3.5 Linear and Nonlinear Relations

GOAL

Recognize whether a relation is linear or nonlinear.

YOU WILL NEED

- grid paper

LEARN ABOUT the Math

Mario is playing a video game in which you gain extra lives by capturing pots of gold. Mario can choose one of two options. He thinks he can capture at least six pots of gold.

Option 1	
Pots of Gold	**Lives Gained**
1	5
2	10
3	15
4	20

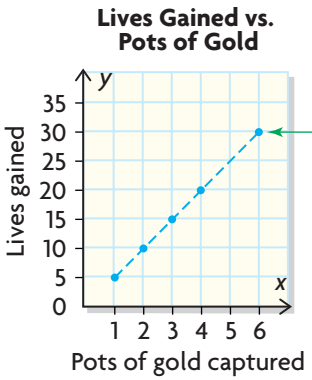
Option 2	
Pots of Gold	**Lives Gained**
1	1
2	4
3	9
4	16

? Which option should Mario choose?

EXAMPLE 1 Applying properties of linear and nonlinear relations

Determine which option Mario should choose to gain the most extra lives.

Mario's Solution: Thinking about graphs



I graphed Option 1. The points lay on a straight line, so it is a linear relation.

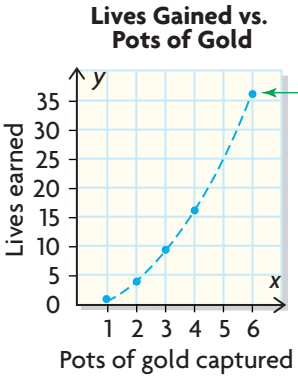
I extrapolated. Option 1 gives me 30 lives for six pots.

extrapolate

to predict a value by following a pattern beyond known values

nonlinear relation

a relation whose graph is not a straight line



I graphed Option 2. The points did not lie on a straight line, so it is a **nonlinear relation**. I extrapolated. Option 2 gives about 35 or 36 lives for six pots.

I'll choose Option 2 because it gives me more lives.

You can also determine whether a relation is linear or nonlinear from a table of values.

Mika's Solution: Thinking about tables of values

Option 1		
Pots of Gold	**Lives Gained**	**Δy**
1	5	
		5
2	10	
		5
3	15	
		5
4	20	

The first differences in Option 1 are constant, so this relation is linear.

Option 2		
Pots of Gold	**Lives Gained**	**Δy**
1	1	
		3
2	4	
		5
3	9	
		7
4	16	
		9
5	25	
		11
6	36	

The first differences in Option 2 are not constant, so this relation is nonlinear.

Mario should choose Option 2.

Mario gets 30 lives with Option 1 and 36 lives with Option 2.

You can also decide if a relation is linear or not from the degree of its equation.

Louisa's Solution: Thinking about the equations

Let p represent the number of pots of gold.
Let L represent the number of lives gained.

I chose pots of gold as the independent variable and lives gained as the dependent variable.

Option 1: lives $=$ $5 \times$ pots of gold
$L = 5p$
$L = 5(6)$
$= 30$

Option 1 gives five lives for each pot.

I created an equation for Option 1. It is of degree 1, so the relation is linear.

I solved for L when $p = 6$. With Option 1, Mario gets 30 lives.

Option 2: lives $=$ (pots of gold)^2
$L = p^2$
$L = 6^2$
$= 36$

I created an equation for Option 2. It is of degree 2, so the relation is nonlinear.

I solved for $p = 6$. With Option 2, Mario gets 36 lives.

Mario should choose Option 2, because it gives more lives.

Reflecting

A. Why would you expect the first differences to be constant for a linear relation but not constant for a nonlinear relation?

B. How can you tell from a table, a graph, and an equation if a relation is linear or nonlinear?

APPLY the Math

EXAMPLE 2 Using an algebraic strategy to identify a linear relation

The circumference of a circle is the diameter multiplied by π. Identify the relation between circumference and diameter as linear or nonlinear.

Kee's Solution

Let d represent the diameter and C represent the circumference.
Then, $C = \pi d$. ← I created an equation for the relation.
The equation is of degree 1, so the relation is linear.

EXAMPLE 3 Using an algebraic strategy to identify a nonlinear relation

The volume of a cube is the length of one side cubed. Identify the relation between volume and side length as linear or nonlinear.

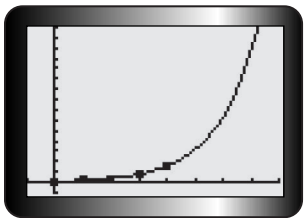
Joe's Solution

Let s represent the side length of a cube and V represent the volume.
Then, $V = s^3$. ← I created an equation for the relation.
The equation is of degree 3, so the relation is nonlinear.

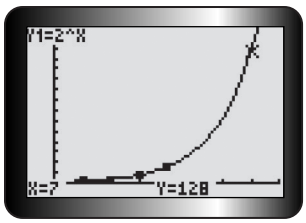
EXAMPLE 4 Evaluating a relation

The number of bacteria, y, in a dish double every hour. An equation for this is $y = 2^x$, where x is time in hours. There is one bacterium at time 0. Predict the number of bacteria at 7 h.

Eva's Solution: Using a strategy involving graphing technology



I graphed the relation. The calculator drew a curve, but really the points shouldn't be connected because the set of data is discrete.



I determined the point at which $x = 7$.

There should be 128 bacteria at 7 h.

Wes's Solution: Using a substitution strategy

$y = 2^x$
$y = 2^7$
$y = 128$

I substituted 7 for x in the equation, since this was the number of hours that had gone by.

Time (h)	Number of Bacteria
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

I used a table of values to see if my answer was reasonable.

I doubled the number of bacteria each hour.

I extended the table to 7 h.

At 7 h, there will be 128 bacteria.

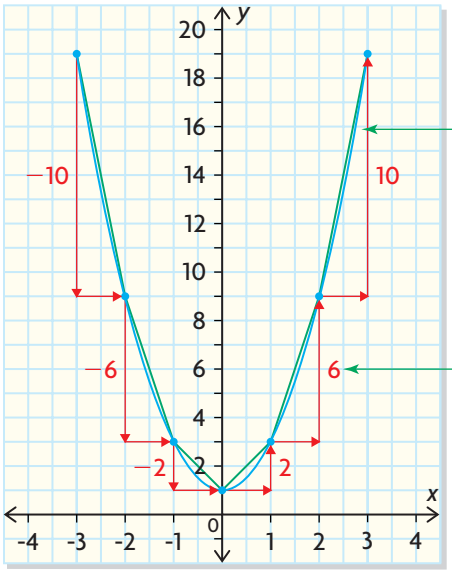
Tech *Support*

For help determining the value of a relation on a graphing calculator, see Appendix B-4.

EXAMPLE 5 Thinking about slopes of nonlinear relations

Calculate the slopes of some line segments joining points on the graph of $y = 2x^2 + 1$. How do these compare to the slopes of segments joining points on the graph of a line?

David's Solution



I drew line segments joining points on the graph that had *x*-coordinates 1-unit apart. That meant that for each segment, the run was 1.

Then I calculated the rise for each line segment. Since the run is 1 in each case, the rise is equal to the slope of each segment.

The slopes are not constant and that is different from the slopes of segments joining points on a linear relation.

I knew that the slopes of any line segment on a linear graph would be constant.

For this relation, the farther pairs of points are from 0, the steeper the slope becomes.

In Summary

Key Ideas

- Some relations are nonlinear.
- If a relation is nonlinear, then the following are true:
  - The graph is not a straight line.
  - The first differences are not constant.
  - The degree of its equation is not 1.

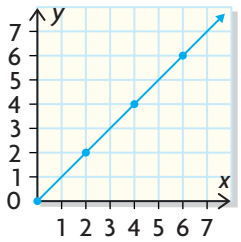
Need to Know

- In a nonlinear relation, the slope between pairs of points is not constant.

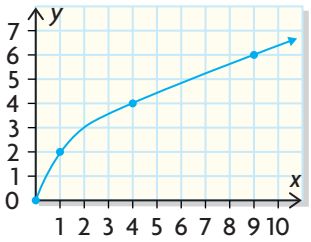
CHECK Your Understanding

1. Identify each relation as linear or nonlinear. Explain how you know.

a)



b)



2. The area of a circle of radius r is $A = \pi r^2$. Identify this relation as linear or nonlinear. Explain.

PRACTISING

3. Identify each relation as linear or nonlinear.

a)

x	y
−3	9
−2	4
−1	1
0	0

b)

x	y
5	1
6	2
7	3
8	4

c)

x	y
1	0.25
2	0.50
3	0.75
4	1.00

4. Josie ran these distances while training for a marathon.

K

Time (min)	Distance Run (km)
0	0
10	2
20	4
30	6

a) Would you choose time or distance as the independent variable? Explain.

b) Identify the relation between distance and time as linear or nonlinear. Explain how you know.

5. This pattern is made from squares with sides of 1 cm.

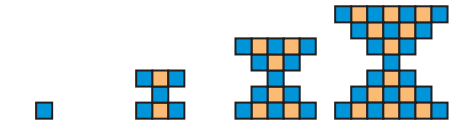


figure 1 figure 2 figure 3 figure 4

a) Use a table to show the perimeter of each figure in terms of its figure number.
b) Determine the perimeter of figure 12. Explain your reasoning.
c) Identify the relation in part a) as linear or nonlinear. Explain how you know.
d) Use a table of values to show the number of blue squares in terms of the figure number.
e) Determine the number of blue squares in figure 12. Explain your reasoning.
f) Identify the relation in part d) as linear or nonlinear. Explain how you know.

6. Identify each relation as linear or nonlinear. Explain your reasoning.

a) the relation between the number of circles in each figure and the figure number

figure 1 figure 2 figure 3

b) the relation between the number of stars in each figure and the figure number

figure 1 figure 2 figure 3 figure 4

7. The relation between kilometres driven, k, and the amount of gasoline, G, (in litres) in the tank of a hybrid car is $G = 80 - 0.2k$.
a) Identify this relation as linear or nonlinear. Explain how you know.
b) Use either a graph or a table to confirm your answer in part a).

8. The amount of cucumbers you can grow in a season depends on the amount of rainfall you get. This relation is represented by the equation $C = 0.006(R + 20)$, where R is the rainfall in millimetres and C is the cucumber yield in kilograms per square metre.
a) Identify this relation as linear or nonlinear. Explain how you know.
b) Use either a graph or a table of values to confirm your answer in part a).

9. When a piece of paper is folded in half, one crease line and two sections of paper are created. The paper is then folded in half again and again, each time increasing the number of crease lines by 1. Identify the relation between the number of creases and the number of sections of paper as linear or nonlinear. Justify your answer.

10. A large hailstone falls from a cloud 5000 m above the ground. This table shows its altitude at different times. About how many seconds will it take for the hailstone to hit the ground? How do you know?

Time (s)	0	5	10	15	20
Altitude (m)	5000	4875	4500	3875	3000

11. Each pattern represents a relation between the figure number and the number of red triangles needed to make it.

a) Which of these patterns are linear and which are nonlinear relations?

b) Explain how you know.

	Pattern 1	Pattern 2	Pattern 3
Figure 1	♦	♦	♦
Figure 2	♦ ♦	♦ ♦ ♦ ♦	♦ ♦
Figure 3	♦ ♦ ♦	♦ ♦ ♦ ♦ ♦ ♦ ♦ ♦ ♦	♦ ♦ ♦ ♦

Extending

12. Calculate the first, second, and third differences for each relation. What is the connection between the degree of the equation and the differences?

a) $y = x^2$

b) $y = x^3$

c) $y = -2x^2$

d) $y = 4x^3$

e) $y = \frac{1}{3}x^2 + 2x - 1$

f) $y = -2x^3 + x$

13. Identify each relation as linear or nonlinear. Use a graph or a table to justify your answer.

a) $y = (x + 1)(x - 2)$

b) $y = 2(x - 1)(x + 3)$

c) $y = x^{\frac{1}{2}}$

d) $y = \sqrt{x}$