5.3 Slope of a Line

YOU WILL NEED

- grid paper
- ruler

GOAL

Determine the slope of a line.

INVESTIGATE the Math

Julien's parents are hiring a caterer for his brother's graduation party.

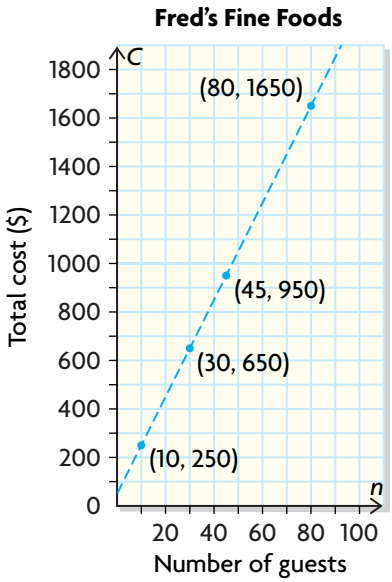
They found the advertisement to the right in the newspaper.

They wonder what the cost per person would be if they used Fred's catering service.

Fred's Fine Foods

Number of Guests	Total Cost ($)
10	250
30	650
45	950
80	1650

? How can Julien's parents determine the cost per person?



A. Copy the graph into your notebook.

B. Select two of the four points given on the graph.

C. Draw a right triangle to illustrate the rise and run between the points.

D. Calculate the slope and express it in lowest terms.

E. Record your results in the following table.

First Point	Second Point	Rise	Run	Slope

F. Repeat parts B to E with a second and a third pair of points.

G. How could you have calculated the rise and run without drawing the graph?

H. What is the cost per person if Julien's parents hire Fred's Fine Foods?

Reflecting

I. Why is the slope the same for any pair of points that you choose on the line?

J. If (x_1, y_1) and (x_2, y_2) are points on a line, what formula could you write to calculate the slope of the line?

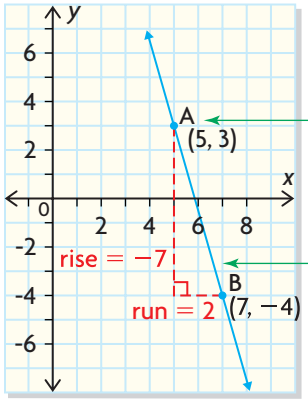
K. Explain why it does not matter which point is (x_1, y_1) and which is (x_2, y_2) when calculating the slope of a line.

APPLY the Math

EXAMPLE 1 Selecting a strategy to determine slope

Calculate the slope of the line passing through points A(5, 3) and B(7, −4).

Rory's Solution: Reasoning by using a graph



I plotted A and B on a grid and drew a line through them. Then, I drew a right triangle to determine the rise and run.

The line sloped down to the right, so I knew that the slope had to be negative.

My answer seemed reasonable because the line was decreasing and steep.

The slope of the line is $m = -\frac{7}{2}$.

Communication Tip

AB often is used to indicate the line segment joining points A and B.

m_{AB} often is used to represent the slope of the line or line segment through points A and B.

Chong Sun's Solution: Calculating using a formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

I decided to use the slope formula.

$$A(5, 3) \quad B(7, -4)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

I chose point A(5, 3) to be (x_1, y_1) and point B(7, −4) to be (x_2, y_2).

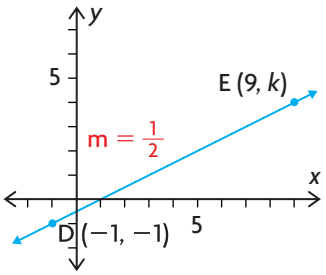
$$m_{AB} = \frac{(-4) - (3)}{(7) - (5)}$$
$$= \frac{-7}{2}$$

I substituted the values into the slope formula.

Since the *y*-value decreased a lot and the *x*-value increased only a little, I knew that my answer was reasonable.

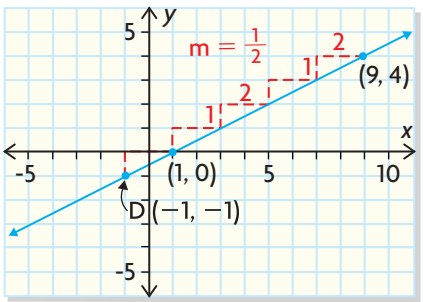
The slope of the line is $m = -\frac{7}{2}$.

EXAMPLE 2 Selecting a strategy to determine an unknown coordinate



Determine the value of *k* in point E using the graph to the left.

Galen's Solution: Reasoning by using a graph



I started at point D. Since the slope was $\frac{1}{2}$, the rise was 1 and the run was 2.

I went up 1 unit from D and to the right 2 units and I got to (1, 0). I continued to go up 1 and right 2 until I got to point E where *x* was 9.

The value of *k* is 4.

The point $(x, y) = (9, 4)$ was on the line with a slope of $\frac{1}{2}$ through points D and E.

Pierce's Solution: Calculating using a formula

The slope of segment DE is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{(k) - (-1)}{(9) - (-1)}$$

$$\frac{1}{2} = \frac{k + 1}{9 + 1}$$

$$\frac{1}{2} = \frac{k + 1}{10}$$

$$10\left(\frac{1}{2}\right) = 10\left(\frac{k + 1}{10}\right)$$

$$5 = k + 1$$

$$5 - 1 = k + 1 - 1$$

$$4 = k$$

The value of k is 4.

I substituted $\frac{1}{2}$ for m because it was given.

I chose point D as (x_1, y_1) and point E as (x_2, y_2).

I substituted the x- and y-values into the formula.

I simplified the equation.

I multiplied both sides of the equation by 10 so that I could work with integers instead of fractions.

I solved the equation using inverse operations.

Point E must have the coordinates (9, 4).

EXAMPLE 3 Using the slope formula to calculate a rate of change

A bathtub is filling with water at a constant rate. After 3 min the water is 7.5 cm deep, and after 8 min the water is 15 cm deep. At what rate is the depth of water increasing?

Quinn's Solution

Time (min)	3	8
Depth (cm)	7.5	15

I organized the given information in table form.

$$m = \frac{15 - 7.5}{8 - 3}$$

$$= \frac{7.5}{5}$$

$$= 1.5$$

I knew that the slope of a line gave the rate of change of the relation, so I calculated the slope using the given values.

The water depth is increasing 1.5 cm/min.

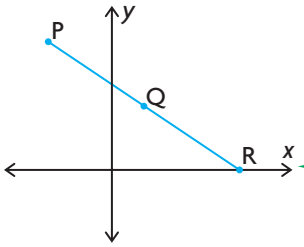
collinear

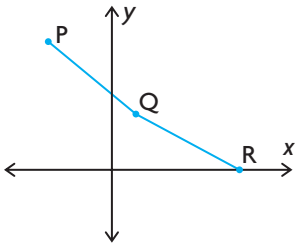
three or more points are collinear if they lie on the same line

EXAMPLE 4 Using reasoning involving slope to determine collinearity

Determine if the points P(−6, 12), Q(3, 6), and R(12, 0) are **collinear**.

Angus's Solution





I started with a rough sketch of the three points, but I couldn't tell if they were on a straight line or not.

I knew that if the points were on a straight line then the graph would look like my first sketch. Otherwise, the graph would look like my second sketch.

Need to determine:
slope of $\overline{PQ}$ and slope of $\overline{QR}$

I noticed that in the second sketch, the slopes of $\overline{PQ}$ and $\overline{QR}$ were different. So, I wanted to calculate the slopes of those two line segments.

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{12 - 6}{(-6) - 3}$$
$$= \frac{6}{-9}$$

I calculated the slope of line segment $\overline{PQ}$.

Q(3, 6) P(−6, 12)
x_1 y_1 x_2 y_2

The slope of segment $\overline{PQ}$ is $-\frac{2}{3}$.

$$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - 6}{12 - 3}$$
$$= \frac{-6}{9}$$

I calculated the slope of line segment $\overline{QR}$.

Q(3, 6) R(12, 0)
x_1 y_1 x_2 y_2

The slope of segment $\overline{QR}$ is $-\frac{2}{3}$.

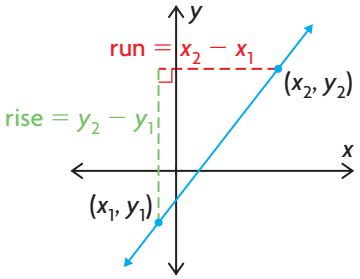
Since $m_{PQ} = m_{QR}$, P, Q, and R are collinear.

Since the slopes were the same for $\overline{PQ}$ and $\overline{QR}$, I knew that an accurate graph would look like my first rough sketch. My points had to be collinear.

In Summary

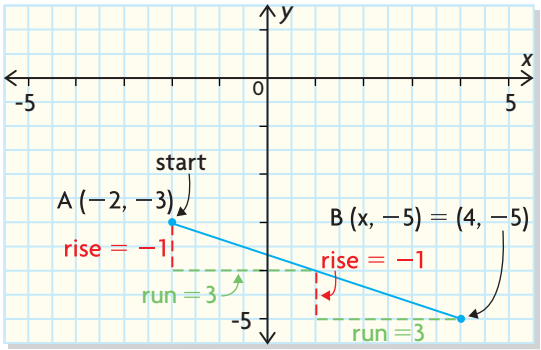
Key Idea

- You can use the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope of a line if you know the coordinates of two points on the line.



Need to Know

- When you use the coordinates of two points to calculate the slope of a line, either point can be (x_1, y_1).
- Points A, B, and C are collinear if the slopes of any pair of line segments (e.g., $\overline{AB}$, $\overline{BC}$, $\overline{AC}$) are equal.
- If you are given the slope and one point on a line, you can determine an unknown coordinate of another point on the line in two ways:



  - Draw an accurate sketch by starting at the known point, and then use the rise and run repeatedly until you reach the required point. This method only works easily if the coordinates are all integers. For example: Determine the value of x if the slope of $\overline{AB}$ is $-\frac{1}{3}$ with A$(-2, -3)$ and B$(x, -5)$. The value of x is 4.
  - Substitute both points and the given slope into the slope formula and use inverse operations to solve for the unknown coordinate.

CHECK Your Understanding

1. Calculate the slope of the line through each pair of points.

a) A(3, 8) and B(10, 15)

b) C(9, −2) and D(8, 4)

2. The point $(-2, -3)$ lies on a line with slope $\frac{2}{3}$. Determine the y-coordinate of the point on the line with x-coordinate 13.

PRACTISING

3. Calculate the slope of the line through each pair of points.

K **a)** A(−2, 5) and B(4, −8)

b) C(0, 5) and D(−2, 3)

c) E(5, 10) and F(5, −4)

d) G(−7, 8) and H(4, 8)

e) I(3.5, 4.8) and J(1.4, 6.2)

f) K(32, 630) and L(58, 1020)

4. Write the coordinates of one other point that would be on the line passing through the point A(2, 5) with each of the following slopes.

a) $-\frac{1}{4}$ **b)** $\frac{8}{3}$ **c)** -4 **d)** 0

5. For the points J, K, and L, the slope of segment $\overline{JK}$ is -4 and the slope of segment $\overline{KL}$ is -2. Explain how you know that J, K, and L are not collinear.

6. Determine whether the given points are collinear.

a) A(−8, 0), B(−6, 1), and C(4, 6)

b) D(−5, 17), E(−12, 40), and F(−42, 128)

c) G(−30, −70), H(−15, −38), and I(17, 26)

d) J(−9, 1), K(−12, 3), and L(6, −9)

7. a) Plot the points $(-3, 8)$ and $(5, 8)$ and draw the line that passes through them.

b) Calculate the slope of the line using the slope formula.

c) What can you conclude about the slope of a horizontal line?

8. a) Plot the points $(4, 10)$ and $(4, -1)$ and draw the line that passes through them.

b) Calculate the slope of the line using the slope formula.

c) What can you conclude about the slope of a vertical line?

9. a) Is the rise equal to zero for a vertical line or a horizontal line? Explain.

b) Is the run equal to zero for a horizontal line or a vertical line? Explain.

10. How can you tell from the coordinates of two points if the line passing through them is horizontal, vertical, or slanted?

11. Estimate the slope of the red line in the "No cell phones" sign. (Assume the grid is square.) Explain how you determined your estimate.

C

12. Nolen was cycling toward his home. After 2 h of cycling he was 55 km from home, and after 4.5 h of cycling he was 17.5 km from home. Assuming he was cycling at a constant rate, how fast was he cycling?

A

13. Manpreet works for Vision Optical where she earns an hourly rate and receives a fixed amount each week to cover her expenses. This table shows her earnings for various hours worked.

Hours Worked	20	25	30	40	50
Weekly Earnings ($)	390	450	510	630	750

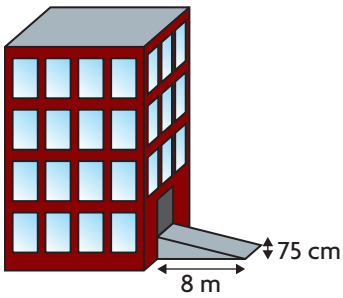
a) Draw a graph of weekly earnings vs. hours worked.
b) Calculate her rate of pay per hour.
c) How long would Manpreet have to work in order to earn $900?
d) Do you think it is likely that Manpreet will earn $900 in a single week? Explain.

14. A house worth $150 000 in 1999 increased by a constant rate to its value of $255 000 in 2007. Calculate the home's annual rate of increase in value.

15. A wheelchair ramp should have a slope of $\frac{1}{12}$ or less.

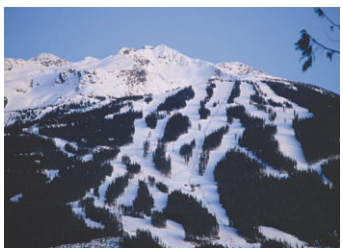
T

a) Does this wheelchair ramp meet the requirements?
b) If a second ramp is to be built with a rise of 90 cm, what is the shortest length that will still meet the building code?



16. Information about the three most popular runs at a ski resort is shown in the following table.

Name of Run	Vertical Drop (feet)	Length of Slope (feet)
Snowbowl	256	890
Bear Claw	480	4824
The Vortex	510	3438

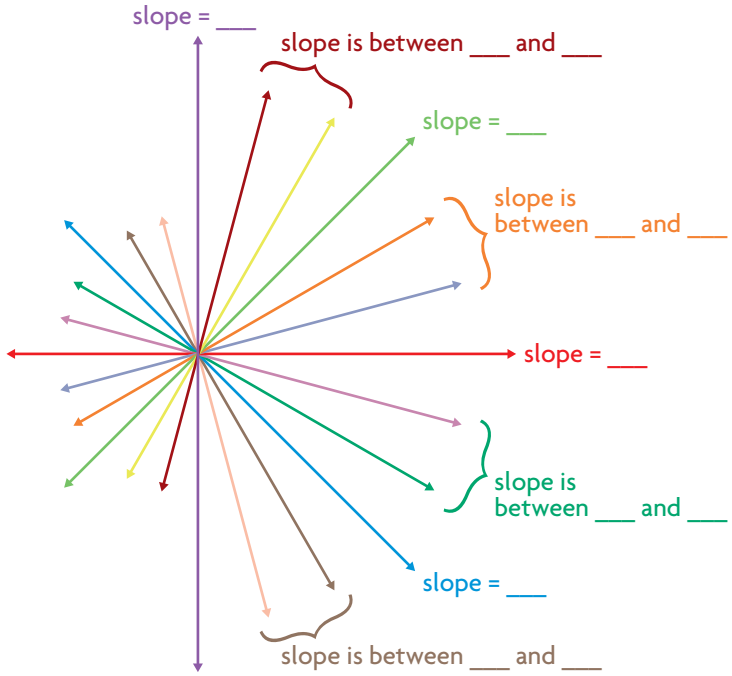


a) Use the Pythagorean theorem to determine the horizontal "run" for each ski slope. Assume each ski slope is perfectly straight (no dips or moguls).
b) Use slopes to rank each ski run from easiest to most difficult.

17. If you hire Daminga's Delicious Dinners to cater a party, it will cost $450 for 20 guests and $675 for 35 guests. If the company charged a fixed rate per guest, calculate the cost per person.

18. Determine the value of k if the points X(3, 2), Y(k, 8), and Z($k + 7$, 29) are collinear.

19. Complete the following picture in your notebook to summarize what you know about slopes of lines.



Extending

20. Consider the points A(7, k), B(11, 4), and C(13, $1 - 3k$).

a) If A, B, and C are collinear, determine the value of k.

b) If A, B, and C are collinear, determine the coordinates of A and C.

c) Determine a possible value for k for which the points would not be collinear.

21. For each situation, write an equation of the line in the form $y = mx + b$.

a) The slope is 5 and the y-intercept is 2.

b) The slope is -4 and the point $(4, -3)$ is on the line.

c) The slope is $\frac{2}{3}$ and the point (6, 4) is on the line.

d) The line is vertical and passes through the point (2, 5).

e) The line is horizontal and passes through $(-1, -2)$.