The table shows the population of a bacterial colony growing in a test tube at various times.

Time (h)	0	1	2	3	4	5	6	7	8
Population (thousands)	1	1.4	2.0	2.7	3.8	5.4	7.5	10.5	14.8

a) Use a graph to describe the growth of the colony.
b) Use a graph to estimate the population of the colony after 7.5 h.

8. A herd of caribou is moved to a small, remote island where they have no predators. Data on the population of the herd were collected for 6 years.

Time (years)	0	1	2	3	4	5	6
Population	24	35	51	74	104	151	225

a) Sketch the data on a scatter plot.
b) Draw a line or curve of best fit through the plotted points. Explain which is more appropriate.
c) Describe the growth of the herd.
d) Predict the population of the herd after seven years.

9. In the Kingdom of Petrodalla, natural gas is the primary resource. The table shows the amount of natural gas produced each year.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Natural Gas Produced (millions of m^3)	1.6	2.1	3.0	4.1	4.3	4.4	3.6	2.1	0.5

a) Sketch the data on a scatter plot.
b) Draw a line or curve of best fit through the plotted points. Explain which is more appropriate.
c) Describe how the production of natural gas changes over time.
d) Predict when natural gas production will decrease to zero.

10. T Consider the carbon dioxide emission data from Example 2, which are repeated here:

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

a) Instead of using a curve of best fit, as is done in the example, use a line of best fit to estimate carbon dioxide emissions in 2012.
b) Compare your estimate in part a) with the result of Example 2. Which result do you think is more reliable? Justify your choice.

11. **a)** How do you know when to use a curve of best fit, and when to use a line of best fit?
b) How can you use a curve of best fit for interpolation or extrapolation if you don't know the equation for the curve?
c) How do you know how far you can reasonably extrapolate?

Extending

12. Consider the carbon dioxide emission data in Example 2, which are repeated here.

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

a) Suppose that the carbon dioxide emissions are reduced in each year after 2006 by a constant rate of 10% per year. Will the emission target of no more than 70 tonnes be reached by 2012?
b) Repeat part a) if the reduction is a constant 5% per year.
c) What is the minimum yearly percentage reduction after 2006 that will guarantee that the emission target will be reached by 2012?

13. The data given in Example 1 are shown in the following table.

Length, *L* (m)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Period, *P* (s)	0.64	0.90	1.10	1.27	1.42	1.55	1.70	1.80	1.90	2.01
Squared Period, P^2 (s^2)	0.41	0.81	1.21							

a) Complete the new table, in which the periods are squared.
b) Plot the squared periods versus length.
c) Draw a line of best fit.
d) Calculate the slope of the line of best fit.
e) It's possible to show that the slope of the line of best fit should be equal to $\frac{4\pi^2}{g}$, where g is the acceleration due to the Earth's gravity. Use this formula and the results of part d) to calculate g.
f) Collect your own data on the period of a pendulum. Then, follow the steps in this exercise to obtain your own experimental value for the acceleration due to earth's gravity. How does your experimental value compare with values obtained in professional experiments?

14. Dorothy has 20 m of fencing to make a rectangular enclosure for her dog.
a) Draw all possible rectangular enclosures using whole numbers from 1 to 9 for the dimensions.
b) Organize the data in table of values with headings Length, Width, and Area.
c) Graph the relation area vs. length. Draw the line or curve of best fit.
d) What dimensions give the maximum area for a perimeter of 20 m.
e) Write an equation that relates the area of the enclosure to its length.