2.2 Multiplying and Dividing Powers

GOAL

Develop and apply exponent principles to multiply and divide powers.

INVESTIGATE the Math

Amir thought there was a way to simplify $\frac{(3^6)(3^9)}{3^{12}}$ without using a calculator.

? How could Amir simplify this expression without a calculator?

A. Draw a table like the one shown. Use it to record the products of powers that have the same base but different exponents.

Extend the table until you see a pattern. Use the pattern to determine how to quickly multiply powers with the same base.

Multiplication	Expanded Form	Product Expressed as a Single Power
$(3^1)(3^2)$	$(3)(3 \times 3)$	3^3
$(3^2)(3^2)$	$(3 \times 3)(3 \times 3)$	3^4
$(3^3)(3^2)$		

B. How is the exponent of the product related to the exponents of the factors?

C. Test your answer to part B using these expressions. Check using a calculator.

a) $(5^2)(5^4)$ **b)** $(2^2)(2^3)(2^4)$

D. Draw a new table like the one shown. Use it to record quotients with the same base and different positive integers for exponents.

Continue to add rows to the table until you see a pattern. Use the pattern to determine how to quickly divide powers with the same base.

Division	Expanded Form	Quotient Simplified and Expressed as a Single Power
$3^2 \div 3^1$	$\frac{(3 \times 3)}{3}$	$\frac{(3 \times \cancel{3})}{\cancel{3}} = 3^1$
$3^4 \div 3^2$	$\frac{(3 \times 3 \times 3 \times 3)}{(3 \times 3)}$	$\frac{(3 \times 3 \times \cancel{3 \times 3})}{(\cancel{3 \times 3})} = 3^2$

E. How is the exponent of the quotient related to the exponents of the terms in the division statement?

F. Use your results in parts B and E to simplify $\frac{(3^6)(3^9)}{3^{12}}$ as a single power with base 3. Check your answer by calculating the value of the original and simplified expressions.

Reflecting

G. How can you determine the exponent of the product of powers with the same base?

H. How can you determine the exponent of the quotient of powers with the same base?

principle
a basic truth or rule about the way something works

I. Why do the **principles** in parts G and H only apply to products and quotients of powers with the same base?

APPLY the Math

EXAMPLE 1 Representing an expression involving powers

Simplify $\frac{(x^7)(x^3)}{x^6}$.

Tony's Solution: Making a conjecture based on numeric examples

$$\frac{(2^7)(2^3)}{(2^6)}$$

I substituted 2 for x and calculated the value of the expression.

$$= \frac{(128)(8)}{(64)}$$
$$= \frac{1024}{64}$$
$$= 16$$
$$= 2^4$$

16 is $2 \times 2 \times 2 \times 2$, which equals 2^4.

$$\frac{(3^7)(3^3)}{(3^6)}$$

Then, I substituted 3 for x and calculated the value of the expression.

$$= \frac{2187 \times 27}{729}$$
$$= \frac{59049}{729}$$
$$= 81$$
$$= 3^4$$

81 is $3 \times 3 \times 3 \times 3$, which equals 3^4.

I think that $\frac{(x^7)(x^3)}{x^6} = x^4$

In both cases, the final power had an exponent of 4.
I thought the same pattern would work for any base.

Danny's Solution: Reasoning using the definition of a power

$$\frac{(x^7)(x^3)}{x^6} = \frac{(xxxxxxx)(xxx)}{x^6}$$

I looked at the numerator and wrote out the powers in expanded form. This showed me that the numerator was x^{10}.

$$\frac{(\overset{1}{\cancel{xxxxxx}})(xxxx)}{\underset{1}{\cancel{xxxxxx}}}$$

I wrote out the denominator in expanded form. I simplified the fraction by dividing 6 of the xs in the numerator and denominator.

$$xxxx = x^4$$

This left 4 xs multiplied together, which I wrote as x^4.

So, $\frac{(x^7)(x^3)}{x^6} = x^4$.

Patty's Solution: Reasoning using exponent principles

$$\left(\frac{x^7}{x^6}\right)x^3 = (x)(x^3)$$

I divided x^7 by x^6 using the exponent principle for division. Subtracting 6 from 7, I got 1. I didn't need to write the exponent because x^1 is the same as x.

$$(x)(x^3) = x^4$$

I used the exponent principle for multiplication. Since 1 + 3 is 4, my final answer was x^4.

$$\frac{(x^7)(x^3)}{x^6} = x^{7+3-6} = x^4$$

I saw that I could have reached the same answer in one step by just adding the exponents in the numerator and subtracting the exponent in the denominator.

Communication *Tip*

Any variable without a visible exponent is understood to have an exponent of 1. For example, $x = x^1$, $2y = 2y^1$, and $-4c = -4c^1$.

EXAMPLE 2 Selecting a strategy to evaluate an expression with two variables

Simplify $\frac{(x^4y^3)(x^3y^5)}{x^5y^5}$ and evaluate when $x = 3$ and $y = -2$.

Kathryn's Solution: Using an algebraic strategy to first simplify the expression

$$\frac{(x^4y^3)(x^3y^5)}{x^5y^5}$$
$$= \frac{(x^4x^3)(y^3y^5)}{x^5y^5}$$
$$= \frac{x^7y^8}{x^5y^5}$$

Everything in the numerator was multiplied together. I rewrote it so that powers of the same base were side by side. Then, I simplified by adding the exponents of the powers having the same base.

$$= x^{7-5}y^{8-5}$$
$$= x^2y^3$$

I divided powers having the same base. I did this by subtracting the exponent of the denominator from the exponent of the numerator.

$$= 3^2(-2)^3$$
$$= 9(-8)$$
$$= -72$$

I substituted 3 for *x* and −2 for *y*, and evaluated. The result was −72.

Jeremy's Solution: Using a substitution strategy

$$\frac{(x^4y^3)(x^3y^5)}{x^5y^5}$$
$$= \frac{[3^4(-2)^3][3^3(-2)^5]}{3^5(-2)^5}$$

I substituted 3 for *x* and −2 for *y* into the question.

$$\frac{[81(-8)][27(-32)]}{243(-32)}$$
$$= \frac{[-648][-864]}{-7776}$$

I calculated the expressions inside each set of brackets.

$$= \frac{559\,872}{-7776}$$
$$= -72$$

I did the final multiplication and division.

EXAMPLE 3 Applying exponent principles to simplify expressions where the base has multiple factors

Simplify $\dfrac{\left(-\frac{2}{5}xy\right)^5}{\left(-\frac{2}{5}\right)^3 xy}$.

Miranda's Solution: Connecting exponent principles to fraction operations

$$\frac{\left(-\frac{2}{5}xy\right)^5}{\left(-\frac{2}{5}\right)^3 xy}$$

$$= \frac{\left(-\frac{2}{5}\right)^5 x^5y^5}{\left(-\frac{2}{5}\right)^3 xy}$$

I expressed the numerator as three separate powers because I knew that the exponent 5 outside of the brackets referred to each of the factors inside.

$$= \left(-\frac{2}{5}\right)^{(5-3)} x^{(5-1)}y^{(5-1)}$$

Then, I divided powers having the same base by subtracting their exponents. I knew that x and y in the denominator each had an exponent of 1.

$$= \left(-\frac{2}{5}\right)^2 x^4y^4$$

$$= \frac{4}{25}x^4y^4$$

I evaluated $\left(-\frac{2}{5}\right)^2$ by multiplying.

$\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) = \frac{4}{25}$.

If each rational number in an algebraic expression can be expressed as a terminating decimal, you can write each number as such and then simplify the equivalent expression.

Communication Tip

You can rewrite a product or quotient raised to an exponent by applying the exponent to each of the terms. For example, $(xy)^2 = x^2y^2$ and $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$.

Lee's Solution: Connecting exponent principles to decimal operations

$$\frac{(-0.4xy)^5}{(-0.4)^3xy}$$

$$= \frac{(-0.4)^5x^5y^5}{(-0.4)^3xy}$$

I converted each fraction to its decimal equivalent. Then, I expressed the numerator as three separate powers because I knew that the exponent 5 outside the brackets referred to each of the factors inside.

$$= (-0.4)^{(5-3)}x^{(5-1)}y^{(5-1)}$$

I divided powers having the same base by subtracting their exponents. The *x* and *y* in the denominator each had an exponent of 1.

$$= (-0.4)^2x^4y^4$$

$$= 0.16x^4y^4$$

I evaluated $(-0.4)^2$. The final result was $0.16x^4y^4$.

EXAMPLE 4 Using exponent principles to solve a problem involving large numbers

The M31 galaxy in the constellation of Andromeda is about 2.4×10^{19} km away. Light travels at about 9.5×10^{12} km/year. Estimate how long it would take light to reach Earth from M31.

Kyle's Solution

I used the formula $t = \frac{d}{s}$.

I needed to divide the distance to the M31 galaxy by the speed of light to find the time in years.

$d = 2.4 \times 10^{19}$ km

$s = 9.5 \times 10^{12}$ km/year

$$t = \frac{2.4 \times 10^{19}}{9.5 \times 10^{12}}$$

I estimated this to be about

$$\frac{2 \times 10^{19}}{10 \times 10^{12}}.$$

2.4×10^{19} is close to 2×10^{19}, 9.5×10^{12} is close to 10×10^{12}, and 10×10^{12} is 10^{13}.

$$= \frac{2 \times 10^{19}}{10^{13}}$$

I used the exponent principle for quotients to simplify the expression.

$$= 2 \times 10^{19-13}$$

$$= 2 \times 10^6$$

$$\frac{2.4 \times 10^{19}}{9.5 \times 10^{12}} \doteq 2\ 000\ 000$$

$10^6 = 1\ 000\ 000$, so $2 \times 10^6 = 2\ 000\ 000$.

The light from M31 takes about 2 000 000 years to reach Earth!

In Summary

Key Idea

- When two powers have the same base, these principles can be used to simplify their product or quotient:

Exponent Principle for Products	Exponent Principle for Quotients
$(a^m)(a^n) = a^{m+n}$	$(a^m) \div (a^n) = a^{m-n}\ (a \neq 0)$

For example, $(2^2)(2^3) = 2^{2+3} = 2^5$ and $3^4 \div 3^2 = 3^{4-2} = 3^2$.

Need to Know

- These exponent principles only work when the powers involved have the same base.
- It is more efficient to simplify an algebraic expression involving powers before substituting to evaluate it.
- An exponent applied to a product or quotient can be written by applying the exponent to each of its terms. In general $(ab)^m = a^m b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.

CHECK Your Understanding

1. Simplify.

a) $(2^2)(2^3)$ **b)** $(x^4)(x^3)$

2. Simplify.

a) $\frac{2^5}{2^2}$ **b)** $\frac{y^6}{y^3}$

3. Simplify if possible, and then evaluate.

a) $\frac{(2^7)}{(2^5)}$ **b)** $\frac{(5^5)(3)(3^4)}{(3^3)(5^4)}$

4. Simplify, and then evaluate for $x = 2$ and $y = 5$.

a) $\frac{(x^4)(x^3)}{x^6}$ **b)** $\frac{y^6x^4}{x^3y^3}$

PRACTISING

5. Simplify.

a) $(5^2)(5^8)$

b) $(m^4)(m^2)$

c) $(7^3)(7)(x^4)(x^2)$

d) $\left(\frac{2}{5}\right)^3\left(\frac{2}{5}\right)^2\left(\frac{2}{5}\right)^4$

6. Simplify.

a) $(n^5)(w^6)(n^3)(w^7)$

b) $(m^3)(m^4)(r^8)(m^2)(r^2)$

c) $2^5(p^3)2^2(p^2)(p^8)$

d) $(b^5)3^4(b^3)3^2(b^7)$

e) $(x^4)(-2)(x^5)(-2)^3$

f) $(a^5)(3^2)(a^4)(a)(3)$

7. Why do you get the same result for each of these expressions?

a) $(5^7)(5^4)$

b) $(5^6)(5^5)$

c) $(5^4)(5^2)(5^5)$

d) $(5^3)(5)(5^5)(5^2)$

8. Simplify.

a) $\frac{5^7}{5^2}$

b) $\frac{m^4}{m^2}$

c) $\frac{(2^5)(x^3)}{(2^4)(x^2)}$

d) $\frac{(-5)^3y^{10}}{(-5)(y^6)(y^3)}$

9. Simplify.

a) $\frac{(7^6)(a^3)(7^2)}{(7^3)a}$

b) $\frac{(10^{10})x^4y^5}{(10^8)xy}$

c) $\frac{(xy)^5}{x^4y^3}$

d) $\frac{x^2y^4}{x^3y}$

10. Create four different expressions involving exponents that simplify to 7^8.

11. Simplify if possible, and then evaluate.

K

a) $\frac{2^8}{2^5}$

b) $\frac{(4^5)(4^6)}{4^7}$

c) $\frac{(7^3)(3^2)(3^4)(7)}{(3^3)(7^2)}$

d) $\frac{(4.2^3)(4.2^5)}{4.2^7}$

e) $\frac{\left(\frac{2}{7}\right)^4}{\left(\frac{2}{7}\right)^2}$

f) $\frac{\left(\frac{4}{5}\right)^5\left(\frac{4}{5}\right)^4}{\left(\frac{4}{5}\right)^6}$

12. Simplify, and then evaluate for $x = 2$ and $y = 5$.

a) $\frac{(x^5)(x^4)}{x^8}$

b) $\frac{(y^6)(y^4)}{(y^8)(y)}$

c) $\frac{(y^6)(x^4)}{(x^3)(y^3)}$

d) $\frac{250y^6}{125y^3}$

e) $\frac{6(x^4)(y^6)}{3(x^3)y^3}$

f) $\frac{\left(\frac{3}{4}xy\right)^3}{\left(\frac{3}{4}\right)^2xy^2}$

13. If you know that the product of two powers is 7^{10} and that the quotient is 7^2, what could the two powers be? How could you verify your answer?
A

14. Scientists estimate that there are 50×10^{12} cells in the average human. There are approximately 6×10^9 humans in the world. Approximately how many cells do all the humans on Earth have? Write your answer using a power with base 10.
T

15. Explain why it is necessary for the bases to be the same in order to apply the multiplication and division principles for exponents.
C

Extending

16. a) Complete the table to show the relationship between the metric units of length. Express each relationship as a power with base 10.

	Millimetres	Centimetres	Metres	Kilometres
Millimetres				
Centimetres				
Metres				
Kilometres				

b) Determine the number of centimetres in 5 km.
c) Determine the number of millimetres in 4 m.

17. A piece of steel plate is used to make a railway car. The plate is 2.5 m wide, 3.2 m long, and 0.5 cm thick. Determine the volume of steel in cubic centimetres.

18. The annual worldwide production of all grains is about 9×10^{12} kg. How much grain is produced per person if there are approximately 6×10^9 people in the world?

19. a) Evaluate $\frac{3^5}{3^5}$.

b) Simplify $\frac{3^5}{3^5}$ using the exponent principle for quotients.

c) Use the meaning of the powers in $\frac{3^5}{3^5}$ to simplify the expression.

d) Discuss what 3^0 might mean.

e) Discuss whether a^0 would have a similar meaning for any value of a.