2.6 Simplifying Polynomial Expressions

GOAL

Expand and simplify polynomial expressions in one variable.

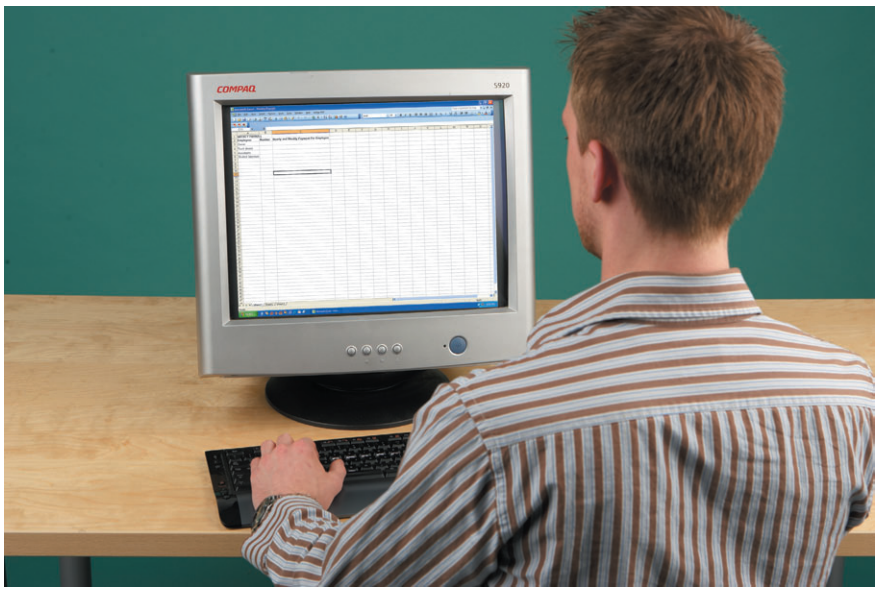
YOU WILL NEED

- algebra tiles

LEARN ABOUT the Math

Todd has a landscaping business. He employs 4 truck drivers, 3 assistants, and 15 student labourers. The pay structure for the business is shown below.

Employees	Number	Hourly and Weekly Payment per Employee
Owner	1	$800/week
Truck drivers	4	$17/h plus $150 for gas
Assistants	3	$12/h plus $50 for expenses
Student labourers	15	$10/h



Todd wants to use a spreadsheet to determine his weekly payroll. He assumes that in any one week all of the employees will work the same number of hours, but also that the number of hours they work from week to week will vary.

? What formula can Todd use to represent his total weekly payroll in terms of number of hours worked?

EXAMPLE 1 Using algebraic reasoning to simplify a polynomial expression

Todd's Solution

Let h represent the number of hours each employee works in a week.

Employees work the same number of hours each week, so I can use the same variable to represent hours worked.

Truck drivers are paid $17h + 150$.
Assistants are paid $12h + 50$.
Student labourers are paid $10h$.

I wrote an algebraic expression to represent each type of worker's weekly pay.

Pay for truck drivers: $4(17h + 150)$
Pay for assistants: $3(12h + 50)$
Pay for labourers: $15(10h)$
Pay for owner, Todd: 800/week

I multiplied the representation for each position's weekly pay by the number of people employed in that position.

Total weekly payroll

I added the weekly payrolls for all of the positions to describe the total weekly payroll.

$$P = 800 + 4(17h + 150) + 3(12h + 50) + 15(10h)$$

$$P = 800 + 4(17h + 150) + 3(12h + 50) + 15(10h)$$
$$= 800 + 68h + 600 + 36h + 150 + 150h$$
$$P = 254h + 1550$$

I simplified this using the distributive property, and then collected like terms.

I can use the formula $P = 254h + 1550$ to represent the total payroll per week.

Communication *Tip*

Computer spreadsheets use "formulas" differently than mathematics does. For example, in mathematics the formula $P = 2l + 2w$ is used to determine the perimeter of a rectangle. When a spreadsheet is used, only $= 2l + 2w$ is entered as the formula in a spreadsheet cell.

Reflecting

A. What mathematical principles did Todd use to simplify his formula?

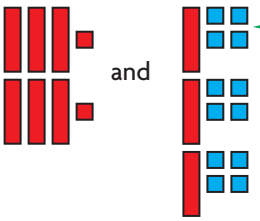
B. What did the only variable in Todd's polynomial expression represent? How did using only one variable help him to expand and simplify the expression to determine the simplified payroll formula?

APPLY the Math

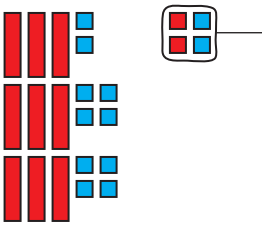
EXAMPLE 2 Simplifying a sum of products of polynomials

Simplify $2(3x + 1) + 3(x - 4)$.

Jaspal's Solution: Reasoning from an algebra tile representation



I used algebra tiles to represent each product.



I combined like tiles and used the zero principle to simplify the polynomial.

$2(3x + 1) + 3(x - 4) = 9x - 10$

I wrote a summary statement by counting the remaining tiles in my representation.

Katerina's Solution: Reasoning using the distributive property

$2(3x + 1) + 3(x - 4)$

I used the distributive property to expand each product.

$= 6x + 2 + 3x - 12$

$= 9x - 10$

I collected like terms.

$2(3x + 1) + 3(x - 4)$
$= 9x - 10$

EXAMPLE 3 Simplifying polynomial expressions

Determine the missing factor:

$(4x^2 - 3x + 2) - (x^2 - 9x - 1) = \blacksquare(x^2 + 2x + 1)$

Jordan's Solution: Reasoning using the distributive property

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$

$(4x^2 - 3x + 2) + (-1)(x^2 - 9x - 1)$

I simplified the polynomials on the left side of the equal sign. The negative sign in front of the second bracket represents (-1), so I used the distributive property to expand, and then collected like terms.

$= 4x^2 - 3x + 2 - x^2 + 9x + 1$

$= 3x^2 + 6x + 3$

$\blacksquare(x^2 + 2x + 1) = 3x^2 + 6x + 3$

Then, I looked at the original expression on the right side of the equal sign. I thought about what factor outside the brackets would give me $3x^2 + 6x + 3$ when I used the distributive property.

$\blacksquare x^2 = 3x^2$

$\blacksquare = 3$

I started with the first term, $3x^2$. To get this, I would have to multiply x^2 by 3, so I chose the number 3 to fill the box.

$3(x^2 + 2x + 1) = 3x^2 + 6x + 3$

I checked by using the distributive property, and it worked.

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$

$= 3(x^2 + 2x + 1)$

Danika's Solution: Connecting to adding opposites

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$

$= 4x^2 - 3x + 2 + (-x^2) + 9x + 1$

$= 3x^2 + 6x + 3$

I simplified the polynomials on the left side of the equal sign. The negative sign in front of the second bracket meant I had to add the opposite of each term inside the bracket. I rewrote the problem and collected like terms.

$3x^2 + 6x + 3 = \blacksquare(x^2 + 2x + 1)$

I looked at the simplified expression. I thought about what factor was common to each of the terms.

$\frac{3x^2}{3} = x^2$,

$\frac{6x}{3} = 2x$, and

$\frac{3}{3} = 1$

$\blacksquare = 3$

Each term had a factor of 3. When I divided each term by 3, the results were the terms of the trinomial on the right hand side.

$(4x^2 - 3x + 2) - (x^2 - 9x - 1)$

$= 3(x^2 + 2x + 1)$

EXAMPLE 4 Connecting exponent principles to simplifying a polynomial expression

Expand and simplify $3x^2(4x^3 - 2x^2 + 6x) - (x^5 + 5x^4 - 4x^3) + 7x^5$.

George's Solution

$3x^2(4x^3 - 2x^2 + 6x) - (x^5 + 5x^4 - 4x^3) + 7x^5$

$= 12x^5 - 6x^4 + 18x^3 - x^5 - 5x^4 + 4x^3 + 7x^5$

I used the distributive property to expand the product for each bracketed expression. To simplify the first set of brackets, I had to apply the exponent principles for multiplying powers. To simplify the second set of brackets, I multiplied each term by the factor (-1). The last term wasn't multiplied by anything.

$= 12x^5 - x^5 + 7x^5 - 6x^4 - 5x^4 + 18x^3 + 4x^3$

$= 18x^5 - 11x^4 + 22x^3$

I simplified the expression by grouping then collecting like terms.

EXAMPLE 5 Solving a problem by simplifying a polynomial expression

A sporting goods company provides skis and snowboards to instructors at ski resorts. Determine an expression that represents the combined value of the equipment after y years of use. Use this value to determine the combined value of all the equipment after 2 years of use.

	Number Provided	Original Value	Yearly Drop in Value
Skis	200	\$600	\$50
Snowboards	300	\$800	\$60

Mark's Solution

After y years, the values of the skis and snowboards will be:

- skis: $600 - 50y$
- snowboards: $800 - 60y$

I wrote algebraic expressions to represent the future value of each item.

$$V = 200(600 - 50y) + 300(800 - 60y)$$

I multiplied each expression by the number of skis or snowboards. This gave me an equation for the combined value V after y years.

I used the distributive property, and then simplified the expressions.

$$= 120\,000 - 10\,000y + 240\,000 - 18\,000y$$
$$= 360\,000 - 28\,000y$$

$$V = 360\,000 - 28\,000(2)$$
$$= 360\,000 - 56\,000$$
$$= 304\,000$$

I substituted 2 for y to determine their combined value after 2 years.

After 2 years, the combined value will be \$304 000.

In Summary

Key Idea

- You can use the distributive property and collect like terms to simplify a sum or difference of products of polynomials.

Need to Know

- Use the order of operations to determine the sequence in which operations must be performed.

CHECK Your Understanding

1. Simplify using the tool or strategy of your choice. Verify using a different tool or strategy.

a) $3(x - 1) + 2(2x + 2)$

b) $2(y^2 - 3) - 2(y^2 - 1)$

2. Simplify.

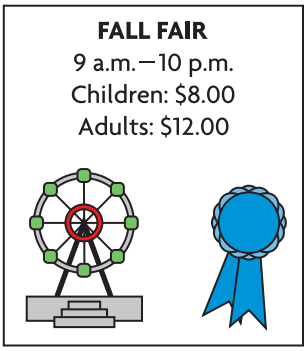
a) $3x^2(x^3 - 1) + 2x^3(2x^2 + 2)$

b) $2(y^2 - 3y^5) - 3(y^2 - y^5)$

3. On average, the following numbers of adults and children pay to enter the fall fair.

	Adults	Children
At 9 a.m.	20	25
Each Hour After 9 a.m. Until Closing	95	120

Determine an expression for the total entrance fees collected h hours after opening.

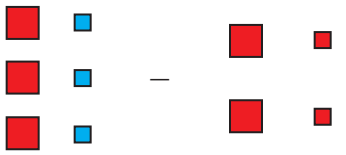


PRACTISING

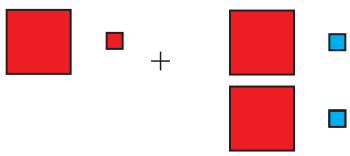
4. C Write an algebraic representation that corresponds to the algebra tile models shown. Simplify the expression using the strategy of your choice.

a)

b)



c)



5. Simplify the following expressions using the tool or strategy of your choice. Verify using a different tool or strategy. K

a) $6(8 + 3c) + 4(10 + 2c)$

b) $5(2x - 3) - 4(3x + 6)$

c) $2(x^2 - 3x + 6) - 3(2x^2 - 4x - 1)$

d) $-y(y^2 + 5y + 4) + 3y(2y^2 - y + 6)$

e) $2(3y^2 + 4y) - 3(2y^2 - y)$

f) $2(4x^3 - 3x + 6) + 3(2x^5 + x^3 - 4x)$

6. Simplify each expression, and then evaluate for $a = 3$.

a) $6(2a + 4) - 3a$

b) $15 - 2(a - 5)$

c) $-10a - 2(a^2 + 7)$

d) $-(2a - a^3) - a^2$

7. Simplify the following polynomial expressions.

a) $3(2x^2 - 1) + 6(2x - 3) - (2x^2 - 5x)$

b) $6(x + 5) - 2(x + 4) + 3(x - 5)$

c) $3(2y^2 - 1) + 6(2y - 3) - (2y^2 - 5y)$

d) $3(4p^2 - 2p + 6) + 6(4p - 2) - (7p^2 + 5p + 1)$

8. Simplify.

a) $4x^2(x^3 - 2x^2) + 2x^3(2x^2 + 2x)$

b) $2y^2(y^3 - 3y^5) - 3(y^2 - y^5)$

c) $3m^3(2m^2 - 5m + 3) - 4m(m^4 + 2m^3 - m^2)$

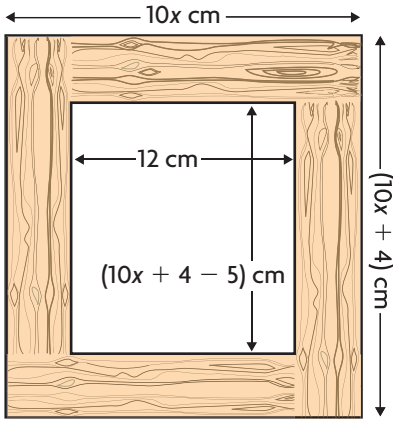
d) $-5x(x^3 - 2x^2) + 2x^2(3x^2 - 5x) - 4x^3(x - 2)$

9. Expand and simplify.

a) $\frac{3}{5}\left(2\frac{1}{3}a - 2\frac{1}{2}\right) - \frac{1}{2}\left(2\frac{1}{5}a + 3\frac{2}{3}\right)$

b) $\frac{1}{6}\left(3\frac{1}{5}a + \frac{2}{3}b\right) + \frac{1}{3}\left(\frac{1}{2}a - \frac{1}{2}\right)$

c) $-1.25(3.1m + 2.2) - 2.15(1.2m - 3.2)$



10. Mary is making rectangular picture frames to the proportions shown.

a) Determine a simplified expression for the outside perimeter of the frame.

b) Determine the outside perimeter when $x = 5$.

c) Determine a simplified expression for the area of one picture frame.

d) Determine the area of one frame when $x = 5$.

e) Determine a simplified expression for the number of square centimetres of wood needed to make 20 frames the same size. Assume there is no waste.

11. A company purchased two kinds of cars for its sales force. The following expressions give the value of each vehicle after it has lost value for x years.

A

- sedans: $V = -2400x + 19\,600$
- sport utility vehicles (SUVs): $V = -3100x + 24\,500$

a) The company has 12 sales representatives who drive sedans and 3 executives who drive SUVs. Write an expression that represents the combined value of the company's automobile fleet after x years.

b) Create a spreadsheet that will determine the cost of each vehicle type and the combined value of all the cars each year for 0 to 6 years.

c) What did the company pay for the SUVs? the sedans?

d) Which vehicle type is losing value at a faster rate?

12. Simplify.

a) $15 - 10(x - 4) - (3x + 3)$

b) In part a), explain why the 10 is not subtracted from the 15 before you expand.

13. **a)** Is the following statement always true, sometimes true, or never true? "Algebra tiles can be used to represent and simplify algebraic expressions that require the distributive property."

T

b) If you chose "always true" or "never true," explain how you know. If you chose "sometimes true," provide an example that shows when it is true, and another that shows when it isn't true.

Extending

14. Simplify the following.

a) $5(2x - 3y) - 4(3x + 6y)$

b) $x(x + y) + 2x(x - y)$

c) $2(x^2 - 3xy + 6y) - 3(2x^2 - 4xy - y)$

15. Apply the distributive property to simplify the following.

a) $(x + 3)(2x + 4)$

b) $(y + 2)(y + 1)$

c) $(2x + y)(x + y)$

16. Simplify.

a) $(x - 3)(x + 4) + 3(x^2 - x + 2)$

b) $(2x + y)(x - y) - (x^2 + y^2)$

c) $(3x + 5)(2x - 4) + (x + 1)(2x + 5)$