3.3 Investigating Properties of Linear Relations

YOU WILL NEED

- grid paper

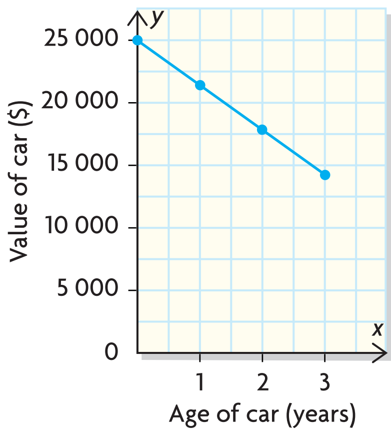
GOAL

Identify properties of linear relations.

INVESTIGATE the Math

Cole bought a new car for \$25 000. This graph shows its value over the first three years.

Value of Car by Age



? When will Cole's car be worth \$0?

A. Calculate the amount by which Cole's car decreased in value between years 1 and 2.

B. Calculate the **rate of change** in the car's value between years 1 and 3.

C. Calculate the **slope** of the graph between years 1 and 3.

How does the slope compare to your answer in part A?

D. Copy the following table. Complete the **first difference** column. How do the first differences compare to the slope in part C?

Age of Car, x	Value of Car, y	First Difference, Δy
0	25 000	
		21 425 − 25 000 = ■
1	21 425	
		17 850 − 21 425 = ■
2	17 850	
3	14 275	

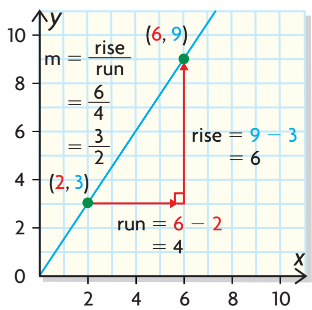
rate of change

the change in one variable relative to the change in another

slope

a measure, often represented by m, of the steepness of a line; the ratio comparing the vertical and horizontal distances (called the rise and run) between two points;

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$



first difference

the difference between two consecutive y-values in a table in which the difference between the x-values is constant

E. Copy and complete the following table.
Why are the first differences different than in part D?

Age of Car, x	Value of Car, y	First Difference, Δy
0	25 000	
		17 850 − 25 500 = 🟩
2	17 850	
		🟥 − 17 850 = 🟩
4	🟥	
		🟦 − 🟥 = 🟩
6	🟦	

Communication *Tip*

- The Greek letter "Δ" (delta) represents change, so Δy represent the difference between two y-values.

F. Write an equation for the relation between the car's value and its age. Which parts represent the first differences, the slope, and the y-intercept?

G. Determine the ***x*-intercept** of the graph. Use it to tell when Cole's car will be worth \$0. How do you know?

***x*-intercept**
the value at which a graph meets the x-axis; the value of y is 0 for all x-intercepts

Reflecting

H. What is the connection between the first differences and the slope?

I. When you calculated the slope, did it matter which points you chose? Explain.

J. Use the graph to explain why the first differences were constant.

APPLY the Math

EXAMPLE 1 Applying the connection between slope and rate of change

Andrea and Dana had cycled 30 km after two hours and 60 km after four hours. At what rate were they cycling?

Dana's Solution: Using a table of values

Time in Hours, x	Distance in Kilometres, y	First Difference, Δy
0	0	
		30 − 0 = 30
2	30	
		60 − 30 = 30
4	60	

I made a table of values and calculated the first differences.

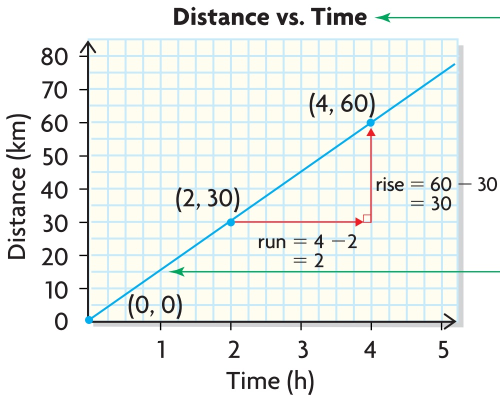
The x-values increased by 2, so $\Delta x = 2$.

$$\text{rate of change} = \frac{\Delta y}{\Delta x} = \frac{30 \text{ km}}{2 \text{ h}} = 15 \text{ km/h}$$

I calculated the rate of change.

We were cycling at a rate of 15 km/h.

Andrea's Solution: Using a graph



The distance depended on time, so I chose time as the independent variable.

I plotted the points and drew a line through them. I assumed they were cycling at a constant rate. I used a solid line because the data are continuous.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{30 \text{ km}}{2 \text{ h}} = 15 \text{ km/h}$$

I calculated the slope of the line.

We were cycling at a rate of 15 km/h.

The slope gives the speed or rate they were cycling.

Communication *Tip*

- We call the horizontal distance between two points the run. The run is positive when it goes to the right and negative when it goes to the left. We call the vertical distance between two points the rise, even if the line goes downward. The rise is positive when it goes upward and negative when it goes downward.

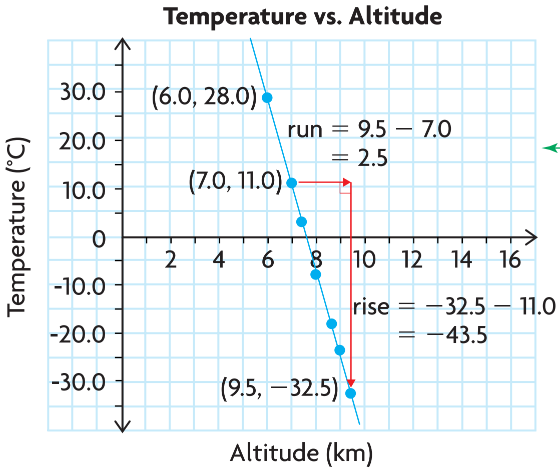
When the data for the independent variable in a table of values do not increase by an equal amount, graphing the data can help you determine if the relationship is linear. If it is, calculating the slope of the line will give you the rate of change of the dependent variable.

EXAMPLE 2 Using a graphing strategy to estimate rate of change

A weather balloon recorded the temperature at these altitudes. Estimate the rate of change of the temperature.

Altitude (km)	6.0	7.0	7.6	8.1	8.7	9.0	9.5
Temperature (°C)	28.0	11.0	2.5	−7.7	−17.9	−23.0	−32.5

Martha's Solution



I didn't use first differences because the altitude was recorded at irregular times.

I graphed the points and connected them with a solid line.

The graph is straight, so this relation is linear.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-43.5\ \text{°C}}{2.5\ \text{km}}$$
$$= -17.4\ \text{°C/km}$$

I calculated the slope, because it has the same value as the rate of change.

The rate of change is -17.4 °C/km.

When the altitude increases by 1 km, the temperature decreases by 17.4 °C.

In Summary

Key Idea

- To determine the rate of change of a linear relation, you can do the following:
  - Calculate the first differences in a table in which the *x*-values increase or decrease by 1.
  - Calculate the slope, $\frac{\text{rise}}{\text{run}}$, using any two points on a graph of the relation. The rate of change has the same value as the slope.

Need to Know

- If the independent values in a table change by a constant amount other than 1, the ratio of the first differences to the change in x, $\frac{\Delta y}{\Delta x}$, is the slope, or the rate of change.
- Δy is the change in y and is equivalent to the rise.
  Δx is the change in x and is equivalent to the run.

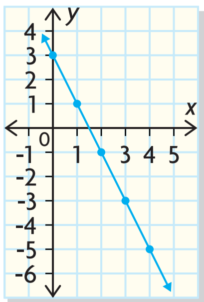
CHECK Your Understanding

1. Which of these relations are linear? How do you know?

a)

x	*y*
1	3
2	6
3	9
4	12

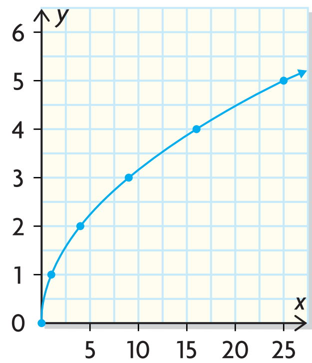
b)



c)

x	*y*
1	1
2	4
3	9
4	16

d)



2. Determine the rate of change for each linear relation in question 1.

PRACTISING

3. Determine the rate of change in each linear relation.

a)

x	*y*
2	11
4	17
6	23
8	29
10	35

b)

x	*y*
5	0
4	2
3	4
2	6
1	8

c)

x	*y*
0	0
0.25	2
0.5	4
0.75	6
1	8

d)

x	*y*
1	−2
4	−8
5	−10
3	−6
2	−4

4. a) Why might this table mislead you about whether the relation is linear?

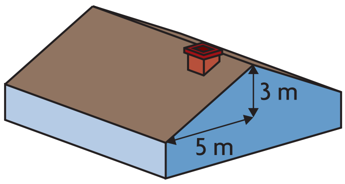
Age of Car in Years, *x*	0	3	1	2
Value of Car in Dollars, *y*	15 000	6 000	12 000	9 000

b) Graph the data.

c) What is the slope and what does it mean?

d) What is the *x*-intercept? Does this seem realistic? Explain.

5. What is the slope of this roof?

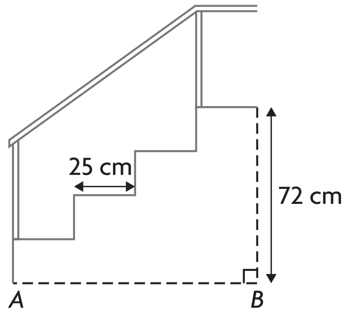


6. There are three steps from the ground to a front porch 72 cm above the ground, as shown.

a) What is the rise of each step?

b) The horizontal distance across each step is 25 cm. Determine the length of *AB*.

c) Determine the slope of the handrail.



7. Determine the slope of the line that passes through each pair of points.

K **a)** (3, 5) and (0, 2)

b) (3, 3) and (−2, 2)

c) (21, −10) and (20, 24)

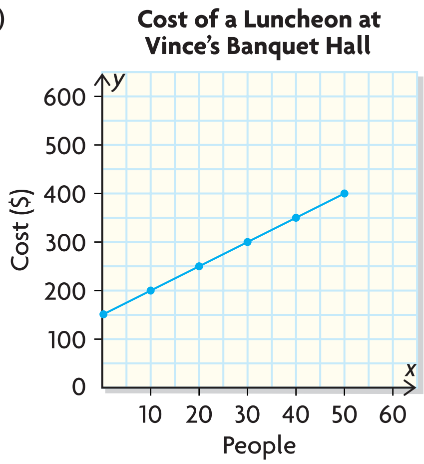
d) (4, 0) and (6, 18)

e) (1, −1) and (2, 2)

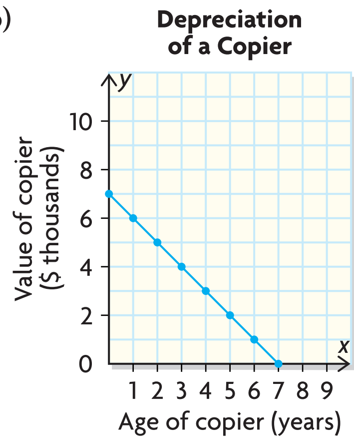
f) (−3, −8) and (−5, −6)

8. Use the title and axis labels of each graph to tell what the *y*-intercept and slope mean in each case.

a)



b)



9. Determine two more ordered pairs for each relation. Explain your reasoning.

a) rise is 2, run is 3; (2, 5) lies on the line

b) rise is −3, run is 4; (0, −2) lies on the line

c) rise is 5, run is 1; (1, −6) lies on the line

d) rise is −2, run is 1; (−2, −3) lies on the line

Altitude (km)	Air Pressure (Pa)
1	80 000
3	60 000
6	40 000
16	20 000
22	10 000
30	5 000

10. a) Graph the data in the table to the left.

A **b)** How does the graph show the rate of change?

c) Estimate the air pressure at an altitude of 20 km.

11. Graph each relation and state the slope.

a) $y = 3x$ **c)** $y = \frac{3}{4}x + 1$ **e)** $y = -x$

b) $y = -2x$ **d)** $y = -\frac{1}{5}x + 1$ **f)** $y = \frac{2}{3}x - 4$

12. An equation for a house's value is $y = 7500x + 125\ 000$, where y is the value in dollars and x is the time in years, starting now.

a) What is the current value of the house?

b) What is the value of the house 2 years from now?

c) Determine the value of the house in 7 years.

d) At what rate is the house value changing from year to year?

13. The amount of money in Alexander's account is $y = 4000 - 70x$, where y is the amount in dollars and x is the time in weeks.

a) Which variable is independent and which is dependent?

b) How do you know the relation is linear?

c) Determine the rate of change of the money in Alexander's account.

d) What does the rate of change mean?

e) How does the rate of change relate to the equation?

f) When will Alexander's account be empty?

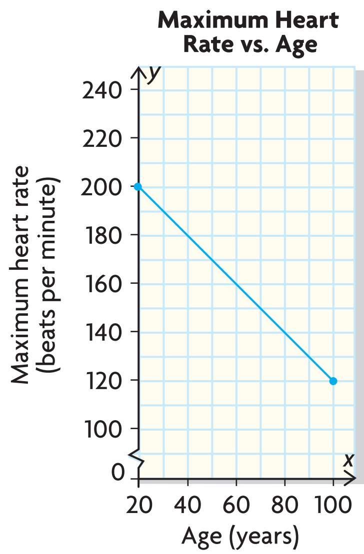
14. This graph shows the maximum heart rate a person should try to achieve while exercising.

a) What does the y-intercept mean?

b) What does the slope represent?

c) Write an equation for the line.

d) Estimate the maximum heart rate for a 58-year-old.



15. Jae-Ho works at a clothing store. He earns a weekly salary of $300 and
T 5% commission on his total weekly sales. He thinks that if his commission doubles, so will his earnings. Is he right? Justify your answer.

16. Marie earns $1 for every 4 papers she delivers.
C

a) Show that the relation between papers delivered and money earned is linear, using a graph and a table of values.
b) What do the first differences mean?
c) What is the rate of change of Marie's earnings?
d) Predict Marie's earnings for delivering 275 papers using an equation.

Extending

17. Write a linear equation for each line.

a) a line with a y-intercept of 2 and a slope of $\frac{3}{5}$
b) a line with a y-intercept of 0 and a slope of -4
c) a line that passes through (0, 5) and (3, 6)
d) a line with a slope of 2 that passes through (4, 1)
e) a line with a slope of $-\frac{1}{2}$ that passes through (3, 0)
f) a line with a slope of $\frac{4}{7}$ that passes through (7, 2)

18. **a)** Make a table of values and a graph for each relation.
C **b)** Copy and complete the table using your results from part a).
c) What connections do you see between each equation and the values in the table?

Equation	Slope	y-intercept	x-intercept
$2x + 5y = 10$			
$4x - 2y = 7$			
$x + y = -2$			