2.5 Multiplying a Polynomial by a Monomial

GOAL

Apply the distributive property to polynomials.

YOU WILL NEED

- algebra tiles
- algebra tile frame

LEARN ABOUT the Math

Judy has been asked to determine the product $3(2x + 4)$.

? How might Judy think about this operation in order to determine the product?

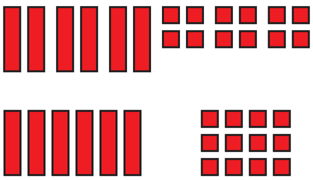
EXAMPLE 1 Multiplying a monomial by a polynomial

Determine the product $3(2x + 4)$.

Judy's Solution: Representing the product using algebra tiles

I knew that multiplying a number by 3 is the same as adding 3 copies of that number. I decided to show the same repeated addition strategy using algebra tiles.

I gathered enough algebra tiles to show 3 sets of $2x + 4$.



There were 3 sets of 2 x tiles and 3 sets of 4 unit tiles. That meant altogether there were 6 x tiles and 12 unit tiles.

$3(2x + 4) = 6x + 12$

Tamara's Solution: Representing the product using an area model

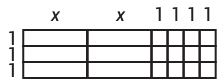
I knew that the area of a rectangle is the product of its length and width. I used algebra tiles to represent the length $2x + 4$ and the width of the rectangle.

I filled in the rectangle with algebra tiles that matched each section of the area inside the rectangle.

$$3(2x + 4)$$
$$= 3(2x) + 3(4)$$
$$= 6x + 12$$

The inside of the rectangle had 3 rows of tiles. Each row contained 2 *x* tiles and 4 unit tiles. Altogether the area was made up of 6 *x* tiles and 12 unit tiles.

Sue's Solution: Representing the product using a diagram



I imagined that the factors were the length and width of a rectangle that was divided into sections. I calculated the area of each section separately and added them together to get the total area. The total area was $6x + 12$.

$$3(2x + 4)$$
$$= 3(2x) + 3(4)$$
$$= 6x + 12$$

I noticed that there were 3 rows of sections and each row had 2 sections with area of *x* and 4 with area of 1.

Shania's Solution: Comparing the product to a product of numbers

$$20 \times 23 = 20 \times (20 + 3)$$
$$= 20 \times 20 + 20 \times 3$$
$$= 400 + 60$$
$$= 460$$

Sometimes, when I have to calculate a product, I split one of the factors into parts and use the distributive property.

distributive property or law

the property that states that when a sum is multiplied by a number, each value in the sum is multiplied by the number separately and the products are then added; for example, $4 \times (7 + 8) = (4 \times 7) + (4 \times 8)$

$$3(2x + 4) = 3(2x) + 3(4)$$
$$= 6x + 12$$

I decided to use the same strategy to multiply with a polynomial.

Reflecting

A. Which student's approach would you use? Why?

B. How does each student's way of thinking about the problem involve an application of the distributive property?

APPLY the Math

EXAMPLE 2 Connecting the distributive property to products of polynomials of degrees greater than 1

Multiply $(2x^3 + 4x - 5)3x^2$.

Peng Bo's Solution

$$(2x^3 + 4x - 5)3x^2$$

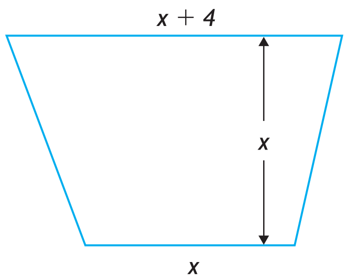
I used the distributive property to multiply each term in the trinomial factor by the factor $3x^2$.

$$= (2x^3)3x^2 + (4x)3x^2 - (5)3x^2$$
$$= 6x^5 + 12x^3 - 15x^2$$

I multiplied the coefficients of the terms to get the coefficients of the product. I added the exponents to determine the exponents of the variable terms.

EXAMPLE 3 Using reasoning and the distributive property to expand a product

Parm is an artist who works in metal and ceramics. He is building a sculpture in the shape of a large trapezoid. The sculpture's proportions are shown on the diagram. To order materials, he needs to determine the trapezoid's area.



Barry's Solution

$$A = \frac{1}{2}h(a + b)$$
$$= \frac{1}{2}x[x + (x + 4)]$$
$$= \frac{1}{2}x(2x + 4)$$

I remembered the formula for the area of a trapezoid. I used x for the height, x for the length of one parallel side, and $x + 4$ for the length of the other side.

$$A = \frac{1}{2}x(2x + 4)$$
$$= \frac{1}{2}x(2x) + \frac{1}{2}x(4)$$
$$= x^2 + 2x$$

I knew that with numbers, I could use the distributive property to multiply each number inside the brackets by the number outside the brackets. I decided to simplify the formula further by multiplying each term inside the brackets by the factor $\frac{1}{2}x$.

Communication *Tip*

The words "expand," "use the distributive property," and "remove brackets" all mean to multiply a polynomial by a factor.

EXAMPLE 4 Using reasoning and the distributive property to determine a missing factor

Complete and verify this equation: $\blacksquare(2x^2 - 5) = 8x^3 - 20x$.

Mohab's Solution

$\blacksquare(2x^2 - 5) = 8x^3 - 20x$ ← Using the distributive property, I knew that whatever factor was represented by the blank box would be multiplied by $2x^2$ and result in $8x^3$.

$4x(2x^2) = 8x^3$ ← $4x$ multiplied by $2x^2$ gives me $8x^3$.

$4x(2x^2 - 5) = 8x^3 - 20x$ ← To check, I replaced the box with the factor $4x$ and multiplied to make sure that the other term was $-20x$.

In Summary

Key Idea

- You can determine the product of a monomial and a polynomial by using the distributive property to expand it.

  For example: $2x(3x^2 + 5) = 2x(3x^2) + 2x(5)$
  $= 6x^3 + 10x$

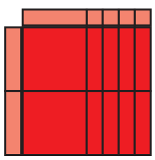
Need to Know

- You can sometimes determine the product of a monomial and a polynomial using the area model. This can be represented with concrete materials or diagrams.

CHECK Your Understanding

1. State the factors and product represented in each model as an algebraic equation.

a)



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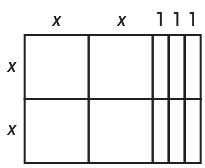


2. Expand.

a) $2a^3(4a^2 - a)$ **b)** $-2(y^2 - y - 1)$

3. Expand using the tool or strategy of your choice. Verify your answer using a different tool or strategy.

a) $2(3x + 4)$ **b)** $3x(x + 2)$

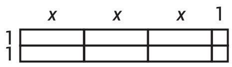
PRACTISING

4. Expand using a different tool or strategy for each.

a) $x(2x + 1)$
b) $4(5 + x)$
c) $(3x + 5)(2x)$

5. What multiplication equation does each model represent?

a)



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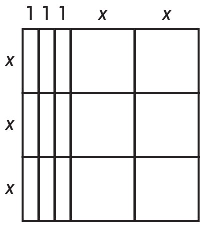

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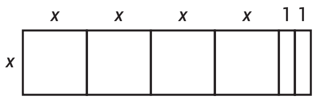


6. Expand.

a) $2(-y^2 - y - 1)$
b) $b^2(2b^3 - 4b + 1)$
c) $3m^3(5m^2 + 6m - 4)$
d) $-(x^2 - 3x + 7)$
e) $-4x(x^2 - 3x)$
f) $-2n^2(3n - 5 + 4n^3)$

7. Determine the missing factor and verify.

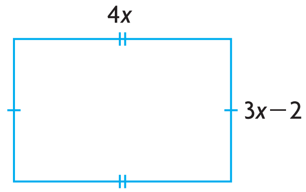
a) $\blacksquare(2x - 10) = 6x - 30$
b) $\blacksquare(x^3 - 5x - 4) = 2x^5 - 10x^3 - 8x^2$
c) $-4a^3(\blacksquare + \blacksquare + \blacksquare) = -12a^7 + 4a^4 - 8a^3$

8. Evaluate each statement for $p = 5$ once before you expand it and once after you expand it.

a) $4(3p + 2)$
b) $3p(6 - p)$
c) $2p^2(4p + 3)$
d) $p(3p^2 - 4p + 4)$
e) Explain how you know that you should get the same result both before you expand each statement and after you expand it.
f) Was it always easier to evaluate after you expanded? Explain.

9. Write simplified algebraic expressions for the perimeter, P, and area, A,
A of the following figures.

a)



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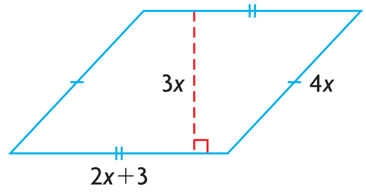
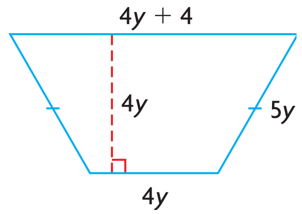


c)



d) Evaluate the perimeter and the area of the rectangle in part a) if $x = 4$ cm.

10. You could evaluate 20×47 by doing the calculation
C $20 \times 40 + 20 \times 7$. How is this like using the distributive property to simplify $x(2x + 7)$?

Extending

11. Expand.

a) $2x(y - 3z)$ **b)** $-3x(xy + yz)$

12. Fill in the missing information to make the statements true.

a) $20x + 15 = 5(\blacksquare + \blacksquare)$

b) $5x^2 + 25x = 5\blacksquare(\blacksquare + \blacksquare)$

c) $4x^5 + 8x^3 - 2x^2 = \blacksquare x^2(\blacksquare + \blacksquare + \blacksquare)$

d) How could you check your answers for parts a), b), and c) to see if they are correct?

13. These polynomials were expanded using the distributive principle. Restate them as a product of two factors.

a) $12x^2 - 6x$

b) $21y^3 + 7y^2 - 14y$

c) $5x^2 - 10xy + 30x$