4.2 Solving Linear Equations Using Inverse Operations

GOAL

Solve equations by working backward.

LEARN ABOUT the Math

Michelle delivers paper waste to a recycling centre. She had net earnings of $23.90 on her first trip.

Michelle's Costs	Michelle's Earnings
$8.00/trip for gas	$72.50/tonne of paper

Communication *Tip*

Using inverse operations is the same as balancing.

? How much paper did Michelle deliver?

EXAMPLE 1 Using inverse operations as a strategy to solve an equation

Determine the amount of paper waste Michelle delivered on her first trip to the recycling centre.

Michelle's Solution

$\$72.50 \times (\#\text{ of tonnes}) - \$8.00 = \$23.90$ ← I created a word equation to represent the situation.

Let p represent the number of tonnes of paper I delivered.

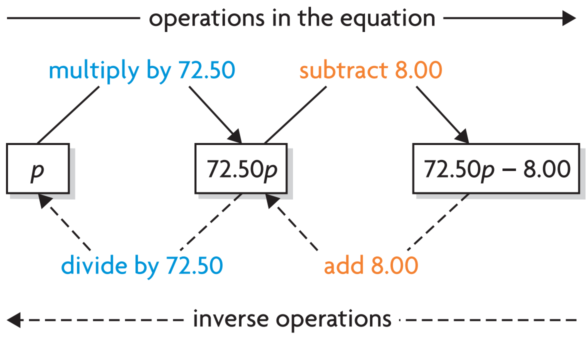
$72.50p - 8.00 = 23.90$ ← I wrote a linear equation that used a variable instead of words to represent the situation.

Try $p = 1$. ← I used guess-and-check to get an idea of the amount of paper.

$72.50(1) - 8.00$

$= 72.50 - 8.00$

$= 64.50$ ← 64.50 is much greater than 23.90, so my guess was too high. There must have been less than 1 tonne of paper.



I used a diagram to show the operations in the equation and the **inverse operations** I needed to undo them.

inverse operations
operations that undo, or reverse, each other, for example: addition is the inverse of subtraction; multiplication is the inverse of division

$$72.50p - 8.00 + 8.00 = 23.90 + 8.00$$
$$72.50p = 31.90$$

I undid the last step by adding 8.00. This **isolated the term** 72.50*p*. Since I got the new equation by doing the same operation to both sides of the original, I knew they were **equivalent equations**.

isolating a term or a variable
performing math operations (e.g., addition, subtraction, multiplication, division) to get a term or a variable by itself on one side of an equation

equivalent equations
equations that have the same solution

$$72.50p \div 72.50 = 31.90 \div 72.50$$
$$p = 0.44$$

I isolated *p* by dividing both sides of the equation by 72.50 and solved the equation.

Check:

Left Side	Right Side
$72.50(0.44) - 8.00$	23.90
$= 23.90$	

When I checked my answer, it gave me the correct amount.

I delivered 0.44 tonnes of paper.

Reflecting

A. How did Michelle's estimate of 1 tonne help her determine the inverse operations needed to solve the equation?

B. Why is "isolating the term or variable" a good name for the process used to solve the equation?

C. How would doing the inverse operations in a different order affect Michelle's solution?

APPLY the Math

EXAMPLE 2 Using an inverse operation strategy to solve an equation

Solve $-2x + 4 = 14$.

Chelsea's Solution

The calculation steps are
- multiply by -2
- add 4

I listed the operations on x in the equation following order of operations.

The reverse steps to solve are
- subtract 4
- divide by -2

Then, I listed the inverse operations that would undo each operation in the equation.

$$-2x + 4 - 4 = 14 - 4$$
$$-2x = 10$$

I isolated the term $-2x$ by subtracting 4 from each side to undo $+ 4$.

$$-2x \div (-2) = 10 \div (-2)$$
$$x = -5$$

Then, I isolated the variable x by undoing multiplication by -2.

Check:

Left Side	Right Side
$-2(-5) + 4$	14
$= 10 + 4$	
$= 14$	

I substituted my solution into the equation to make sure that it worked.

$x = -5$ is the correct solution.

EXAMPLE 3 Using an inverse operations strategy to solve an equation with fractional coefficients

Solve $\frac{w}{3} - 13 = 7$.

Drake's Solution

$$\frac{w}{3} - 13 = 7$$

$$\frac{w}{3} - 13 + 13 = 7 + 13$$

I performed the inverse operations. The inverse operation of subtracting 13 is adding 13.

$$\frac{w}{3} = 20$$

$$3 \times \frac{w}{3} = 3 \times 20$$

$\frac{w}{3} = w \div 3$

The inverse operation of dividing by 3 is multiplying by 3.

$$w = 60$$

$w = 60$ is the correct solution.

I used mental math to check that $\frac{60}{3} - 13$, or $20 - 3$, equals 7.

EXAMPLE 4 Solving a problem represented by linear equation

A photographer charges a sitting fee of \$100. The first four prints are free. Each additional print costs \$5.25. How many prints can you buy with \$257.50?

Asad's Solution

$$\$100 + \$5.25 \times (\text{\# of prints} - 4 \text{ free prints}) = \$257.50$$

$$100 + 5.25(P - 4) = 257.50$$

I wrote a word equation, and then, an algebraic equation to describe the situation. I used *P* to represent the number of prints ordered.

$$100 - 100 + 5.25(P - 4) = 257.50 - 100$$

$$5.25(P - 4) = 157.50$$

I used inverse operations to isolate the term with the variable in it.

$$5.25(P - 4) \div 5.25 = 157.50 \div 5.25$$

$$1(P - 4) = 30$$

Then, I used other inverse operations to isolate the term in brackets.

$$P - 4 + 4 = 30 + 4$$

$$P = 34$$

I used inverse operations and added 4.

Check:

$100 + 5.25(34 - 4)$

$= 100 + 157.50$

$= 257.50$

I checked to see if 34 prints would actually cost \$257.50.

You can order 34 prints.

In Summary

Key Ideas

- You can write an equation that is equivalent to a given equation by applying the same operation to both sides.
- You can use inverse operations to isolate individual terms or variables.

Need To Know

- You can substitute a value for the variable to get a sense of the inverse operations to use to solve an equation.
  For example, for the equation $3x - 17 = 34$, you might try $x = 10$. Since $3 \times 10 - 17 = 13$, is too low, you know the solution is greater than 10. You also know that the operations you have to undo are subtracting 17 and multiplying by 3.
- An equation in which the variable appears on only one side can be solved by
  - listing the operations that can be used to evaluate the expression in the order in which they would be used
  - performing the inverses of the operations one at a time, in their opposite order, until the variable is isolated

  For example:

$$\begin{aligned} 3x - 4 &= 2 \\ 3x - 4 + 4 &= 2 + 4 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

- You can check your solution by substituting it into the original equation. The solution is correct if both sides evaluate to the same number.
- There is sometimes more than one way to solve an equation using inverse operations. As long as you perform the same operations on both sides of the equation, the solution will be correct.

Communication *Tip*

It's common practice to line up equal signs when solving equations. This makes it easier to check the results of each step. Only one equal sign appears in each line of a solution and it separates the left side from the right side.

CHECK Your Understanding

1. List the inverse operations and the order in which you would apply them to isolate the variable in each equation.

a) $-3x + 2 = 15$

b) $12.4x - 3.2 = 21.5$

c) $\frac{x}{2} + 5 = 11$

2. Solve each equation in question 1. Show all steps.

3. An author is paid \$5000. In addition, he receives a royalty of \$1.25 for every book sold.

a) Write an equation to represent the number of books that have to be sold for the author to earn \$10 000.

b) Solve the equation using inverse operations. Show all steps.

c) Verify your solution.

PRACTISING

4. List the operations you would use to isolate the variable in each equation.

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a) $6b - 10 = -2$	**d)** $6 - 2d = 4$
b) $2.5c + 1.0 = 1.5$	**e)** $6 - 2e = 6$
c) $3f - 4 = 10$	**f)** $-3 - h = -2$

5. Solve each equation in question 4. Show all steps and verify each solution.

6. The relation $C = 8.00 + 0.50T$ represents the cost of a pizza in dollars. T represents the number of toppings ordered.

a) Write an equation that represents a \$10 order.

b) Solve the equation in a) to determine the number of toppings. Show all steps.

7. A submarine is currently submerged at a depth of 600 m. It rises at a rate of 4 m/s.

a) Write a linear relation that shows the relationship between the depth of the submarine and the number of seconds it has been rising.

b) Write the equation you must solve to determine when the submarine will reach a depth of 486 m.

c) List the inverse operations you need to use to isolate the variable and solve the equation.

d) Solve the equation. Show all the steps.

e) Verify your solution.

8. Caroline told Marc that using balancing and solving linear equations was different from using inverse operations to isolate the variable. Was Caroline right? Explain.

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9. A hot-air balloon is at a height of 500 m. It develops a steady leak and begins to descend at a rate of 60 m/min. Write and solve an equation to determine how long it takes for the balloon to reach a height of 20 m.

10. Solve each equation.

a) $\frac{x}{4} + 1 = 3$

b) $\frac{x}{2} - 10 = 3$

c) $5 - \frac{y}{3} = 3$

d) $10 + \frac{b}{5} = -1$

e) $\frac{w}{3} + 5 = 1$

f) $3 - \frac{d}{6} = -1$

11. Liz was testing Jane on solving with equations. She gave Jane the following problem:

"I am a number such that when you divide me by 7, and then, add 13 you get 32. What number am I?"

Write and solve an equation to determine Liz's number.

12. Solve each equation.

a) $3(x + 1) = 12$

b) $2(x - 4) = 4$

c) $\frac{(w + 3)}{4} = 2$

d) $\frac{(y - 5)}{3} = 6$

e) $\frac{(2a + 3)}{3} = 5$

f) $-2 = \frac{-2c}{5} + 1$

13. T Jack's Restaurant charges $22.95 for brunch but allows one person per table to eat free. To figure out how many people attended the Sunday brunch, Jack collected the information in this table.

Table Number	Bill Total ($)
1	137.70
2	68.85
3	160.65
4	91.80
5	91.80

a) Why is it reasonable that Jack used the equation $22.95(x - 1) = T$ to determine the number of people at each table? What do the variables x and T represent?

b) Create and solve the equation for each table number.

c) How many people in total sat at the five tables?

14. The relationship between Celsius and Fahrenheit is represented by

A $C = \frac{5}{9}(F - 32)$.

a) Determine the Celsius temperature that is equivalent to 58 °F.
b) List the operations you used to calculate the Celsius temperature.
c) List the inverse operations you would use to isolate F.
d) Determine the Fahrenheit temperature that is equivalent to 25 °C.

15. When you use an inverse operation to isolate a variable, why can you say that the equation you get at each step is equivalent to the original one?

Extending

16. Solve each equation.

a) $2x + 3 = 4(1 - x) + 5$ **b)** $\frac{1}{2}x - \frac{1}{4} = \frac{x}{3} + \frac{1}{6}$

17. The intercepts of a graph are the points at which the line crosses the x-axis and the y-axis. Consider the relation $3x + 5y = 15$.
a) List the inverse operations you would use to express the relation in the form $y = mx + b$.
b) Use your answer from part a) to solve the relation for y.
c) List the inverse operations you would use to express and isolate the x-variable.
d) Use your answer from part c) to solve the relation for x.
e) Explain how you can use your answers to parts b) and d) to quickly graph this relation.

18. The members of a scout troop held a car wash for charity. They washed 49 vehicles. They charged $4 per car and $6 per truck and earned a total of $230. How many of each type of vehicle did they wash?
a) Write an algebraic relation that models the number of vehicles washed.
b) Write an algebraic relation that models the total money earned.
c) Graph the relations and use the graphs to solve the problem.