

# 5.4

## Using Points to Determine the Equation of a Line

### GOAL

Determine the equation of a line given information about related points.

### YOU WILL NEED

- graphing calculator

### LEARN ABOUT the Math

Ken's Kanine Kennel provides suites that dogs in the same family can share. Ken's charges a room fee for the family plus an additional amount for each dog. One day's stay costs \$71 for 2 dogs and \$113 for 5 dogs. Julie wants to know the daily cost to board her 3 dogs.

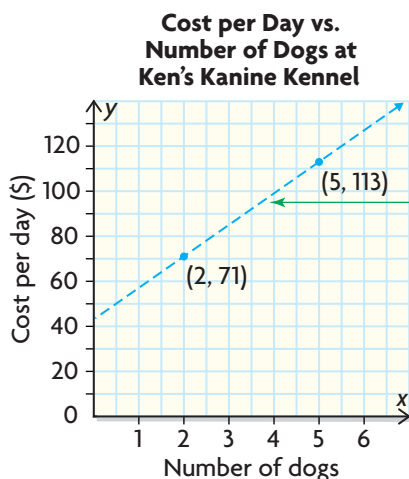
**?** How can you determine the equation of this relationship?

#### EXAMPLE 1 Using a strategy involving the slope formula and equation solving

Determine the equation that describes the relationship between the number of dogs and the daily boarding cost.

#### Katerina's Solution

Let  $x$  represent the number of dogs and  $y$  represent the daily cost.



I chose  $x$  to represent the number of dogs because it was the independent variable, and  $y$  to represent the cost because the cost depended on the number of dogs.

I sketched the given information so that I could get an idea of what was happening.

I knew that the equation of a line can have the form  $y = mx + b$ . I needed to determine the slope ( $m$ ) and the  $y$ -intercept ( $b$ ).

$$y = mx + b$$



**Determine the slope:**

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{113 - 71}{5 - 2} \\
 &= \frac{42}{3} \\
 &= 14
 \end{aligned}$$

I used the slope formula and the coordinates of the two points.

$$\begin{array}{cc}
 (2, 71) & (5, 113) \\
 \uparrow \quad \uparrow & \uparrow \quad \uparrow \\
 x_1 \quad y_1 & x_2 \quad y_2
 \end{array}$$

$$y = 14x + b$$

I substituted the value of the slope into my equation.

**Determine the  $y$ -intercept:**

$$\begin{aligned}
 71 &= 14(2) + b \\
 71 &= 28 + b \\
 71 - 28 &= 28 - 28 + b \\
 43 &= b
 \end{aligned}$$

From my sketch, I could tell the  $y$ -intercept was between 40 and 50. I also knew that the point  $(2, 71)$  was on the line, so if I substituted 2 for  $x$  in the equation, then the  $y$ -value had to be 71. That gave me enough information to solve for  $b$ .

$$y = 14x + 43$$

Since I knew the values for both  $m$  and  $b$ , I could write the equation.

Check:

Left Side	Right Side
	$14x + 43$
113	$= 14(5) + 43$
	$= 70 + 43$
	$= 113$

I checked to see if boarding 5 dogs would cost \$113 using my equation. Since both sides were equal, I knew that my equation was correct.

The relationship between the daily boarding cost and the number of dogs is  $y = 14x + 43$ . This means that Ken's charges a room fee of \$43, plus an additional \$14 per dog per day.

**Reflecting**

- A. How was Katerina able to determine the slope and the exact value of the  $y$ -intercept of the line?
- B. Explain why Katerina could have used the point  $(5, 113)$  to determine a value for  $b$ .
- C. Katerina determined the value of  $m$  before she determined the value for  $b$ . Could she have determined  $b$  before  $m$ ? Explain.

## APPLY the Math

### EXAMPLE 2 Using the slope and one point to determine the equation of a line

Determine the equation of the line that has a slope of 4 and passes through the point (2, 6).

#### Sherif's Solution

$$y = mx + b$$

$$y = 4x + b$$

I was given a value for the slope, so I substituted 4 for  $m$ .

**Determine the  $y$ -intercept:**

$$6 = 4(2) + b$$

$$6 = 8 + b$$

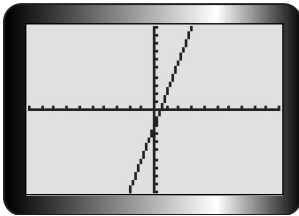
$$6 - 8 = 8 - 8 + b$$

$$-2 = b$$

(2, 6) was a point on the line, so I substituted its coordinates for  $x$  and  $y$ .

The equation of the line is  $y = 4x - 2$ .

I solved the equation for  $b$ .



X	Y1
-1	-6
0	-2
1	2
2	6
3	10
4	14
5	18

X=2

I graphed my equation and used the table to verify that my line passed through the point (2, 6).

### EXAMPLE 3 Reasoning from properties of a line to determine its equation

Determine the equation of the line that has a slope of  $-\frac{1}{3}$  and the same  $y$ -intercept as the line  $2x - 4y + 7 = 0$ .

#### Anayis's Solution

**Determine the slope:**

$$y = mx + b$$

$$y = -\frac{1}{3}x + b$$

I substituted the given value of  $-\frac{1}{3}$  for the slope,  $m$ .

**Determine the  $y$ -intercept:**

$$2x - 4y + 7 = 0$$

I needed to determine the  $y$ -intercept of  $2x - 4y + 7 = 0$  because the lines had the same  $y$ -intercept.

$$2(0) - 4y + 7 = 0$$

I knew that the  $x$ -coordinate was 0 for the point where the line crossed the  $y$ -axis.

$$-4y + 7 = 0$$

$$-4y + 7 - 7 = 0 - 7$$

$$-4y = -7$$

$$\frac{-4y}{-4} = \frac{-7}{-4}$$

$$y = \frac{7}{4}$$

$$y = \frac{7}{4}$$

I used inverse operations to solve the equation for  $y$ .

$$b = \frac{7}{4}$$

I knew that the  $b$  in the equation represented the  $y$ -intercept.

The equation of the line is

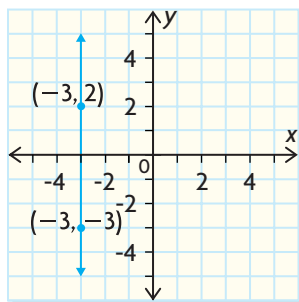
$$y = -\frac{1}{3}x + \frac{7}{4}$$

#### EXAMPLE 4

Using reasoning to determine the equation of a line through two points with the same  $x$ -coordinate

Determine the equation of the line that passes through the points  $(-3, -3)$  and  $(-3, 2)$ .

#### Omar's Solution



This line is vertical. The equation of the line is  $x = -3$ .

From the sketch of the graph, I could see that the line was vertical and that every point on the line had  $-3$  for its  $x$ -coordinate.

**EXAMPLE 5****Using the equation of a line to solve a problem**

The speed of sound in air can be calculated based on temperature using a linear relation. At  $10^\circ\text{C}$  the speed of sound is  $337.4\text{ m/s}$ , and at  $21.5^\circ\text{C}$  the speed is  $344.3\text{ m/s}$ . What is the speed of sound when the temperature is  $32.3^\circ\text{C}$ ?

**Suhaila's Solution**

Let  $T$  represent the temperature and  $S$  represent the speed of sound.

I chose variables to represent each unknown quantity.

$$S = mT + b$$

I wanted to find an equation for the relationship first. Using  $y = mx + b$ , I replaced  $y$  with  $S$  since sound is the dependent variable and  $x$  with  $T$  since temperature is the independent variable.

**Determine the slope:**

$$\begin{aligned} m &= \frac{344.3 - 337.4}{21.5 - 10} \\ &= \frac{6.9}{11.5} \\ &= 0.6 \end{aligned}$$

I substituted the points  $(10, 337.4)$  and  $(21.5, 344.3)$  into the slope formula.

This means the speed increases  $0.6\text{ m/s}$  for each Celsius degree increase in temperature.

**Determine the S-intercept:**

$$S = 0.6T + b$$

I substituted the slope value  $0.6$  into my equation.

$$337.4 = 0.6(10) + b$$

$$337.4 = 6 + b$$

$$337.4 - 6 = 6 - 6 + b$$

$$331.4 = b$$

I used the point  $(T, S) = (10, 337.4)$  in my equation to solve for  $b$ .

This means that when the temperature is  $0^\circ\text{C}$  the speed of sound is  $331.4\text{ m/s}$ .

$S = 0.6T + 331.4$  represents the speed of sound  $S$  at temperature  $T$ .



$$S = 0.6(32.3) + 331.4$$

$$S \doteq 350.8$$

At  $32.3^\circ\text{C}$ , the speed of sound is  $350.8\text{ m/s}$ .

I substituted the given temperature  $32.3^\circ\text{C}$  into my equation to solve the equation.

## In Summary

### Key Ideas

- You can determine the equation of a line in the form  $y = mx + b$  if you know two points on the line or one point and the slope.

### Need to Know

- You can determine the equation of a line as follows:
  - If the slope is not given, and you know two points on the line, use the coordinates of the points to calculate the slope.
  - Substitute the value of the slope for  $m$  and the coordinates  $(x, y)$  of a point on the line into  $y = mx + b$  and solve for  $b$ .
  - Use the values of  $m$  and  $b$  to write the equation of the line.

## CHECK Your Understanding

1. Complete the table on the right by determining the missing values.

	Slope	y-intercept	Equation
a)	3	5	
b)			$y = 5x + 1$

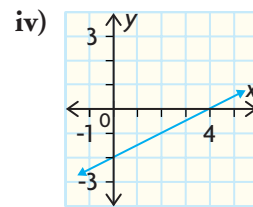
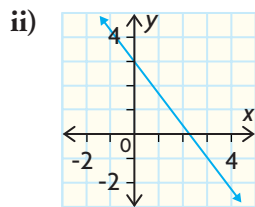
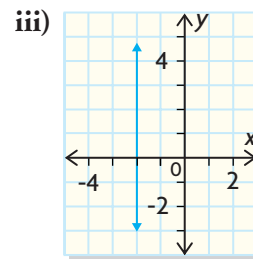
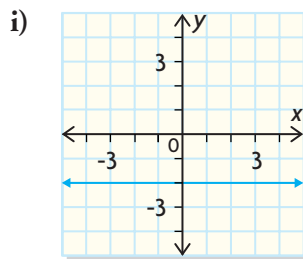
2. Match each equation to its corresponding graph.

a)  $x = -2$

c)  $y = -2$

b)  $x - 2y = 4$

d)  $y = -\frac{4}{3}x + 3$



3. Determine the equation of the line with the following characteristics.
- has a slope of  $-2$  and passes through the point  $A(5, 2)$
  - passes through the points  $B(4, 6)$  and  $C(1, -3)$

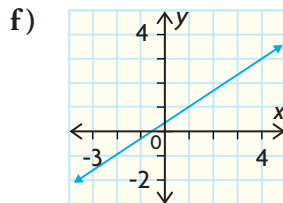
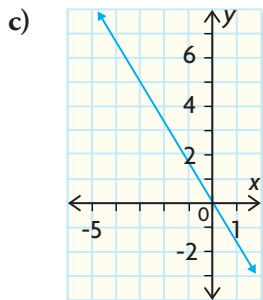
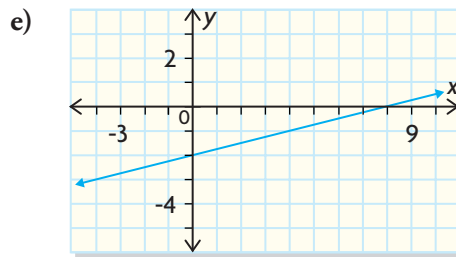
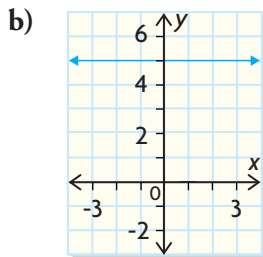
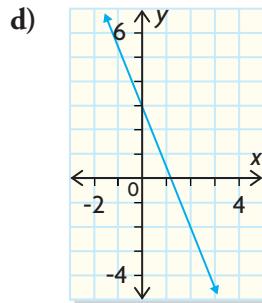
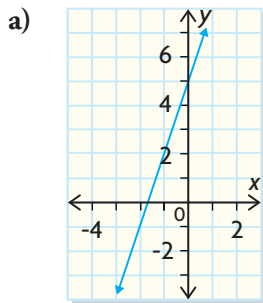
## PRACTISING

4. Complete the table shown below by determining the missing values.

**K**

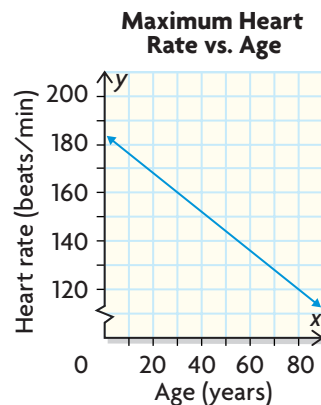
	Slope	y-intercept	Equation
a)	$-5$	$3$	
b)			$y = \frac{4}{3}x - 2$
c)	$0$	$2$	
d)			$y = \frac{1}{2}x$

5. Determine the equation of each line shown below.



6. If the equation  $y = 3x + b$  represents a line that passes through the given point, determine the value of the  $y$ -intercept,  $b$ .
- a)  $(4, 1)$                       b)  $(-3, 2)$                       c)  $(1, -3)$
7. If the equation  $y = mx + 3$  represents a line that passes through the given point, determine the slope value,  $m$ .
- a)  $(2, 4)$                       b)  $(-3, 7)$                       c)  $(8, 2)$
8. Determine the equation of each line described below.
- a) passing through the point  $A(0, 4)$ , with a slope of  $-\frac{8}{9}$
- b) passing through the point  $A(3, -5)$ , with a slope of  $\frac{1}{5}$
- c) has an  $x$ -intercept of 4 and a  $y$ -intercept of  $-3$
- d) has an  $x$ -intercept of 6 and passes through the point  $(6, 4)$
9. Determine the equation of the line passing through each pair of points.
- a)  $A(1, 9)$  and  $B(1, -7)$                       d)  $G(6, 18)$  and  $H(-12, 3)$
- b)  $C(-8, -3)$  and  $D(8, 27)$                       e)  $I(0, 5)$  and  $J(0, 12)$
- c)  $E(-12, 7)$  and  $F(4, 7)$                       f)  $K(-5, -1)$  and  $L(15, 1)$
10. Determine the equation of the line that has the same  $x$ -intercept as the line described by  $x - 5y + 10 = 0$ , and the same  $y$ -intercept as the line  $3x + 2y - 6 = 0$ .
11. The LeBlanc family is driving home. The LeBlancs are using cruise control so their speed is constant. After 3 h, they are 350 km from home. After 5 h, they are 130 km from home.
- a) Write an equation to represent this distance–time relationship.
- b) What do the slope and  $y$ -intercept of your equation mean in this situation?

12. A stress test evaluates the health of a patient's heart. While riding on a stationary bike or running on a treadmill, a patient has his or her heart rate measured by a technician and compared with a safe maximum heart rate. This safe heart rate is based on the patient's age as shown in the graph.



- a) What does the  $y$ -intercept represent in this situation?
- b) What does the slope of the graph represent?
- c) Determine the equation for the line.
- d) Ellen is 14 years old. Using your equation, determine her maximum safe heart rate.



13. The local fall fair charges a flat fee for admission plus an additional cost for ride tickets. Last year, Kelsey purchased 15 tickets and spent a total of \$19.50. His brother Quinn purchased 36 tickets and spent a total of \$30.00 at the fair.
- Determine an equation to represent the relationship between the total amount of money spent and the number of tickets purchased.
  - A ride pass, which gives a person entrance to the park and unlimited use of the rides, costs \$21. Write the equation for the relationship between the total amount spent on a ride pass and the number of rides it can be used for.
  - Last year, Erin used 25 tickets at the fall fair. Should Erin purchase tickets again this year, or buy a ride pass? Explain.
  - Heather only likes the fun house, which requires one ticket. She went on this ride 10 times last year. How much money would Heather save by purchasing tickets instead of a ride pass?
14. Lori downloads music from the Music Genie site, which charges a monthly membership fee plus an amount for each song downloaded. A three-month record of her activity on the site is shown.

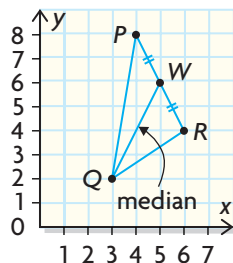
Month	Number of Songs Downloaded	Monthly Bill (\$)
January	54	26.90
February	38	25.30
March	21	23.60

- Use two points from the table to determine the equation of the relationship between numbers of downloads and her monthly bill.
  - Verify that the third point from the table also satisfies your equation.
  - Lori's brother thinks she should change to Web Waves, which doesn't have a membership fee and charges \$0.95 per song. Based on your calculations, do you think Lori should change music companies? Explain.
  - Digital Beats charges \$25 for a monthly membership, with unlimited free downloads. Would you recommend Lori change to Digital Beats? Explain.
15. Shawn says he can only figure out the equation of a line if he is given the  $y$ -intercept and the slope of the line. Barb says that she can figure out the equation using the coordinates of any two points on the line. With whom do you agree? Why?

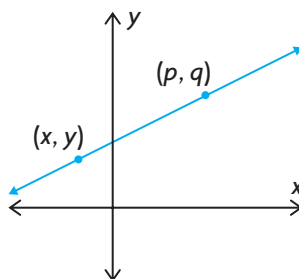


## Extending

16. Determine the equation of the **median** from Q to the midpoint of PR, in triangle PQR, with P(4, 8), Q(3, 2), and R(6, 4).



17. Given any two points on a line, the equation of the line can be determined from the point-slope form of the equation of the line:  
 $y = m(x - p) + q$ .



- Show how the formula can be developed using the slope formula,  

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
- Use the point-slope form of the equation of a line to determine the equation of a line that has a slope of 3 and passes through the point (1, 2).  

$$\begin{array}{c} \uparrow \quad \uparrow \\ p \quad q \end{array}$$
- Determine the equation of the line in part b) using  $y = mx + b$  to verify that the new formula works.
- Use the point-slope form of the equation of a line to determine the equation for each of the following lines.
  - passing through the points (4, -6) and (5, -1)
  - passing through the points (3, -1) and (9, 3)
  - passing through the points (4, 5) and (3, 9)