2.3 Power of a Power

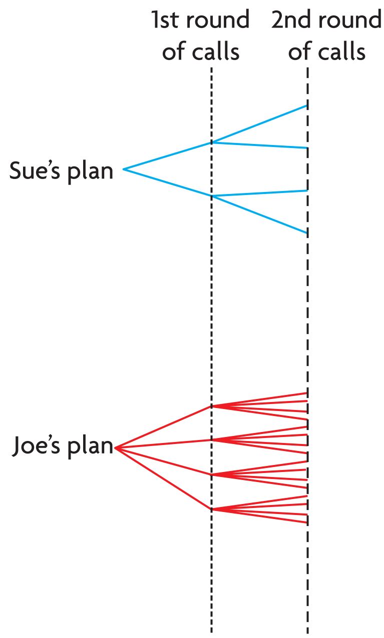
GOAL

Simplify expressions involving a power of a power.

LEARN ABOUT the Math

Sue and Joe want to spread the news about school picture day. Sue suggests that she call 2 people and ask each person called to call 2 more people, and so on. Joe suggests that he call 4 people and ask each person called to call 4 more people, and so on.

Joe says that with his plan, the same number of people would be called on the 4th round of calls as on the 8th round of calls with Sue's plan.



? Is Joe right?

EXAMPLE 1 Representing a power as an equivalent power

Determine and compare the number of people called on the 8th round of calls using Sue's plan and on the 4th round of calls using Joe's plan.

Jaan's Solution: Representing 2^8 as a power with base 4

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

I started by looking at 2^8, since that is the number of people who would be called in the 8th round with Sue's plan.

$$2^8 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$$
$$= (2^2)(2^2)(2^2)(2^2)$$
$$= (2^2)^4$$
$$= 4^4$$

I grouped the 2s in pairs. Each pair could be written as 2^2 and was equal to 4. I now had four 4s multiplied together.

4^4 is how many people would be called after the 4th round with Joe's plan.

2^8 is the same as 4^4. So, Joe is right.

I used my calculator to evaluate the powers. $2^8 = 256$ and $4^4 = 256$, so I knew Joe was right.

Marie's Solution: Representing 4^4 as a power with base 2

$4^4 = 4 \times 4 \times 4 \times 4$

I started by looking at 4^4 because that is the number called in round 4 of Joe's plan.

$$\begin{aligned} &4 \times 4 \times 4 \times 4 \\ &= (2^2)(2^2)(2^2)(2^2) \\ &= (2^2)^4 \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8 \end{aligned}$$

I wrote each 4 as 2^2.

4^4 is the same as 2^8. So, Joe is right.

I used my calculator to evaluate the powers. $2^8 = 256$ and $4^4 = 256$, so I knew Joe was right.

Reflecting

A. How did Jaan and Marie use different representations to show that Joe was correct?

B. Why could 2^8 also have been written as $(2^4)^2$?

C. How could you use Jaan's approach to write 8^4 as a single power of 2? How does this strategy demonstrate a principle for calculating a power of a power?

Communication *Tip*

When expressing a power of a power, use brackets to indicate the base to which the outermost exponent applies. For example, $(2^3)^4$ means $(2^3)(2^3)(2^3)(2^3)$.

APPLY the Math

EXAMPLE 2 Selecting a strategy to simplify a power of a power

Simplify $(x^5)^3$.

Jordi's Solution: Reasoning using products of powers with the same base

$$\begin{aligned} &(x^5)^3 \\ &= (x^5)(x^5)(x^5) \\ &= x^{5+5+5} \\ &= x^{15} \end{aligned}$$

I used the principle for multiplying powers with the same base.

$$\begin{aligned} &(x^5)^3 \\ &= x^{15} \end{aligned}$$

So, x to the power of 5 all to the power of 3 is the same as x to the power of 15.

Parm's Solution: Reasoning using the power-of-a-power principle

$(x^5)^3$
$= x^{5\times 3}$
$= x^{15}$

When I use numbers I can multiply or divide exponents if the bases are the same, so I assumed that this would also be true with variables.

I applied the power-of-a-power principle by multiplying the exponents 5 and 3 together.

$(x^5)^3$
$= x^{15}$

So, *x* to the power of 5 all to the power of 3 is *x* to the power of 15.

EXAMPLE 3 Simplifying an expression when the base is a term with more than one variable

Simplify $\dfrac{(2x^2y^3)^3}{(2xy^2)^2}$.

Teresa's Solution: Reasoning using the exponent principle for products

$$\frac{(2x^2y^3)^3}{(2xy^2)^2}$$
$$= \frac{(2x^2y^3)(2x^2y^3)(2x^2y^3)}{(2xy^2)(2xy^2)}$$

I started by writing the numerator and denominator using repeated multiplication.

$$= \frac{(2)(2)(2)(x^2)(x^2)(x^2)(y^3)(y^3)(y^3)}{(2)(2)(x)(x)(y^2)(y^2)}$$

Then I rearranged the numerator and denominator by putting the factors with the same base side-by-side.

$$= \frac{(2^3)(x^6)(y^9)}{(2^2)(x^2)(y^4)}$$

I rewrote each expression by using the exponent principle for multiplication, adding the exponents where the base was the same.

$= (2x^4y^5)$

I simplified the expression by using the exponent principle for division, subtracting the exponents where the base was the same.

Marty's Solution: Reasoning using the power-of-a-power principle

$$\frac{(2x^2y^3)^3}{(2xy^2)^2}$$

$$= \frac{(2)^3(x^2)^3(y^3)^3}{(2)^2(x)^2(y^2)^2}$$

The exponents outside the brackets in the numerator and the denominator apply to all of the factors inside the brackets. This means that I could write the expression as a product of separate factors.

$$= \frac{(2^3)(x^6)(y^9)}{(2^2)(x^2)(y^4)}$$

To simplify a power of a power, I multiplied the exponents.

$$= 2x^4y^5$$

To divide when the bases are the same, I subtracted the exponent in the denominator from the exponent in the numerator.

In Summary

Key Idea

- When a power is raised to another exponent the following principle can be used to simplify the power.

Exponent Principle for Power of a Power
$(a^m)^n = a^{mn}$

For example, $(a^4)^3 = a^{4 \times 3} = a^{12}$.

Need to Know

- If you have the power of a product, the outer exponent refers to each factor inside the brackets. For example: $(a^mb^n)^p = (a^m)^p \times (b^n)^p = a^{mp}b^{np}$.
- If you have the power of a quotient, the outer exponent refers to each term inside the brackets. For example: $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$ $(b \neq 0)$.

CHECK Your Understanding

1. Express each of the following as a power with a single exponent.

a) $(7^3)^5$ b) $(x^4)^6$ c) $(c^3)^2$

2. Express each of the following as a power with a different base.

a) 16 b) 4^3 c) 9^4

PRACTISING

3. Express each of the following as a power with a single exponent.

a) $(3^4)^2$ c) $(2^5)^3$ e) $(x^2)^3$

b) $(9^4)^3$ d) $(10^6)^6$ f) $(5^2)^4$

4. Express each of the following as a power with the base indicated.

a) 16^2 with a base of 4

b) 16^2 with a base of 2

c) 25^3 with a base of 5

d) 27^3 with a base of 3

5. Explain each principle, and then give a numerical example.

C a) $(a^m)^n = a^{mn}$

b) $(a^m b^n)^p = a^{mp} b^{np}$

c) $\left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$

6. Simplify.

K a) $(3^4)^2(3^5)$ c) $\dfrac{(2^3)^3}{2^4}$ e) $\dfrac{(5^5 \times 5^2)^2}{(5^4 \times 5)^2}$

b) $(5^4)^3(5^4)^3$ d) $\dfrac{(10^4)^2}{(10^2)^3}$ f) $\left(\dfrac{3^5}{3^3}\right)^2$

7. Simplify.

a) $(y^3)^4$ b) $(m^2)^3$ c) $(c^3)^3$ d) $(n^3)^4$

8. Simplify.

a) $(v^2)^2(v)$ c) $\dfrac{(k^5)^3}{k^2}$ e) $\dfrac{(x^2x^3)^4}{(x^5x)^3}$

b) $(n^4)^3(n^2)^3$ d) $\dfrac{(j^8)^2}{(j^5)^2}$ f) $\left(\dfrac{y^6}{y^4}\right)^3$

9. Simplify.

a) $(3a^2)^3$ c) $(-2m^2)^4$ e) $(5a^2 \times 2b^3)^2$

b) $(5x^5)^2$ d) $(4^3p^4)^2$ f) $(3x^4y^2)^3$

10. Simplify.

a) $(4^3 \times 3^2)^2(4^5 \times 3^2)^3$

b) $(2x^3)^4(2x^2)^5$

c) $\dfrac{(2^5 \times 5^2)^2}{(2^4 \times 5)^2}$

d) $\dfrac{(5a^3)^5}{(5a^5b^2)^2}$

11. Simplify.

a) $(2y^3)^4$

b) $(3x^5)^2$

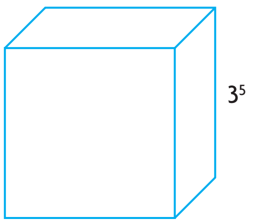
c) $(3a^3)^2(3^3a^5b^2)^2$

d) $\dfrac{(5^3a^4)^5}{(5^4a^3)^2}$

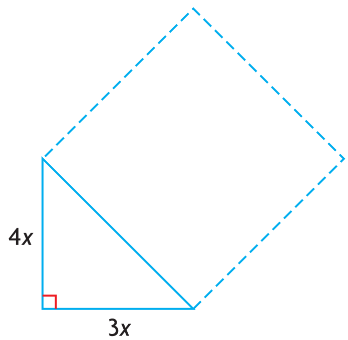
12. Without actually computing the values, explain how you know that
T each expression below is equal to 0.

a) $(3^2)^6 - (3^3)^4$ b) $(10^2)^8 - (10^4)^4$ c) $(-2^3)^2 - (-2^2)^3$

13. The length of the side of a cube is 3^5. Express its surface area (*SA*) and
A volume (*V*) using powers and simplify.



14. Determine an expression for the area of the square drawn on the **hypotenuse**.



15. Evaluate.

a) $\dfrac{(2^3)^4}{(2^2)^5}$

b) $\dfrac{(5^3)^6}{(5^3)^5}$

c) $\dfrac{(6)(2^3)^3}{(2^2)^4}$

d) $\dfrac{(5^2)^3(7^3)^4}{(7^{11})(5^5)}$

e) $\dfrac{(5^2)(6^6)}{(5^1)^4(6^2)^3}$

f) $\dfrac{[(2^4)^2]^3}{[(2^2)^3]^2}$

16. Simplify and evaluate each. Use $a = 2$, $b = -1$, and $c = 4$.

a) $\frac{a^5}{a^2}$

b) $(b^3)^2$

c) $\frac{(c^2)^3}{c^5}$

d) $\frac{a^3b^3}{ab}$

17. Show that 3^{10} is the same as 9^5 using your understanding of exponents.

18. Simplify and evaluate each.

a) $\frac{(x^5)^2(x^7)^3}{(x^4)^6}$ when $x = 2$

b) $\frac{(m)^{11}}{(m^5)^2} + \frac{n^7}{(n^2)^3}$ when $m = 3$ and $n = 4$

c) Explain how using exponent principles helped you to solve these problems.

19. Determine the value of the exponent that makes each statement true.

a) $4^3 = 2^{\blacksquare}$

b) $6^9 = 216^{\blacksquare}$

c) $625^2 = 25^{\blacksquare}$

d) $27^4 = 3^{\blacksquare}$

20. Write each power in simplified form.

a) 4^5 as a power of 2

b) 9^6 as a power of 3

c) 27^4 as a power of 3

d) $(-125)^7$ as a power of (-5)

21. Knowing that 2^3 is 8 and 3^2 is 9, how do you know that $2^{30} < 3^{20}$?

22. Describe the relationship between the power-of-a-power principle and the other exponent principles you know.

Extending

23. Jody's calculator will only input one-digit numbers. The exponent key and the display are working fine. Explain how she can use her calculator to evaluate each of the following.

a) 25^4

b) 16^2

24. Explain why you can write 2^8 as a power having a base of 4 and an integer as an exponent, but cannot do this for 2^7.