GOAL

Represent a relation using a table of values, a graph, or an equation.

LEARN ABOUT the Math

Chris runs a window-washing service. She charges a flat rate of $5, plus $3 per window.

? How can Chris's customers calculate the cost to wash their windows?

EXAMPLE 1	Representing a relation in different ways

Represent the **relation** between the number of windows washed and the cost to wash them.

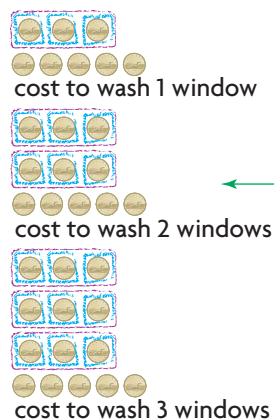### Geri's Solution: Representing the relation with a table of values

Number of Windows	Cost ($)
0	5
1	$3 \times 1 + 5 = 8$
2	$3 \times 2 + 5 = 11$
3	$3 \times 3 + 5 = 14$

I created a table of values.

The customer chooses the number of windows to wash, so this is the **independent variable**.

The cost depends on how many windows are washed, so cost is the **dependent variable**.

Brian's Solution: Representing the relation with a picture

cost to wash 1 window
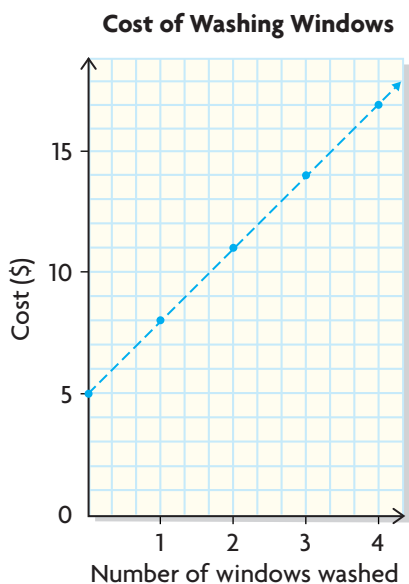
cost to wash 2 windows

cost to wash 3 windows

I created a series of pictures.

$5 is constant, so it is the same in each picture.

I circled the part that increases with each additional window washed.

The picture represents the relation between cost and windows washed.

relation

a description of how two variables are connected

independent variable

in a relation, the variable whose values you choose; usually placed in the left column in a table of values and on the horizontal axis in a graph

dependent variable

in a relation, the variable whose values you calculate; usually placed in the right column in a table of values and on the vertical axis in a graph

Marlene's Solution: Representing the relation with a graph

Cost of Washing Windows

I used Geri's table to graph the relation.

The number of windows washed is the independent variable, so I plotted that on the horizontal axis.

Cost is the dependent variable, so I plotted it on the vertical axis.

I connected the points with a dotted line because this set of data is **discrete**. Washing part of a window does not make sense.

discrete
a set of data that cannot be broken into smaller parts

Theo's Solution: Representing the relation with an algebraic expression

Let *W* represent the number of windows washed.
Let *C* represent the cost in dollars.

I represented the variables.

Chris charges a flat rate of $5, plus $3 per window.

I wrote the relation in words.

cost = (3 × number of windows washed) + 5

Constants go at the end of an equation.

$C = 3W + 5$

I replaced the words with variables to create an equation.

Reflecting

A. How do the students' representations all describe the same relation?

B. Which representation would you use? Why?

APPLY *the Math*

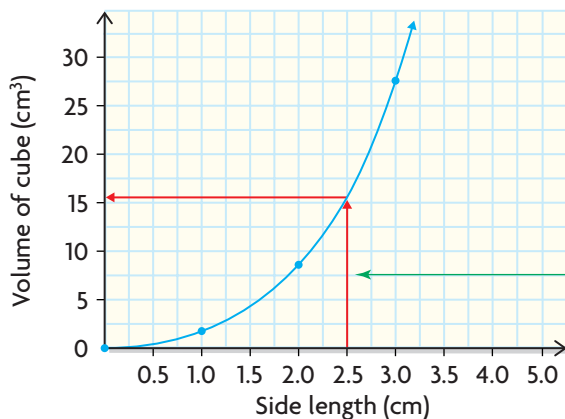EXAMPLE **2** | Solving a problem involving a relation

Determine the volume of a cube with a side length of 2.5 cm.

Andrea's Solution: Using a graph to estimate a value

Side Length of Cube (cm)	Volume of Cube (cm³)
1.0	$1 \times 1 \times 1 = 1^3 = 1$
2.0	$2 \times 2 \times 2 = 2^3 = 8$
3.0	$3 \times 3 \times 3 = 3^3 = 27$

I made a table of side lengths and volumes. I used the side length to calculate volume.

continuous

a set of data that can be broken down into smaller and smaller parts and still have meaning

interpolate

to estimate a value between two known values

Volume of Cube vs. Side Length

I graphed the relation. This set of data is continuous, so I connected the points with a solid line.

I interpolated. I drew a line from 2.5 cm on the horizontal axis to the graph.

I drew a line from that point to the vertical axis.

The volume is about 15 cm³.

My estimate seems reasonable since multiplying $2.5 \times 2.5 \times 2.5$ gives me an exact volume of 15.625 cm³.

Andrea was able to estimate the volume from the graph. Pilar wanted a more exact answer, so she represented the relation using an equation.

Pilar's Solution: Using an equation to determine an exact value

Let x represent the side length of the cube.
Let y represent the volume of the cube.

> The volume depends on the side length.

$y = x^3$

> I described the relation using an equation and calculated the volume for $x = 2.5$.

$y = 2.5^3$
$y \doteq 15.6$

The volume is 15.6 cm^3.

> I answered to the nearest tenth, because that is how the side length is given.

You can use a graphing calculator to graph a relation, if you know its equation. You can use this graph to make accurate estimates.
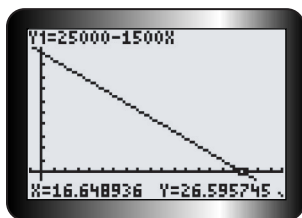
EXAMPLE 3 | Using technology to estimate a value

The equation $V = 25\ 000 - 1500\,T$ represents a car's value after T years. When will the car be worth \$0?

Otto's Solution
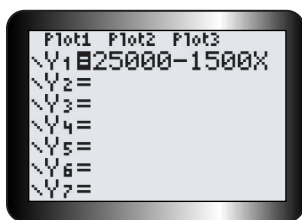
> The independent variable is the car's age. The dependent variable is the car's value.

> I graphed the relation and used the TRACE key to estimate when the car would be worth \$0.

The car will be worth \$0 after about 16.7 years.

Tech | Support

- See Appendix B-5 for information on how to set a calculator window so that a graph is visible.
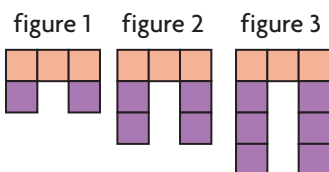- For help using the TRACE key, see Appendix B-4.

CHECK Your Understanding

figure 1   figure 2   figure 3

1. Describe a relation between the figure number and the total number of squares using a table of values, a graph, and an equation.

2. Describe each relation using two of the following: a graph, a table of values, a picture, or an equation. Justify your choice.
   a) the perimeter of an equilateral triangle in terms of its side length
   b) the amount John pays for a taxi ride, if the fare is $0.50/km plus a flat rate of $2.50

PRACTISING

3. Graph each relation.

a)

Time (min)	Distance (km)
0	15
5	18
10	21
15	24

b)

Side Length (cm)	Area (cm²)
1	1
2	4
3	9
4	16

c)

| x	y
−2	−4
−1	−2
0	0
1	2

3.1

4. Describe each relation in the previous question using an equation.

5. Elinor is training for a race. The table shows her times and distances.

Time (min)	Distance (km)
0	0
10	2
20	4
30	6

    a) Which variable is independent and which is dependent?
    b) Estimate the distance Elinor has run after 22 min.
    c) Describe the relation using a graph.
    d) Verify your estimate in part b).

6. Describe each relation with either a table of values or an equation.

    a)

    b)
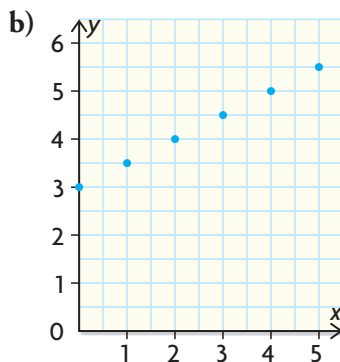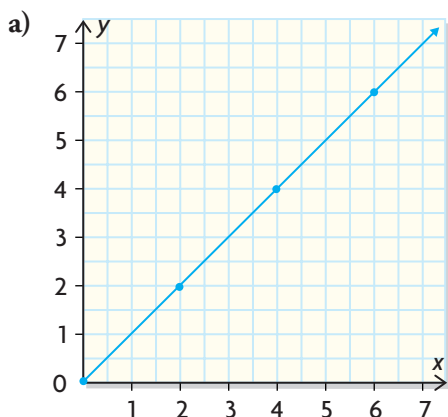
7. This pattern is made of equilateral triangles with sides of 1 cm.

    a) Graph the relation between a figure and its perimeter.
    b) Determine the perimeter of figure 10. Explain your reasoning.
    c) Graph the relation between the figure number and the number of white triangles in the figure.
    d) Determine the number of white triangles in figure 10. Explain.

figure 1   figure 2   figure 3   figure 4

8. The relation between Celsius and Fahrenheit is $C = \dfrac{5}{9}(F - 32)$.

    a) Which variable is independent in this equation? Justify your choice.
    b) Describe the relation using a table of values.
    c) Graph the relation.
    d) Are the data continuous or discrete?
    e) Estimate the Celsius temperature when $F = 100$ using your graph.
    f) Calculate the Celsius temperature when $F = 100$ using the equation.
    g) Why might you predict a value using an equation, instead of a graph or a table?

3.1

9. These ordered pairs show the relation between the amount of cell phone use in minutes and the cost, in dollars: (0, 25), (10, 26), (20, 27)
   a) Explain why cost is the dependent variable and what the ordered pair (0, 25) means.
   b) Graph the relation.
   c) Are the data continuous or discrete?
   d) Describe the relation using an equation.
   e) Would you predict the cost of 100 min using a graph, or using an equation? Explain.
   f) Predict the cost of 100 min.

10. Antwan charges $5/h, plus a flat fee of $8, in his lawn-mowing business.
   a) Describe the relation between earnings and hours using an equation.
   b) Justify your choice for independent and dependent variables.

11. A van's gas tank holds 75 L. The van uses 0.125 L/km.
A a) Describe the relation between the distance the van travels and the volume of gas in its tank.
   b) How far can the van travel on a full tank of gas?

12. a) Which of these ordered pairs are points on the graph of $y = 5x$?
     A. (0, 0)   **B.** (2, 10)   **C.** (4, 15)   **D.** $(-2, -10)$
   b) Which of these ordered pairs are points on the graph of $y = 3x - 6$?
     A. (2, 0)   **B.** (5, 9)   **C.** $(-1, -9)$ **D.** (6, 10)

13. Represent each relation using a table of values and a graph.

   a) $y = x$ c) $y = -x + 1$ e) $y = -\dfrac{1}{2}x$

   b) $y = 2x + 3$ d) $y = 0.25x - 3.5$ f) $y = -\dfrac{2}{3}x + \dfrac{1}{6}$

14. Match each table to its graph and equation. Explain your reasoning.

a)
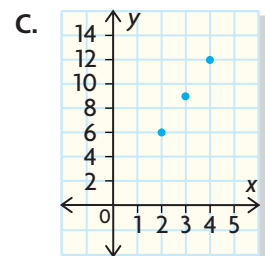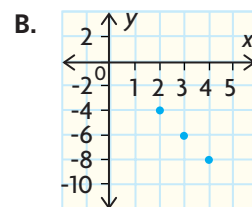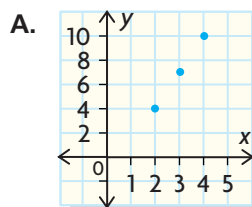| x	y
2	6
3	9
4	12

b)
x	y
2	4
3	7
4	10

c)
x	y
2	-4
3	-6
4	-8

A.

B.

C.

i) $y = -2x$ ii) $y = 3x$ iii) $y = 3x - 2$

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15. Clarise has $50 in her piggy bank. She takes $2.50 from it each week to buy a hot chocolate and a banana from the cafeteria. Create a table of values, a graph, and an equation to describe the amount of money in the piggy bank each week.

16. Jacques surveyed people about their part-time jobs.

Group 1
Abe: $7 per hour (waiting tables)
Beth: $20 per lawn (mowing lawns)
Carl: 10% commission (selling furniture)

Group 2
Anne: $7.50 per hour plus tips of $25.00
Boris: $12 per lawn plus a flat rate of $5
Carol: 7% commission, plus a flat rate of $50

Suppose Abe and Anne each work for 10 h, Beth and Boris each mow 5 lawns, and Carl and Carol each sell $1000 worth of goods. Which group earns more?

17. Describe each relation in words.

C **a)** $I = 2.54c$, where I is inches and c is centimetres

b) $F = \frac{9}{5}C + 32$, where F is degrees Fahrenheit and C is degrees Celsius

c) $k = \frac{p}{2.2}$, where p is pounds and k is kilograms

d) $K = C + 273$, where K is degrees Kelvin and C is degrees Celsius

Extending

18. **a)** Graph $y = 2x$, $y = 2x + 2$, $y = 3x$, and $y = 3x - 1$ on the same axes.
b) How do the equations tell you whether the graph will pass through the origin?

19. The table to the right shows several different heights and areas for triangles with a base of 10 cm.
a) Graph the relation between height and area.
b) Write an equation to relate the area of the triangle to its height.

20. A rocket's height in metres, h, at time t, in seconds, is given by $h = -5t^2 + 3t + 2$. Describe the relation between height and time with a table of values and a graph.

Height (cm)	Area (cm²)
2	10
5	25
10	50
20	100