# 2.4 Adding and Subtracting Polynomials

**YOU WILL NEED**

- algebra tiles

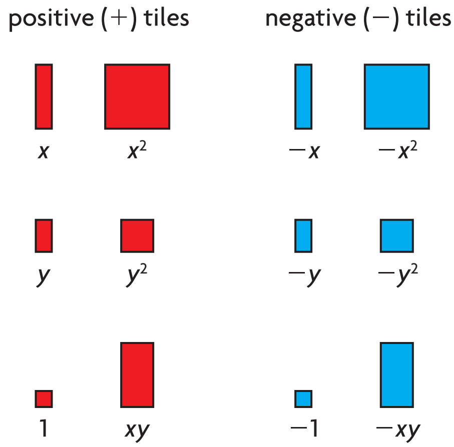
**GOAL**

Add and subtract like terms.

## *LEARN ABOUT the Math*

A class is playing a game with algebra tiles. The game has the following rules:

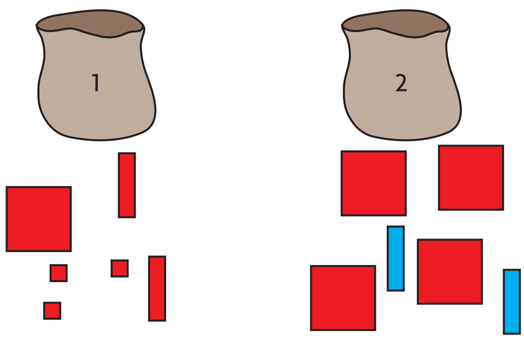
- A player gets two pouches. Each contains six randomly selected algebra tiles.
- A player can use the **zero principle** to add the tiles in the two pouches or subtract the tiles in the second pouch from those in the first.
- The goal is to end up with the fewest tiles.




**?** How can you decide if you should add or subtract?

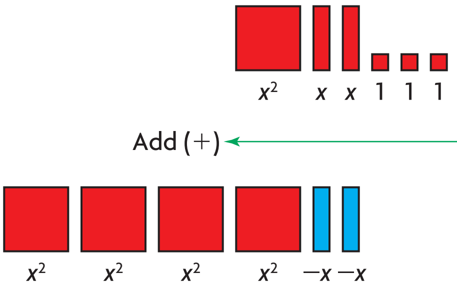
### EXAMPLE 1 Using a concrete model to represent an operation

Farell and Peter received the following tiles in their pouches.



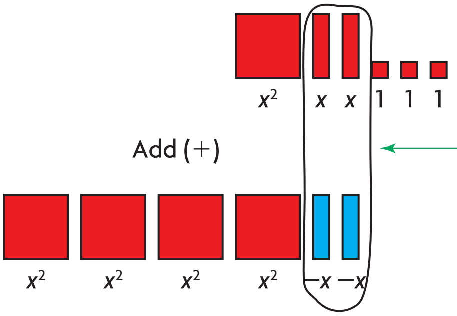

Should they add or subtract to get a result that uses the fewest number of tiles?

## Farell's Solution: Representing and simplifying a sum using algebra tiles

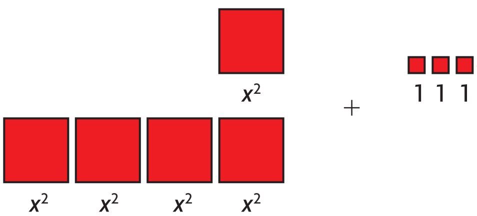



I arranged the algebra tiles from the two pouches and decided to add them by combining **like terms**.

I need to calculate
$(x^2 + 2x + 3) + (4x^2 - 2x)$




I figured positive- and negative-$x$ tiles could be combined to makes zeros using the zero principle, just as with opposite integers. So, I paired the two positive-$x$ tiles with the two negative-$x$ tiles and removed them.



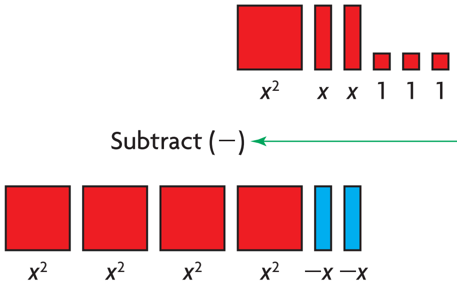

There were 5 $x^2$ tiles and 3 unit tiles left. I couldn't combine the $x^2$ tiles and the unit tiles since they were different things.

The expression I get by adding the two polynomials is $5x^2 + 3$ and that uses 8 tiles.

**like terms**

algebraic terms that have the same variables and exponents apart from their numerical coefficients (e.g., $2x^2$ and $-3x$)

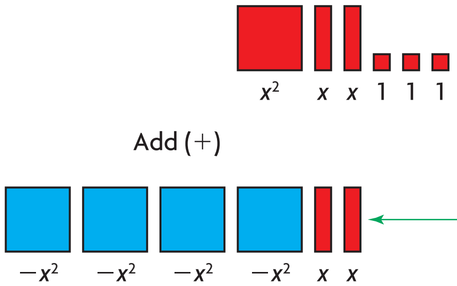
## Peter's Solution: Representing and simplifying a difference using algebra tiles




I decided to subtract the tiles of the second pouch from those in the first pouch.

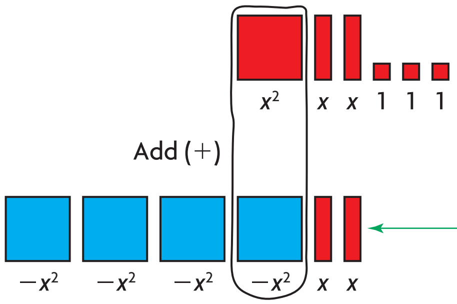
I needed to figure out the difference and how many tiles it would take to represent that difference.

I need to calculate $(x^2 + 2x + 3) - (4x^2 - 2x)$.



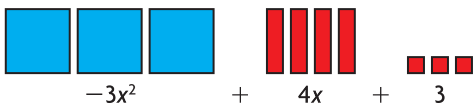

To subtract, I added the opposite of each tile in the second pouch, just like I would do with integers. I replaced red tiles with blue tiles and blue tiles with red tiles, and then added.

The opposite of $4x^2$ is $-4x^2$. The opposite of $(-2x)$ is $2x$.




I saw that I could eliminate an $x^2$ tile and a negative-$x^2$ tile using the zero principle, just as with opposite integers. Then, I gathered together congruent tiles of the same colour.

Since $1 + (-4) = (-3)$,
$x^2 + (-4x^2) = -3x^2$.
Since $2 + 2 = 4$,
$2x + 2x = 4x$.



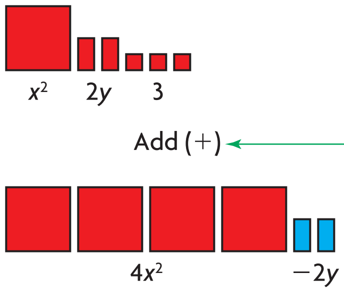

I saw and counted 3 negative-$x^2$ tiles, 4 $x$ tiles and 3 unit tiles.

The expression I get by subtracting is $-3x^2 + 4x + 3$ and that uses 10 tiles. Subtraction resulted in more tiles than addition, so I should use addition.

## EXAMPLE 2 Reasoning about expressions algebraically

Jay and Sierra got two new pouches of tiles. The first pouch contained 1 $x^2$ tile, 2 $y$ tiles, and 3 unit tiles. The second pouch had 4 $x^2$ tiles and 2 $-y$ tiles. They represented their contents using algebraic expressions. Jay added and Sierra subtracted the expressions. Which expression results in the fewest tiles?

### Jay's Solution: Representing and simplifying a sum algebraically




I represented the tiles using algebraic expressions. I expressed the contents of the first pouch as three **monomials** and the contents of the second pouch as two monomials.

$x^2 + 2y + 3 + 4x^2 + (-2y)$

I wrote an algebraic expression that represented the sum.

$= \underline{x^2} + \underline{\underline{2y}} + 3 + \underline{4x^2} + \underline{\underline{(-2y)}}$

I identified the algebraic terms that could be combined by looking at the variable part of each term. If the variable parts and their exponents were identical, the coefficients of the terms could be added using integer arithmetic. The variable part stayed the same.

$= \underline{x^2} + \underline{\underline{2y}} + 3 + \underline{4x^2} \underline{\underline{- 2y}}$

$= 5x^2 + 3$

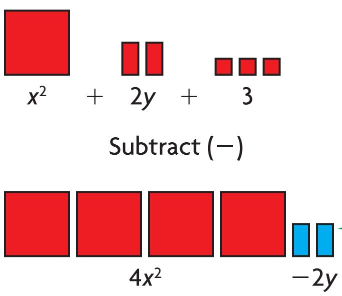
I ended up with $5x^2 + 3$.

This would use 8 tiles.

**monomial**

an algebraic expression with one term; for example, $5x^2$, $4xy$

### Sierra's Solution: Representing and simplifying a difference algebraically




I represented the tiles using algebraic expressions. I represented the contents of the first pouch as a **trinomial** and the contents of the second pouch as a **binomial**.

**trinomial**

an algebraic expression containing three terms; for example, $2x^2 - 6xy + 7$

**binomial**

an algebraic expression containing two terms; for example, $3x + 2$

**polynomial**

an expression that comprises a sum and/or difference of monomials

$(x^2 + 2y + 3) - (4x^2 - 2y)$ ← I wrote this as a difference of two **polynomials**.

$= x^2 + 2y + 3 + (-4x^2) + 2y$ ← To subtract the second polynomial I added the opposite of each term.

$= \underline{x^2} + \underline{\underline{2y}} + 3 \underline{- 4x^2} + \underline{\underline{2y}}$ ← I simplified my expression by combining like terms.

$= -3x^2 + 4y + 3$ ← The result was $-3x^2 + 4y + 3$.

This would use 10 tiles.

### Communication *Tip*

- Simplifying an algebraic expression means representing the expression using as few terms as possible.

## Reflecting

**A.** How did using algebra tiles help Farell and Peter know which terms could be added or subtracted?

**B.** How did the appearance of algebraic terms help Jay and Sierra know which terms could be added or subtracted?

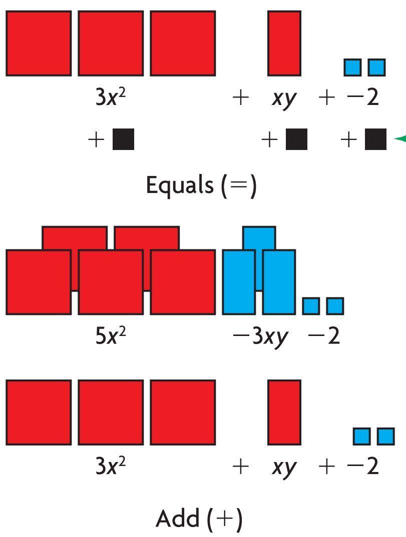
**C.** How did an understanding of integer operations and the zero principle help each student simplify his or her polynomials?

# *APPLY the Math*

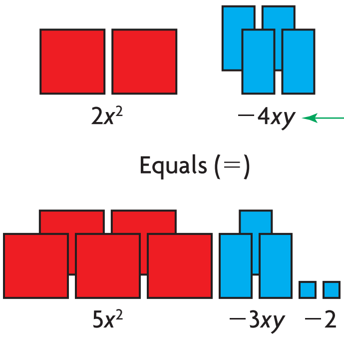
## EXAMPLE 3 Reasoning about like terms to determine missing terms in a polynomial

What polynomial must be added to $3x^2 + xy - 2$ to give the result $5x^2 - 3xy - 2$?

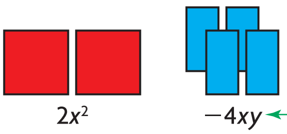
### Barry's Solution




← I used algebra tiles to show $3x^2 + xy - 2$ and $5x^2 - 3xy - 2$. I thought about what tiles needed to be added to the ones in the first line to get the ones in the last line.




I used algebra tiles and integer arithmetic to determine how many and what colour of tiles were needed. I saw that I needed 2 $x^2$ tiles, 4 negative-$xy$ tiles, and no unit tiles.




When I decided on the algebra tiles I needed, I wrote the algebraic representation.

$$3x^2 + 1xy - 2 + 2x^2 - 4xy$$
$$= 5x^2 - 3xy - 2$$

I checked my work by writing my new expression algebraically and combining like terms.

I have to add $2x^2 - 4xy$.

## EXAMPLE 4 Using algebraic reasoning to simplify a difference of polynomials

Simplify $(5x^2y + 4xy) - (2x^2y - xy)$.

### Raman's Solution

$$(5x^2y + 4xy) - (2x^2y - xy)$$
$$= 5x^2y + 4xy + (-2x^2y + 1xy)$$

I knew that subtracting a polynomial is the same as adding its opposite terms.

$$= 5x^2y + 4xy - 2x^2y + 1xy$$
$$= 5x^2y - 2x^2y + 4xy + 1xy$$

I identified the like terms and grouped them together.

$$= (5 - 2)x^2y + (4 + 1)xy$$
$$= 3x^2y + 5xy$$

I combined the coefficients of the like terms by adding or subtracting them.

## EXAMPLE 5 Using polynomials to represent and solve a problem

Joan and Chris both have jobs. They both work the same number of hours per week. Their pay rates and expenses are shown.

| | Pay Rate | Weekly Expenses |
|---|---|---|
| **Joan** | \$15.50/h | \$40 uniform rental |
| **Chris** | \$14/h | \$35 cafeteria charge |

Write an algebraic expression in simplest form to describe Joan and Chris's combined take-home pay each week.

Use this polynomial to determine their combined income if they both work 38 hours in a week.

### Susie's Solution

Joan and Chris each work $h$ hours in a week.

I used $h$ for the number of hours per week that Joan and Chris each worked.

Joan's income for a week: $15.5h - 40$

Chris's income for a week: $14h - 35$

To represent Joan's weekly income, I multiplied her hourly rate by $h$ and subtracted \$40 for her uniform.

To represent Chris's weekly income, I multiplied her hourly rate by $h$ and subtracted \$35 for her meals.

combined income

$$= (15.5h - 40) + (14h - 35)$$
$$= 15.5h + 14h - 40 - 35$$
$$= 29.5h - 75$$

I added the two expressions by combining like terms.

combined income for 38 hours

$$= 29.5(38) - 75$$
$$= 1121 - 75$$
$$= 1046$$

To determine their combined weekly income, I substituted 38 for $h$ and evaluated.

For 38 hours, their combined income was \$1046.

## In Summary

### Key Idea

- You can simplify a sum or a difference of polynomials by adding or subtracting the coefficients of like terms.
  For example: $(2y + 3x^2) + (8y - 5x^2)$
  $= (3x^2 - 5x^2) + (2y + 8y)$
  $= -2x^2 + 10y$

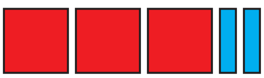
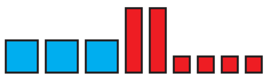
### Need to Know

- Like terms can be combined by adding or subtracting their numerical coefficients.
- The sum or difference of the coefficients of like terms can be calculated using the principles for adding and subtracting rational numbers.
- It often is easier to subtract two polynomials by using the same strategy you use with integers: adding the opposite.
  For example: $(2y - 2x^2) - (3y + 4x^2)$
  $= 2y - 2x^2 + (-3y - 4x^2)$
  $= 2y - 2x^2 - 3y - 4x^2$
  $= -6x^2 - 1y$

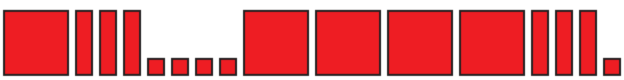
## *CHECK Your Understanding*

**1.** Draw an algebra tile representation of each polynomial.
**a)** $2x^2 - x$ **b)** $x^2 + 3$ **c)** $2y - 2x + 2$

**2.** Copy each question. Identify the like terms in each and circle their coefficients.
**a)** $3x, 4y, -2x$ **b)** $6m, -1.5m, 4n, 3m^2$

**3.** Write an algebraic expression for each algebra tile representation.
**a)**  **b)** 

**4.** Simplify the following algebra tile representation. State your result as a polynomial.



**5.** Simplify the following.
**a)** $2x + 3x$
**b)** $3y^2 - 2y^2 + 4y^2$
**c)** $3x - 2y + 4x$
**d)** $(2x + 3) + (5x - 4)$
**e)** $(3x - 5) + (-2x + 6)$
**f)** $(3x + 2) - (5x + 2)$

## PRACTISING

**6.** Draw an algebra tile representation of each polynomial.

**a)** $x^2 + 3x$
**b)** $2x^2 - y^2$
**c)** $xy + 4x$
**d)** $2x^2 - 3x - 4$

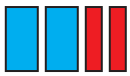
**7.** Copy each question. Identify the like terms in each and circle their coefficients.

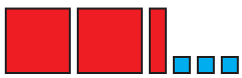
**a)** $-2g, 3f, -5g$
**b)** $-\frac{1}{2}y, -4x, 2\frac{1}{2}y, 6x^2$
**c)** $5x, -2.1y^3, -0.8y^3, 2y$
**d)** $-3.75rs, 3.3r, -5.1s, 4.25rs$

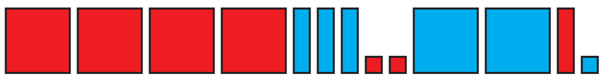
**8.** Write a simplified algebraic expression for each algebra tile representation.

**a)** 

**b)** 

**c)** 

**d)** 

**e)** 

**9.** Simplify the following.

**a)** $3h + 1 + 2h + 5$
**b)** $7y - 3y - x^2 + 4x^2$
**c)** $\frac{3}{4}w^2 - \frac{2}{3}w^2 + \frac{1}{4}w^2 - \frac{4}{3}w^2$
**d)** $\frac{3}{4}a - \frac{1}{5}b - \frac{1}{4}a + \frac{2}{5}b$

**10.** Simplify the following.

**a)** $(2x - 3y) + (3x + y)$
**b)** $(2y^2 - 3y + 4) + (-5y^2 + 5y - 3)$
**c)** $(3x^2 - 4xy + 6y^2) + (6x^2 - 8xy - 3y^2)$

**11.** Simplify the following.

**a)** $(5x - 4y) - (3x + 2y)$
**b)** $(3y^2 - 2y + 1) - (-5y^2 + 2y - 3)$
**c)** $(3x^2 - 4xy + 6y^2) - (6x^2 - 8xy - 3y^2)$

**12.** Simplify each of the following. Check your answer using a different tool or strategy.

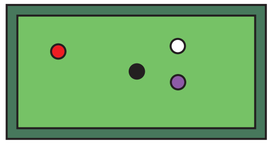
**a)** $2y - 3y - x^2 + 3x^2$
**b)** $(y^2 - 2y + 2) - (3y^2 - 2y + 3)$

**13.** Determine the polynomials that need to be added to each row of the table.

| | Initial Polynomial | Polynomial To Be Added | Final Polynomial |
|---|---|---|---|
| **a)** | $x^2 + 3x$ | | $-x^2 + 5x$ |
| **b)** | $2x^2y^2 - 4y^2$ | | $5x^2y^2 - 3y^2$ |
| **c)** | $-7xy + 4x$ | | $-7xy + 3x - 2$ |
| **d)** | $2x^2 - 3x - 4$ | | $-2x^2 + 3x - 6$ |

**14.** A pool table is always twice as long as it is wide. The Cue Ball Company makes pool tables in many different sizes. Each table top must have rubber bumpers around the outside edge and a felt top. The rubber bumpers cost \$2.25/m and the felt material for the top costs \$28/m$^2$. Determine an algebraic expression that represents the total cost for felt and rubber for the table top. Use this to determine the cost of the materials for a top that has a width of 1.5 m.

A




**15.** Jan is a plumber. She charges \$35 to visit a job site. Her hourly rate is \$43.50. Fred repairs furnaces. He charges \$41 for a service call plus \$38.75/h. Let $x$ represent the number of hours they work.

**a)** Represent Jan's bill as a polynomial.
**b)** Represent Fred's bill as a polynomial.
**c)** Write a new polynomial that represents Jan's and Fred's combined charge, assuming that they both work $x$ hours at a site.
**d)** Calculate their combined charge if they both work 8 h at the same complex.

**16.** Elizabeth and Dragan serve food at different restaurants on a cruise ship. Their earnings are based on tips, as shown, from which they have to pay for room and board.

| | Elizabeth | Dragan |
|---|---|---|
| **Average Weekly Tips** | \$220/table | \$160/table |
| **Room and Board** | \$160/week | \$125/week |

**a)** Write a polynomial to represent Elizabeth's weekly earnings after she pays for room and board.
**b)** Write a polynomial to represent Dragan's weekly earnings after he pays for room and board.
**c)** Dragan and Elizabeth work the same number of tables. Write a single polynomial that combines Dragan's and Elizabeth's earnings.

d) Evaluate the earnings for five tables.

e) Suppose Dragan works seven tables and Elizabeth works five tables. Can the single polynomial in part c) be used to calculate their joint earnings? Explain.

**17.** T In a TV game show, each player begins with \$1000. For each question answered correctly, a player receives \$125. For each one answered incorrectly, a player must pay \$250.

a) Express the total winnings for a player using an algebraic expression.

b) Use the expression from part a) to find the total winnings for the three players if:

- player 1 answered 12 questions correctly and 8 incorrectly
- player 2 answered 10 questions correctly and 2 incorrectly, and
- player 3 answered 15 questions correctly and 5 incorrectly.

**18.** Create two 3-term polynomials such that:

a) When the polynomials are combined there are 5 terms.

b) When the polynomials are combined there are 3 terms.

c) When the polynomials are combined there is 1 term.

**19.** C Describe how your knowledge of the zero principle and of adding and subtracting rational numbers helps simplify a sum or difference of polynomials.

## Extending

**20.** Simplify.

a) $2x + 3y + 4z - 4x + 3y - z$

b) $-4abc - 3ab - 6abc - 4ab$

c) $3xy + 5yz - 2xyz + 6xy - xyz$

**21.** Simplify.

a) $(2x + 3y) + (5x - 4y) + (2x - y)$

b) $-(4ab - 3a) - (6ab - 4a) + (2ab + 6a)$

c) $(3xy + 5y^2) - (3xy + 5y^2) + (3xy + 5y^2)$

**22.** Two polynomials are added and the sum is $3x^2 - y + 4$. For each statement, state whether it is always true, sometimes true, or never true. Explain or provide a counter-example to justify your answer.

a) Both are monomials.

b) Both include a $y$-term.

c) If there is an $x$-term in one polynomial then there must be an $x$-term in the other.

d) Both are binomials.