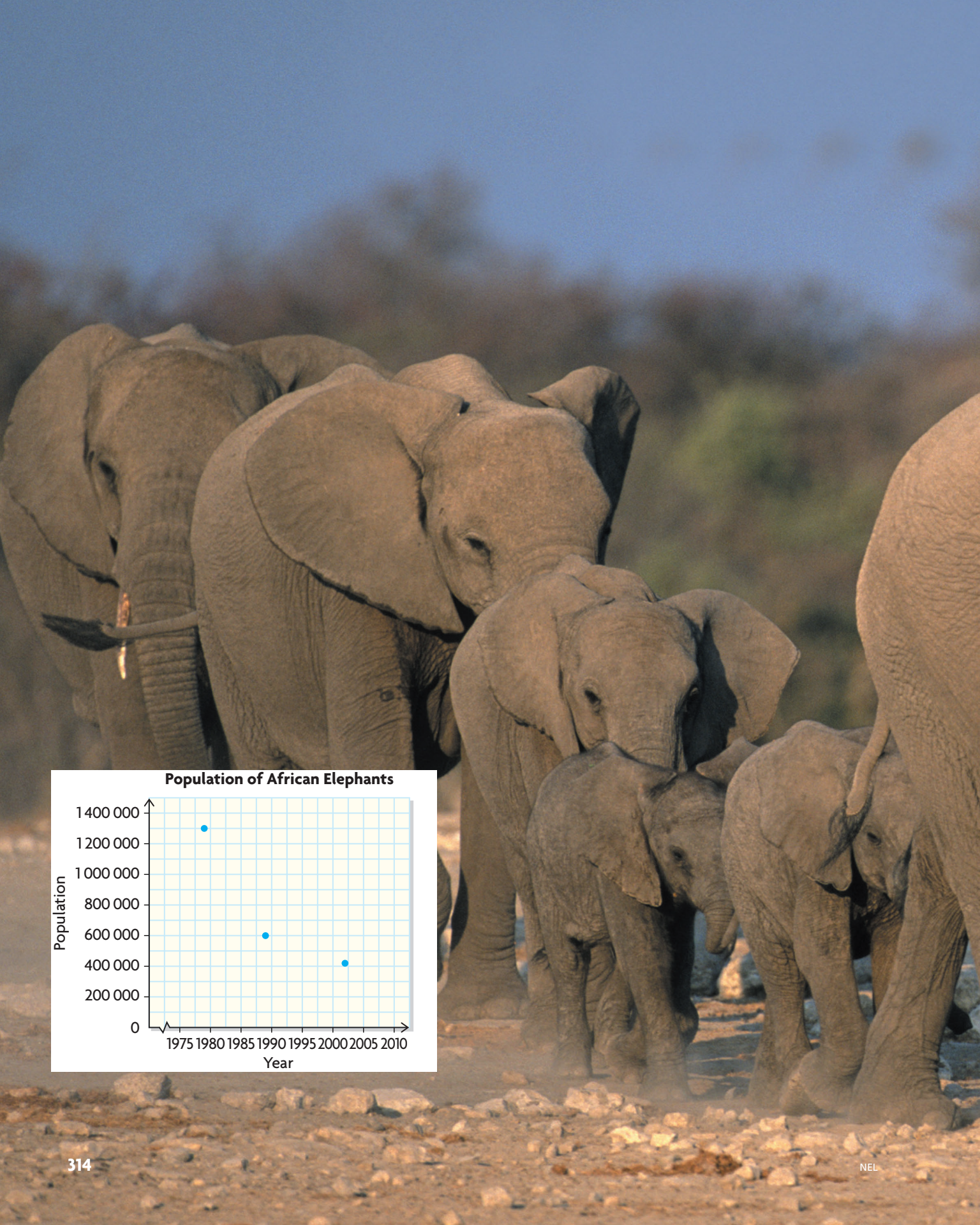

Population of African Elephants
1 400 000
1 200 000
1 000 000
800 000
600 000
400 000
200 000
0
Population
1975 1980 1985 1990 1995 2000 2005 2010
Year

Chapter

6

Investigating Relationships

GOALS

You will be able to

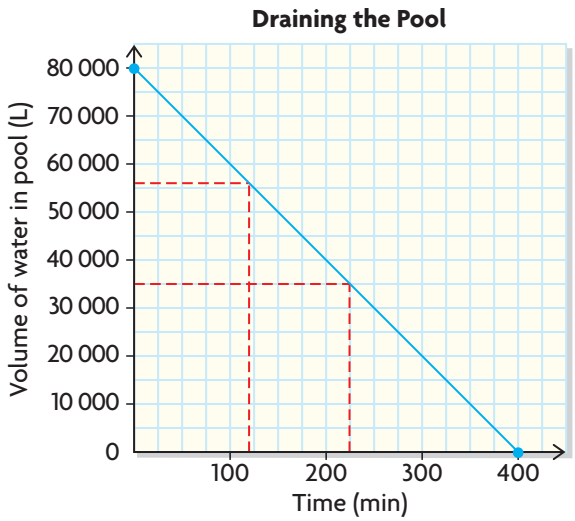
- Use a graph to describe and interpret experimental data
- Draw an appropriate line or curve of best fit
- Determine the equation of a line of best fit
- Identify a trend in a data set and formulate a conjecture
- Describe a situation that is explained by the events shown in a given graph

? The graph shows the population of African elephants for a few years. **If the pattern continues, approximately when might African elephants become extinct?**

6 Getting Started

WORDS YOU NEED to Know

1. The graph shows the amount of water that remains in a pool that is being drained at a constant rate.



Match each phrase to the mathematical term that best represents it.

a) x-intercept
b) y-intercept
c) slope
d) partial variation
e) independent variable
f) dependent variable

i) The amount of water in the pool to start.
ii) The rate at which the volume of the water in the pool is changing.
iii) The time at which the water volume in the pool is recorded.
iv) The amount of water in the pool at a particular time.
v) The time at which the pool is empty.
vi) The kind of algebraic relation that describes how the amount of water in the pool changes with time.

SKILLS AND CONCEPTS You Need

Determine the equation of a line given the slope and *y*-intercept

The equation of a line can be determined by $y = mx + b$, where m represents the value of the slope and b represents the value of the y-intercept.

EXAMPLE

Determine the equation of the line with slope 5 and y-intercept 3.

Solution

$m = 5$ and $b = 3$, so the equation is $y = 5x + 3$.

2. Determine the equation of the line with the following slopes and y-intercepts.

a) slope $= \frac{2}{3}$; y-intercept $= 4$

b) slope $= -2$; y-intercept $= -2.5$

Study Aid

- For help, see Lesson 5.4, Examples 1 and 2.

Determine the equation of a line given the slope and a point on the line

Substitute the value for the slope into m and the coordinates of the given point for x and y in the equation $y = mx + b$. Then, solve the equation for b.

EXAMPLE

Determine the equation of the line with slope 2 that passes through the point (3, 5).

Solution

Substitute $m = 2$, $x = 3$ and $y = 5$ into $y = mx + b$.

$$5 = 2(3) + b$$
$$5 = 6 + b$$
$$-1 = b$$

The equation of the line is $y = 2x - 1$.

3. Determine the equation of each line.

a) slope $= -\frac{3}{5}$; passes through (5, 2)

b) slope $= 1.8$; passes through (10, –1)

Determine the equation of a line through two given points

Use the coordinates of the points to determine the slope of the line. Then, use the slope and one of the points to determine the y-intercept (as shown in the previous example).

EXAMPLE

Determine the equation of the line that passes through the points (–5, 5) and (5, 7).

Solution

Calculate the slope.

$$\begin{aligned} m &= \frac{7 - 5}{5 - (-5)} \\ &= \frac{2}{10} \\ &= \frac{1}{5} \end{aligned}$$

Substitute $m = \frac{1}{5}$ and the coordinates of (–5, 5) into $y = mx + b$.

$$5 = \left(\frac{1}{5}\right)(-5) + b$$

$$5 = -1 + b$$

$$6 = b$$

The equation of the line is $y = \frac{1}{5}x + 6$.

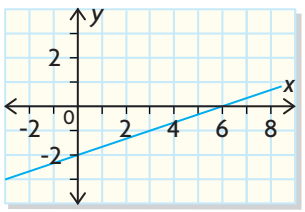
4. Determine the equation of each line.

a) passes through (1, 1) and (–3, 4)

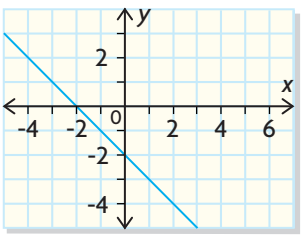
b) passes through (5, 8) and (6, 6)

5. Determine the equation of each line.

a)



b)



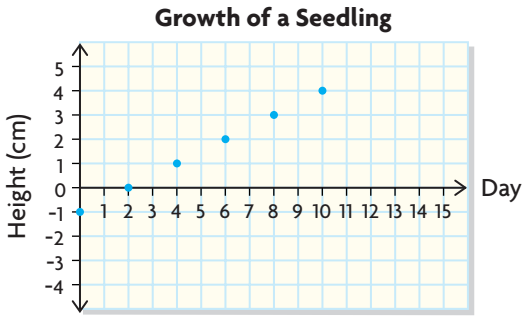
PRACTICE

6. Draw a graph of each equation.

a) $y = 2x + 3$

b) $y = -3x + 4$

7. A seed is placed 1 cm below the surface of the soil. The graph shows the height of the sprout as it grows.



a) Determine an equation for the graph.

b) What does the y-intercept represent?

c) Determine the rate at which the seedling is growing.

d) What does the x-intercept represent?

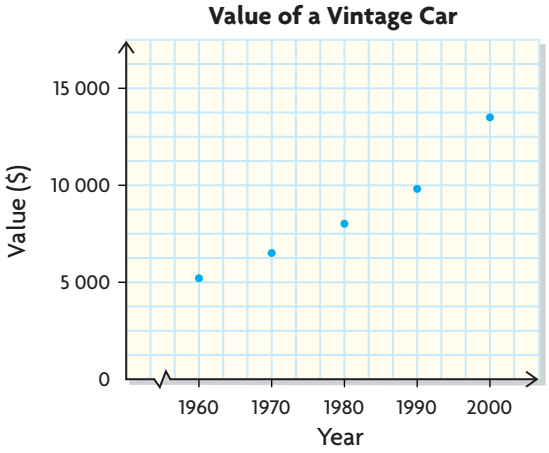
e) What do negative values of y represent? What do positive values of y represent?

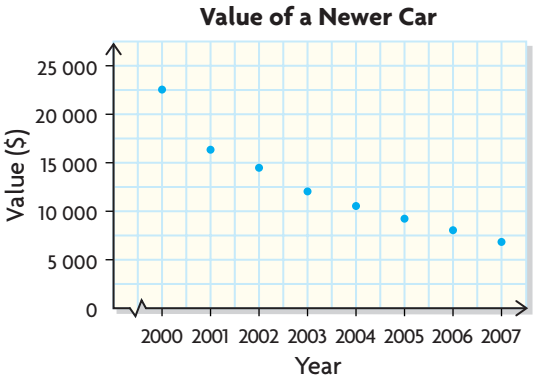
8. The graphs below show how the values of certain cars change with time.

a) For each graph, describe the changes in words.

b) Use the graph to estimate the value of the newer car in 2008.

c) Use the graph to estimate when the value of the vintage car was $9000.



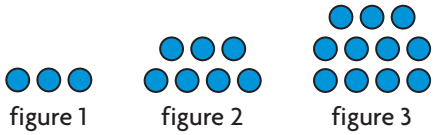


Study Aid

- For help, see the following examples.

Question	Lesson
6	3.4 Examples 2 and 3
7	3.3 Examples 1 and 2
8	3.4 Example 2
9	3.5 Example 1

9. Consider the sequence of figures.



a) State the pattern rule in words.

b) Construct a table of values for the number of circles in each figure. The first two rows are completed for you.

Figure Number	Number of Circles
1	3
2	7
3	
4	

c) Write a formula that you can use to predict the number of circles if you know the figure number.

d) Calculate the first differences and include them in another column of your table of values.

e) If you plotted a graph, would it be linear? How can you tell?

f) Plot the data on a scatter plot.

10. Several students collected this data for 12 triangles. Use it to determine what type of relationship exists between the hypotenuse of a right triangle and the size of one of the acute angles, when the base is fixed at 10 cm.

Angle (°)	5	10	15	20	25	30	35	40	45	50	55	60
Height (cm)	0.9	1.8	2.7	3.6	4.8	5.8	7.0	8.4	10	11.9	14.3	17.3

11. You are given the coordinates of two points on a line. Create a flowchart that summarizes the sequence of steps you would follow to determine the equation of this line.

APPLYING What You Know

Comparing Costs

The table shows how much a traveller paid for a hotel room and dinner in nine different cities.

City	Cost of Hotel Room ($)	Cost of Dinner ($)
1	48	20
2	50	21
3	69	19
4	67	11
5	91	39
6	72	45
7	57	30
8	125	19
9	63	33

? Is there a relationship between the cost of a hotel room and the cost of a dinner?

A. Draw a scatter plot of the data. Use "cost of hotel room" as the independent variable and "cost of dinner" as the dependent variable.

B. Look at the point for City 3. What do the coordinates of the point represent?

C. Describe any pattern that you observe in the data.

D. Identify any points that do not fit the pattern.

E. Suppose you use the graph to predict the cost of a dinner in a city where the hotel room cost is $80. How accurate do you expect your prediction to be?

F. Use the table of values and your scatter plot to help you describe the relationship between the cost of a hotel room and the cost of dinner for these nine cities.

6.1 Interpreting Data

YOU WILL NEED

- ruler and metre stick, or measuring tape
- grid paper

GOAL

Plot and interpret experimental data.

INVESTIGATE the Math

A character on a TV crime show predicted the height of a suspect based on hand span. Robin wants to find out what the relationship is.

? How is hand span related to height?

A. Measure your hand span.

B. Measure your height.

C. Gather the data for all members of your class and put it in a table.

D. Choose one variable as the independent variable and the other as the dependent variable.

Draw a scatter plot to represent the data.

E. Would you say the variables are **continuous** or **discrete**?

F. Are there any data points that don't fit the pattern? If so, explain.

G. How does the scatter plot suggest how hand span and height are related?

Reflecting

H. How did making a scatter plot from your data table help you to determine whether hand span and height are related?

I. Did your choice for the dependent variable in part D affect your conclusion about whether there is a relationship between hand span and height? Explain.

APPLY the Math

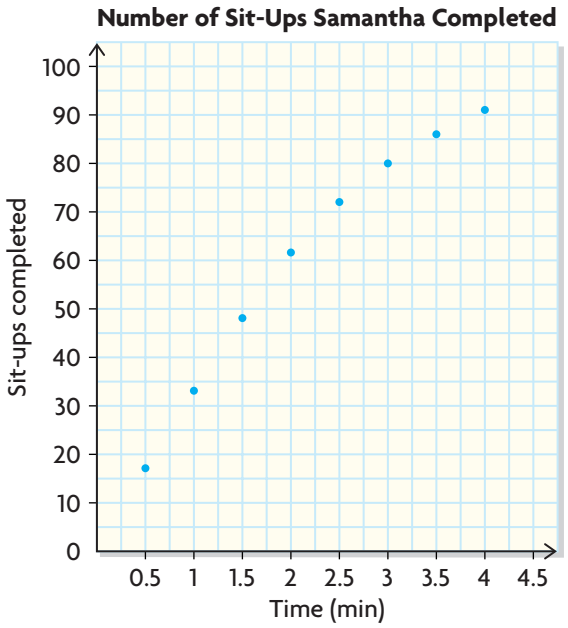
EXAMPLE 1 Representing and interpreting discrete data

The table below shows how many sit-ups Samantha did in gym class.

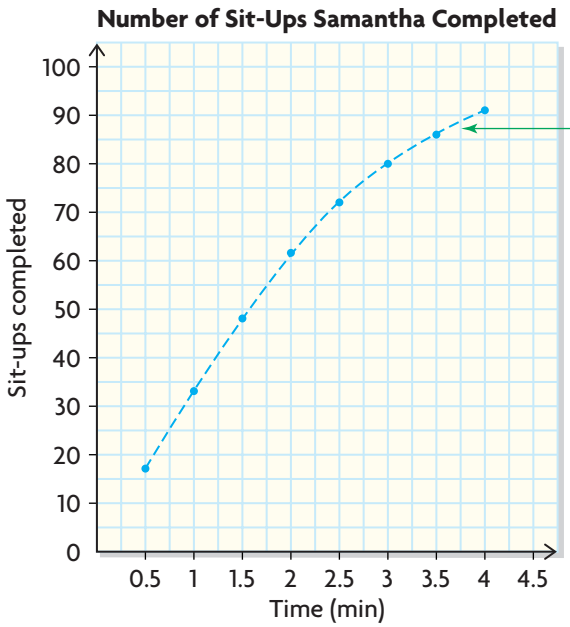
Time (min)	0.5	1	1.5	2	2.5	3	3.5	4
Sit-Ups Completed	17	33	48	62	72	80	86	91

Describe the relationship between completed sit-ups and time.

Hiro's Solution: Reasoning from a hand-drawn scatter plot



I plotted the data on a scatter plot. I chose "sit-ups completed" as the dependent variable, since the number of sit-ups that Samantha does depends on how much time she takes.



I joined the points with a dashed line. The "sit-ups completed" variable is discrete because you can only do a whole number of sit-ups.

The line is nearly straight at the beginning, but bends at the end.

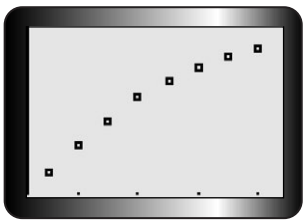
At first, the number of sit-ups goes up by almost the same amount in each half minute.

Toward the end, she must be getting tired because she does fewer sit-ups every half minute.

Nadine's Solution: Reasoning from a graph drawn using technology

Tech *Support*

- For help using your calculator to create a scatter plot, see Appendix B-9.



I plotted the data using the Lists in my graphing calculator. I entered "time" into L1 as the independent variable. I entered "sit-ups completed" into L2 as the dependent variable, since the number of sit-ups that Samantha does depends on how much time she takes.

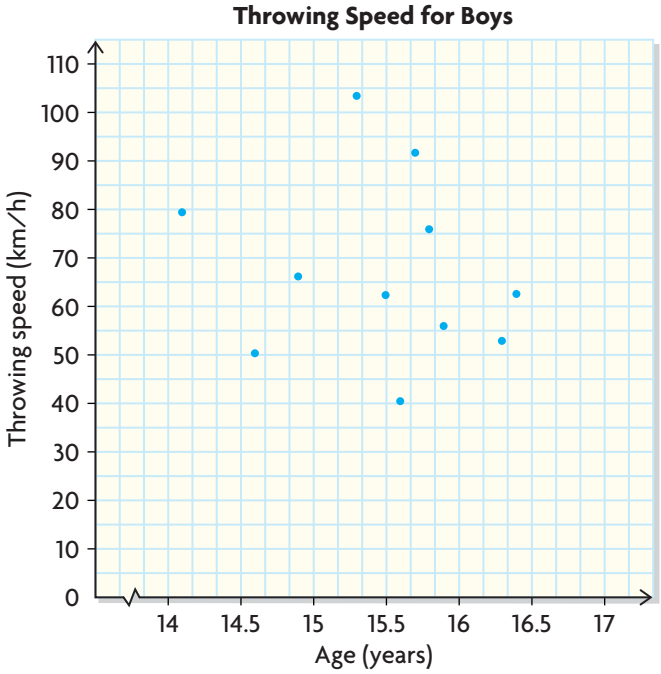
The screen shows the same relationship that Hiro found plotting the data by hand.

EXAMPLE 2 Representing and interpreting continuous data

Students in Grade 9 and Grade 10 are trying out for the junior boys' baseball team at their school. The speeds of their pitches were measured with a hand-held radar gun and are shown in the table below. Determine if there is a relationship between throwing speed and age.

Age (years)	14.1	14.6	14.9	15.3	15.5	15.6	15.7	15.8	15.9	16.3	16.4
Throwing Speed (km/h)	79.3	50.2	66.1	103.3	62.3	40.4	91.6	75.8	55.9	52.7	62.4

Jay's Solution



I didn't know which variable to choose as the independent variable. I remembered that time is usually on the horizontal axis, so I chose "age" as the independent variable.

The data are continuous. I know because any speed and any age between the ones in my data set are valid.

I plotted the data.

There doesn't seem to be any relationship between age and throwing speed for this set of data.

The data points are really scattered. I couldn't see a simple pattern and didn't even try to join data points.

In Summary

Key Ideas

- You can use a table of values to organize numerical data collected from an experiment.
- If the data points on the scatter plot seem to follow a predictable pattern, you might suggest that there is a relationship between the variables.
- Often, the purpose of an experiment is to determine whether the values of the dependent variable actually do depend on the values of the independent variable.

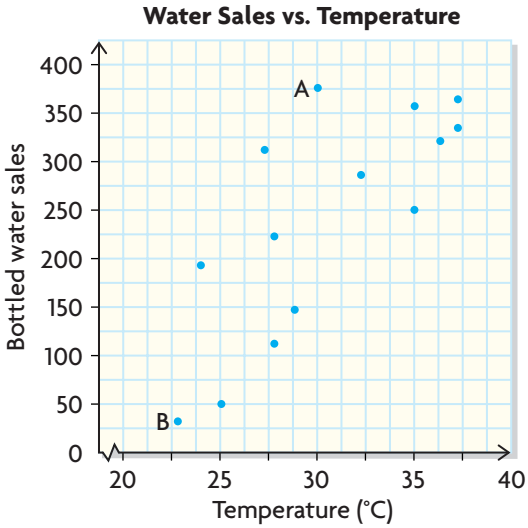
Need to Know

- In some cases, either variable could be chosen as the independent variable, depending on your point of view.
- Sometimes, the points in a scatter plot are approximated by a line or smooth curve. The line or curve may help you see if there is a relationship between the variables.

CHECK Your Understanding

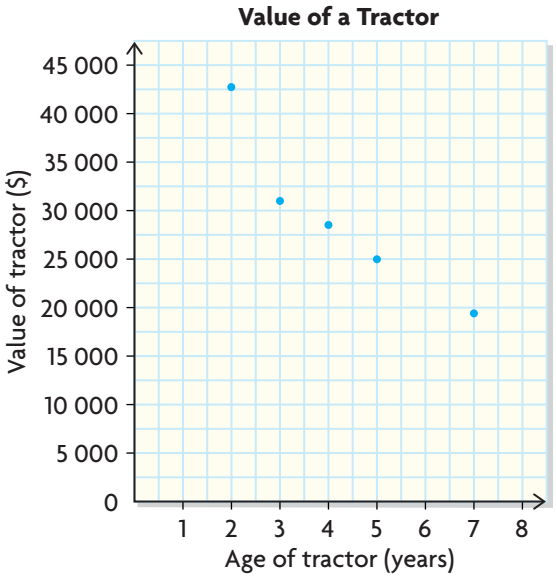
1. Suppose you were to survey your classmates to see if there is a relationship between math marks and the number of hours spent watching TV.
   a) What column headings would you use in a table of values designed to organize the data from your survey?
   b) Which variable would you choose as the independent variable? Explain.
   c) How would you interpret the ordered pair (2, 65) if it were to appear on a scatter plot of your data?

2. The scatter plot shows the sales of bottled water at a refreshment booth at the Canadian National Exhibition in Toronto for different days during a heat wave one summer.
   a) What information does point A represent? What does point B represent?
   b) What does the scatter plot show about the relationship between water sales and temperature?



PRACTISING

3. The scatter plot shows the ages of some tractors and their values.



a) Identify the independent variable and the dependent variable.
b) Would you consider the variables to be discrete or continuous? Would you use a dashed line or a solid line to join the points?
c) Does the scatter plot suggest a relationship between the age of a tractor and its value? Explain.

4. These data show the heights of some Grade 9 boys and their fathers.

K

Height of Grade 9 Boy (cm)	164	168	150	162	159	165	187	152	180	166	148	159
Height of Father (cm)	171	186	164	180	176	177	192	167	189	180	165	172

a) Identify the independent variable and the dependent variable.
b) Would you consider the variables to be discrete or continuous?
c) Would you use a dashed line or a solid line to join the points?
d) Construct a scatter plot for the data.
e) Does the scatter plot suggest a relationship between a boy's height and his father's height? Explain.
f) Is there is a relationship between the variables? Suggest reasons for this.

Use the data below for questions 5 and 6.

Countries and Films Represented at the Toronto International Film Festival

Year	2006	2005	2004	2003	2002	2001	2000
Countries	61	52	55	50	50	54	56
Films	352	335	336	339	345	326	329

5. a) Draw a scatter plot of the number of countries represented each year.
b) Describe any patterns you see in the scatter plot of part a).
c) Draw a scatter plot of the number of films shown each year.
d) Describe any patterns you see in the scatter plot of part c).
e) For each scatter plot, would you consider the variables to be discrete or continuous? Would you use a dashed line or a solid line for the graph?

6. a) Draw a scatter plot using the number of countries and the number of films.
b) Explain how you chose the independent and dependent variables.
c) Describe any patterns you see in the scatter plot.

7. The amount of fuel a hybrid car uses is measured at various speeds as
A shown in the table below.

Speed (km/h)	3	8	11	16	21	26	32	40	50	60	64	67	71	80	90	100	110
Fuel Consumption (L/100 km)	14.9	5.3	4.7	3.8	3.5	3.3	3.2	3.1	3.1	3.1	3.1	3.8	3.9	4.1	4.4	4.9	5.3

a) Draw a scatter plot of the data.
b) Describe any pattern you see.
c) Does the pattern you described in part b) seem reasonable? Explain.
d) Does the pattern you described in part b) suggest how you should drive in order to minimize fuel consumption?
e) Who would want this information? Why?
f) Are the variables discrete or continuous?
g) Which variable did you choose for the independent variable? Explain.

8. This table shows the birth rates in four provinces over the last few years.

T

Number of Births per 1000 People

Year	Alberta	British Columbia	Newfoundland and Labrador	Ontario
2001	12.4	9.8	8.8	10.9
2002	12.8	9.9	8.8	10.8
2003	12.9	9.7	8.9	10.9
2004	12.9	9.7	8.6	10.8
2005	12.7	9.6	8.6	10.6

a) Draw a scatter plot of the number of births for this five-year period for each province on a single grid.

b) Do any provincial data show a strong pattern?

9. Suppose that you have plotted some data on a scatter plot.

C **a)** How could you tell whether there is a relationship between the variables?

b) How could you decide whether to use a solid or dashed line?

Extending

10. The data on the right show the number of car accidents in a year for different age groups.

a) Choose an independent variable and a dependent variable. Explain how you chose.

b) Draw a scatter plot of the data. Use the median age for each age group.

c) Describe any trends you see in the scatter plot.

d) Is it appropriate to connect the plotted points with a line? If so, should the line be solid or broken? Explain.

e) Veera says that young people have more car accidents than old people. Are there sufficient data to support this claim? If not, what further information would be helpful? Explain.

Age Group	Number of Accidents
16–19	6 382
20–24	7 183
25–34	11 733
35–44	8 990
45–54	5 517
55–64	3 307
65–74	2 308

6.2 Lines of Best Fit

YOU WILL NEED

- ruler
- grid paper

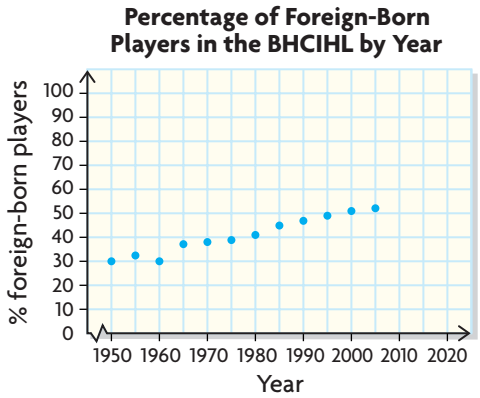
GOAL

Sketch a line of best fit for a given set of data and determine the equation of the line.

LEARN ABOUT the Math

Over the past half century, the percentage of foreign-born players in the Black Horse Corners International Hockey League (BHCIHL) has increased. The data are summarized in the table and on the scatter plot.

Year	Percentage of Foreign-Born Players in the BHCIHL
1950	30
1955	33
1960	30
1965	37
1970	38
1975	39
1980	41
1985	45
1990	47
1995	49
2000	51
2005	52

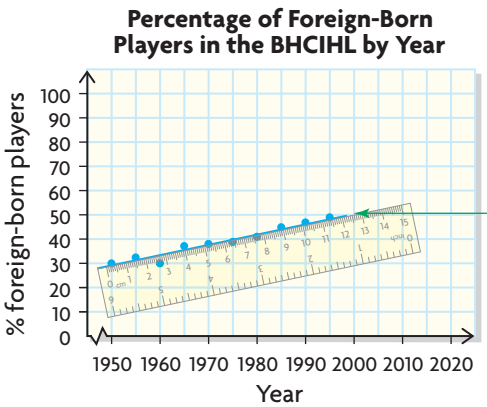


? How might the percentage of foreign-born players in BHCIHL in the year 2020 compare with the percentage in 1993?

EXAMPLE 1 Representing a situation using a line of best fit

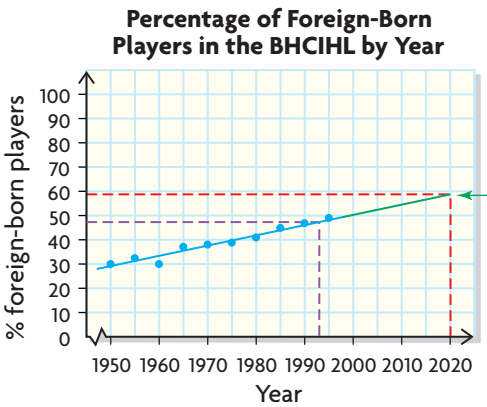
Use the scatter plot to estimate, and then, compare the number of foreign-born players in 2020 and in 1993.

Ryan's Solution: Using a graphing strategy



I drew a **line of best fit** to help me see the **trend**. I placed my transparent ruler over the plotted points so that most of them were close to the edge. I tried to "balance" the points on either side of the line.

I can see that the percentage of foreign-born players in the league is increasing as time passes.



I assumed that this increasing trend would continue and I extended the line of best fit until it went to 2020.

If the trend continues, the percentage of foreign-born players in the BHCIHL in 2020 will probably be about 59%.

I used the extended line of best fit to **extrapolate** the percentage for 2020.

The percentage of foreign-born players in the BHCIHL in 1993 was probably about 48%.

I used the line of best fit to **interpolate** the percentage for 1993.

Based on the trend I see, the number of foreign-born players in the BHCIHL will likely be quite a bit higher in 2020 than it was in 1993.

line of best fit

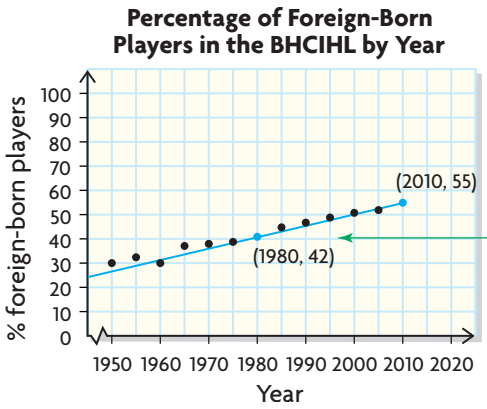
a line that best describes the relationship between two variables in a scatter plot

trend

a relationship between two variables for which the independent variable is time

Omar determined an equation to describe the relationship between the percentage of foreign-born players and time. He then used his equation to make a prediction for the year 2020.

Omar's Solution: Using an algebraic strategy



I used a transparent ruler to draw a line of best fit.

$$(x_1, y_1) = (1980, 42),$$
$$(x_2, y_2) = (2010, 55)$$

I wrote down the coordinates of two points that lie on the line of best fit.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{55 - 42}{2010 - 1980} \\ &= \frac{13}{30} \end{aligned}$$

I used the coordinates to calculate the slope of the line.

$$\begin{aligned} y &= mx + b \\ y &= \left(\frac{13}{30}\right)x + b \\ 55 &= \left(\frac{13}{30}\right)(2010) + b \\ 55 &= 871 + b \\ b &= 55 - 871 \\ b &= -816 \\ y &= \frac{13}{30}x - 816 \\ y &= \left(\frac{13}{30}\right)(2020) - 816 \\ &\doteq 59.3 \end{aligned}$$

I wrote the equation for the line of best fit. I used the coordinates of a known point on the line, (2010, 55), to determine the value of b.

I wrote the final equation for the line of best fit by substituting the slope m and the y-intercept b.

I substituted the value $x = 2020$ to estimate the percentage of foreign-born players that will be in the BHCIHL by the year 2020.

I estimate that in 2020, about 59.3% of players in the BHCIHL will have been born outside of Canada.

$$y = \left(\frac{13}{30}\right)(1993) - 816$$
$$\doteq 47.6$$

I substituted the value $x = 1993$ to estimate the percentage of foreign-born players that were in the BHCIHL in 1993.

I estimate that in 1993, about 47.6% of players in the BHCIHL had been born outside of Canada.

If the trend continues, in 2020 there will be almost 12% more foreign-born players in the league than there were in 1993.

These values are reasonable, since they are very close to the values that I could read off the graph of the line of best fit.

Reflecting

A. Different students may use their rulers to draw different lines of best fit. How might that affect their estimates?

B. Do Ryan's and Omar's solutions seem to give similar results? Explain.

C. Which boy's strategy would you choose? Why?

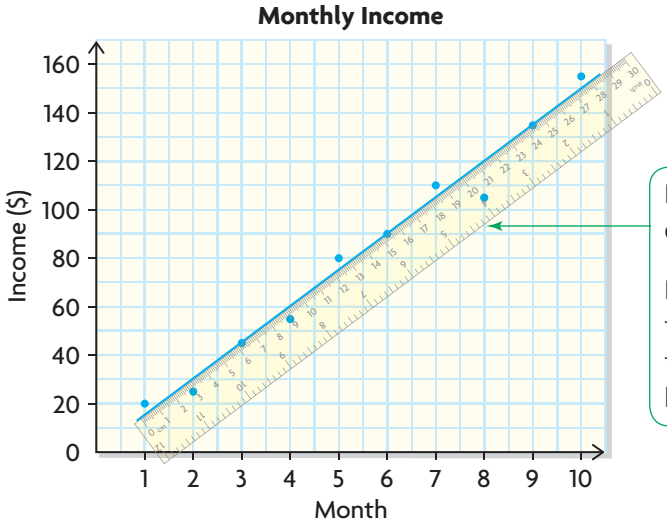
APPLY the Math

EXAMPLE 2 Using a line of best fit to describe a trend

Kajsa and Erika make bead jewellery in their spare time. Their monthly income for 10 consecutive months is shown in the table. Describe the trend in their income.

Month	1	2	3	4	5	6	7	8	9	10
Income ($)	20	25	45	55	80	90	110	105	135	155

Kylie's Solution: Using a graphical representation



I sketched the data on a scatter plot.

I used a transparent ruler to draw a line of best fit.

From the scatter plot and the line of best fit, the trend is that the monthly income increases steadily.

Jasper's Solution: Using an algebraic representation

I sketched a line of best fit.

$$(x_1, y_1) = (1, 20),$$
$$(x_2, y_2) = (6, 90)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{90 - 20}{6 - 1}$$
$$= 14$$

I chose two points on the line and used their coordinates to calculate the slope of the line.

Their monthly income is increasing by \$14 each month.

The slope is the rate of change in their income each month.

$$y = mx + b$$
$$y = 14x + b$$
$$20 = 14(1) + b$$
$$20 = 14 + b$$
$$20 - 14 = b$$
$$6 = b$$
$$y = 14x + 6$$

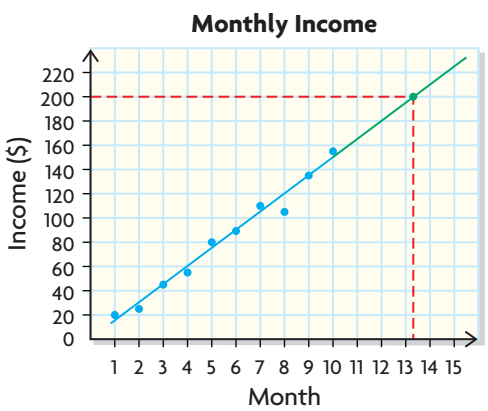
To determine b, I substituted 14 for m and the coordinates of the point (1, 20) into the equation for the line of best fit.

The equation of the line of best fit has a positive slope, so the trend for Kajsa and Erika's income is that their monthly income is increasing steadily.

EXAMPLE 4 Using a line of best fit to solve a problem

Using the data in Example 3, estimate when Kajsa and Erika's income will reach \$200 for a month.

Mathieu's Solution: Using a graphing strategy



I sketched a line of best fit.

I located the *x*-coordinate of the point on the line that represents an income of \$200.

The x-value of this point is about $x = 13$.

Kajsa and Erika's income will reach \$200 after about 13 months.

I rounded the value of x because only whole numbers make sense for this situation.

Cassandra's Solution: Using an algebraic strategy

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{150}{10}$$

$$y\text{-intercept} = 0$$

I used Mathieu's scatter plot and line of best fit to estimate its slope and y-intercept.

$$y = 15x$$

$$200 = 15x$$

$$x = \frac{200}{15}$$

$$\doteq 13.3$$

I used this information to write the equation of this line.

I substituted 200 for y and solved for x.

Kajsa and Erika's income will reach \$200 after about 13 months.

I rounded the value of x because only whole numbers make sense for this situation.

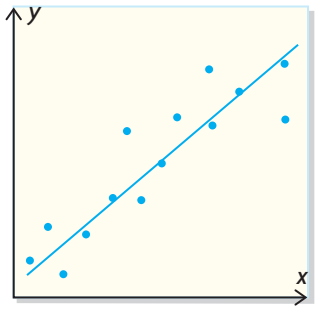
In Summary

Key Ideas

- You can use a line of best fit to make predictions for values not actually recorded or plotted. This is done by interpolating or extrapolating.
- Predictions can be made by reading values off a graph, or by using an equation of the line of best fit.

Need to Know

- If the pattern of points on a scatter plot looks like it follows a straight line, a line of best fit can be used to represent the relationship between the variables.
- When you draw a line of best fit, the points on the scatter plot should be "balanced" on each side of the line.
- You can use the coordinates of two points on the line of best fit to determine its slope and its equation.

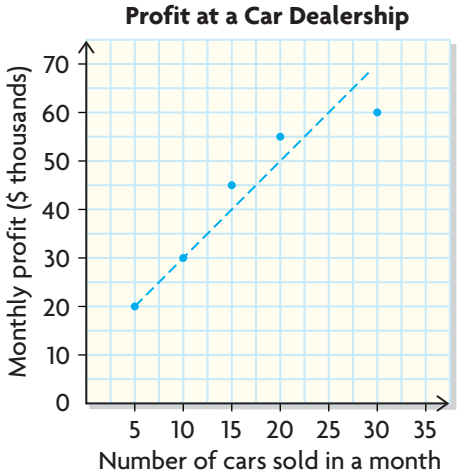


CHECK Your Understanding

1. This scatter plot shows the monthly profit for a car dealership when a certain number of cars are sold.

a) Use the graph to estimate the monthly profit in a month where 23 cars are sold.

b) Use the graph to estimate the number of cars sold in a month where the profit is $67 000.



2. The table shows temperatures at various times of the day.

Time (p.m.)	2	3	4	5	6	7
Temperature (°C)	−1	2	2.5	4	6.5	9

a) Construct a scatter plot for the data in the table.
b) Sketch a line of best fit.
c) Determine an equation for the line of best fit.
d) Predict the value of the temperature at 5:30 p.m.
e) Predict the time when the temperature is 8 °C.

PRACTISING

3. In this table, x represents the number of people enrolled in various classes at a health club, and y represents the number in each class that are male.

x	19	10	6	16	15	9	12	21
y	10	4	2	5	7	3	8	8

a) Construct a scatter plot for the data.
b) Sketch a line of best fit.
c) Use the line of best fit to estimate the value of y when $x = 14$.
d) Use the line of best fit to estimate the value of y when $x = 27$.

4. The following data show the final marks for 10 students in a math class and the average number of hours they studied math per week.

Final Mark	75	81	68	62	88	83	90	77	89	60
Average Number of Study Hours Per Week	3	3	5	1	5	3	6	3	5	2

a) Construct a scatter plot.
b) Sketch a line of best fit.
c) Determine an equation for the line of best fit.
d) Use the equation to estimate the mark for a student who studies an average of 4 h per week.
e) Use the graph to estimate the study time for a student whose final mark is 71.
f) Is there a relationship between final mark and number of hours of study per week? If so, describe it.

5. K A chair company has a contract to build all 1790 seats in a concert hall. The progress over the first week of work is shown in the table.

Number of Days	1	2	3	4	5	6	7
Total Number of Seats Completed	97	204	327	443	539	661	795

a) Estimate the number of seats built after 9 days. How many are built by the middle of day 5?
b) Estimate the number of days needed to build 1252 seats.
c) The company gets a bonus if it is able to finish all of the seats in two weeks or less. If the workers continue to make chairs at about the same rate in the second week, will the company be able to collect the bonus?

6. T Tomas is a member of the school track and field team. His times for running various distances are shown.

Distance (m)	50	100	150	200	250
Time (s)	6.1	12.0	18.3	25.2	31.7

a) Determine an equation for the line of best fit if the data were plotted on a scatter plot.
b) What is Tomas's time for a 175 m run likely to be?
c) Is it reasonable to use the same line of best fit to determine the time needed to run 3000 m? Explain.
d) What does the slope of the line of best fit tell you about how Tomas runs?

7. Kim is on her school basketball team. This table shows her statistics for
A the first 10 games of the season. (Each field goal made counts for two points, and each free throw made counts for one point.)

Game	Minutes Played	Field Goals		Free Throws		Points
		Made	Attempted	Made	Attempted	
1	32	5	13	4	6	14
2	30	4	10	3	3	11
3	24	2	6	1	1	5
4	29	1	3	2	4	4
5	36	3	6	0	1	6
6	19	5	11	2	2	12
7	12	0	3	0	4	0
8	21	1	5	1	2	3
9	18	3	5	1	5	7
10	19	3	7	2	6	8

a) Use a line of best fit to estimate the number of field goals Kim would make if nine were attempted.
b) Use a line of best fit to estimate the number of points Kim would score if she played for 40 min.

8. Suppose that you plotted some data on a scatter plot.
C **a)** How would you draw a line of best fit?
b) How would you determine an equation for the line of best fit?
c) How would you use the line of best fit to interpolate or extrapolate?

Extending

9. The tables at the right show how the world record times for the 100 m sprint have changed over the years for both men and women.
a) Plot both sets of data on the same scatter plot.
b) Describe the trend in each set of data.
c) Sketch lines of best fit for each set of data.
d) Do the lines of best fit in part c) suggest that the women's world record time will someday be less than the men's? If so, predict when this might occur if current trends continue.
e) Do you expect current trends in each data set to continue forever? Explain.

Men:

Year	Time (s)
1960	10.0
1968	9.95
1983	9.93
1988	9.92
1991	9.86
1994	9.85
1996	9.84
1999	9.79
2005	9.77

Women:

Year	Time (s)
1960	11.3
1968	11.1
1976	11.01
1977	10.88
1983	10.81
1984	10.76
1988	10.49

Curious **Math**

Median-Median Line of Best Fit

You can determine a line of best fit without relying on judging by eye. Here's one way.

Use this set of data.

x	1	2	3	4	6	8	9	10	11
y	2	4	3	6	8	10	9	10	8

Step 1: Group the data into equal thirds in order from least to greatest x-value. The lowest third of the x-values go in the Left group, the highest third go in the Right group, and the middle third go in the Middle group.

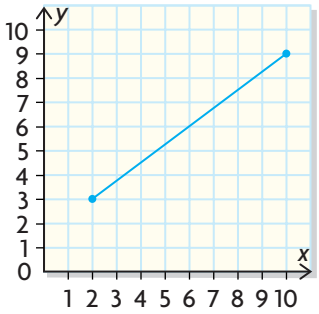
	Left			Middle			Right		
x	1	2	3	4	6	8	9	10	11
y	2	4	3	6	8	10	9	10	8

(If there is one point left over, let the Middle group have an extra point. If there are two points left over, let the Left and Right groups each have one extra point.)

Step 2: Determine the **median** of the x-values for each group (x_L, x_M, and x_R), and the median of the y-values for each group (y_L, y_M, and y_R).

	Left (x_L, y_L)	Middle (x_M, y_M)	Right (x_R, y_R)
Median	(2, 3)	(6, 8)	(10, 9)

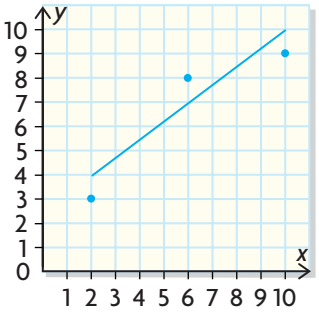
Step 3: Plot the median points (x_L, y_L) and (x_R, y_R). Lightly sketch a line through these two points.



Step 4: Plot the point (x_M, y_M). Move the line from Step 3 vertically one-third of the distance toward (x_M, y_M). The resulting line is called the median-median line of best fit.

For the following data sets,

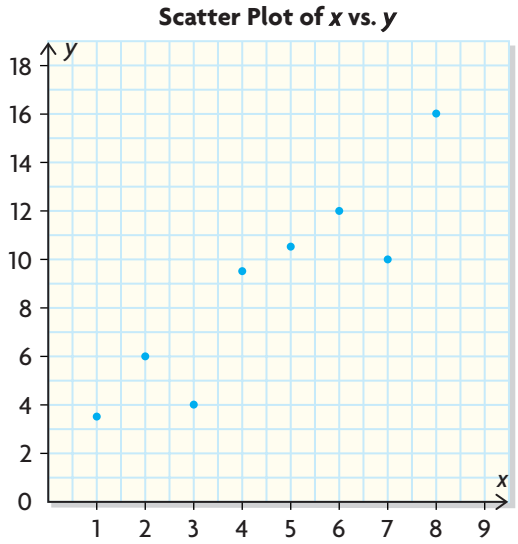
- **i)** Plot all of the points, then draw a line of best fit, as you have been doing throughout the chapter.
- **ii)** On the same grid, draw a median-median line of best fit.
- **iii)** Compare the two lines. Does either line seem to be a better approximation to the data? Explain.
- **a)** (5, 8), (6, 11), (11, 1), (3, 10), (9, 2), (10, 6), (12, 4), (7, 6), (4, 9), (5, 12)
- **b)** (1.3, 1), (2, 2.1), (3, 3), (3.5, 1.5), (5.2, 3), (6, 9), (9.1, 10.8), (10, 4.3), (11.6, 7.0), (12, 9.4)



6 Mid-Chapter Review

FREQUENTLY ASKED Questions

This scatter plot is used for all of the frequently asked questions:



Q: How can you represent numeric data from an experiment involving two variables?

A: If any value between two plotted values of a variable is possible and meaningful, then the variable is continuous. You can organize the data into a table of values. Then, you can use the values to create a scatter plot. If the data points seem to follow a pattern, you can use a line or a curve to represent the pattern.

Use a solid line or curve if both variables are continuous. Use a dashed line or curve if one or both variables are discrete.

Study *Aid*

- See Lesson 6.1, Examples 1 and 2.
- Try Mid-Chapter Review Question 1.

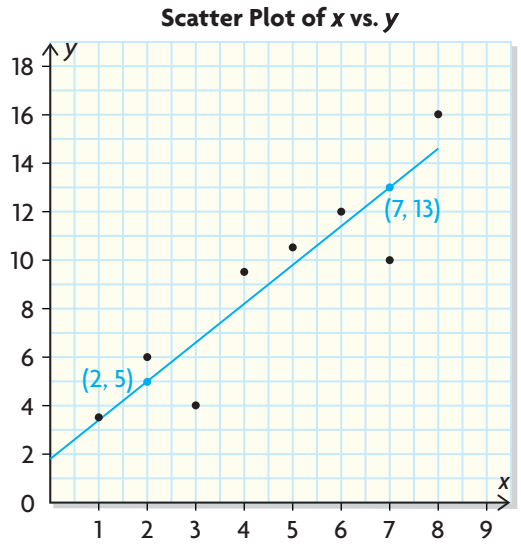
Q: How can you determine a line of best fit?

A: First, plot the data on a scatter plot. Then, position your ruler so that the slope of the line roughly follows the pattern of the plotted points. Also, try to position the ruler so that the plotted points are balanced on either side of the line of best fit. It helps if your ruler is transparent.

Study *Aid*

- See Lesson 6.2, Example 1.
- Try Mid-Chapter Review Question 4.

EXAMPLE



Q: How can you determine an equation for a line of best fit?

A: Choose two points on the line and read off their coordinates. Then, determine the slope and use it and the coordinates of one of the points to determine an equation.

Study **Aid**

- See Lesson 6.2, Example 2.
- Try Mid-Chapter Review Questions 4 and 5.

EXAMPLE

The coordinates of two points on the line of best fit are (2, 5) and (7, 13). The slope of the line is

$$\begin{aligned} m &= \frac{13 - 5}{7 - 2} \\ &= 1.6 \end{aligned}$$

So, the equation of the line is $y = 1.6x + b$. To calculate the value of b, substitute the point (2, 5) into the equation to get

$$\begin{aligned} 5 &= 1.6(2) + b \\ 5 - 3.2 &= b \\ 1.8 &= b \end{aligned}$$

The equation of the line of best fit is $y = 1.6x + 1.8$.

PRACTICE Questions

Lesson 6.1

1. Family doctors record the growth of their young patients. At each checkup, Jennifer's younger sister has her height and mass measured.

Height (cm)	58	60	64	68	73	74
Mass (kg)	5.0	6.3	7.3	8.1	8.8	8.2

a) Graph the data.
b) Should the graph consist of scattered points or can the points be connected by a line? If the points can be connected by a line, then should the line be solid or dashed? Explain.
c) Does the graph show a relationship between height and mass? If so, describe it.

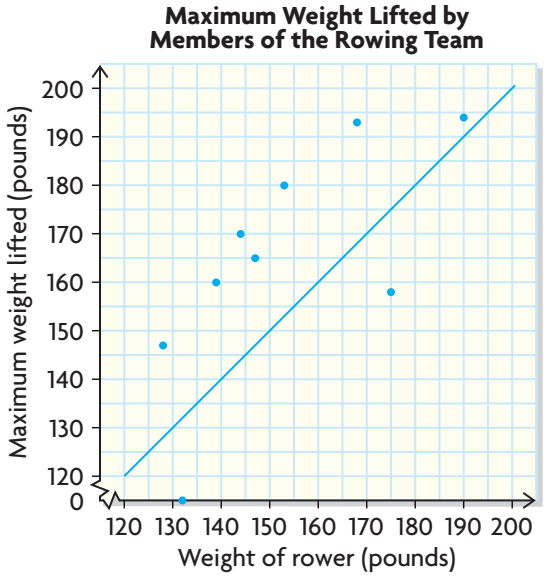
2. The data show how many babies were born at Cook's Mills Hospital on the first 10 days of a month.

Day	1	2	3	4	5	6	7	8	9	10
Number of Births	8	2	5	5	1	9	1	8	7	1

a) Plot the data on a scatter plot.
b) Describe the pattern of the data.
c) Do you think that collecting more data (either including a longer time period or including more hospitals) might change your conclusion in part b)? Explain.

Lesson 6.2

3. The school rowing coach measures the maximum amount of weight that can be lifted by each member of the team.



The results are shown in the above scatter plot. Is the line of best fit appropriate? If so, explain. If not, sketch an appropriate line of best fit.

4. Andrew took a hearing test.

The x-values represent times (in seconds) and the y-values represent loudness levels of a test sound.

x	0	1	2	3	4	5	6	8	10
y	0.0	1.0	1.7	2.1	2.4	2.6	2.8	3.1	3.3

a) Plot the data on a scatter plot.
b) Sketch a line of best fit.
c) Determine an equation for the line of best fit from part b).
d) Estimate the loudness after 3.7 s.
e) If the test followed the same pattern, when would the loudness reach a level of 4.2?

5. Markus and Joelle had the same set of data. Both students created a scatter plot and drew a line of best fit. They then determined the equations of their lines of best fit and compared their results. Should their answers be the same? Explain.

6.3 Curves of Best Fit

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

GOAL

Construct and interpret a curve of best fit for a given set of data.

LEARN ABOUT the Math

Sean and Parminder are studying the motion of a pendulum. Sean says that the pendulum will swing in the same way, no matter how long it is. Parminder is not so sure.

They did an experiment to see whether the length of a pendulum affects its period, which is how long it takes to go back and forth once.

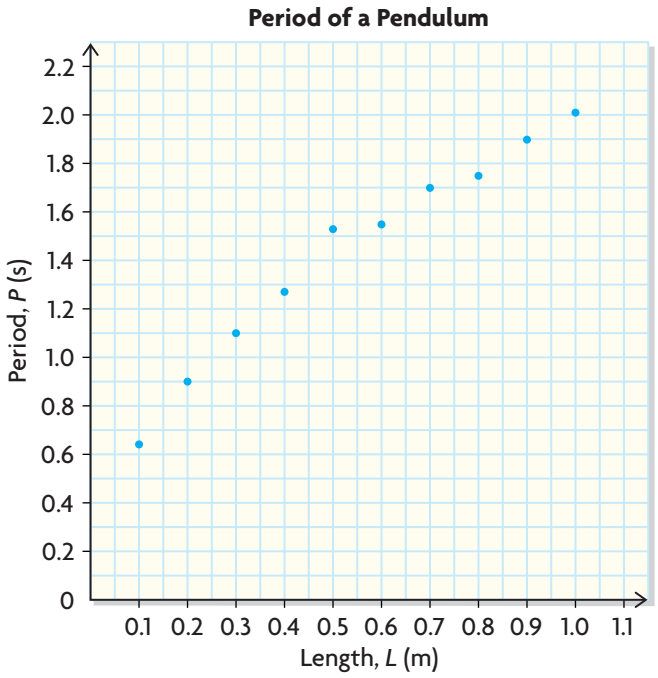
Length, *L*(m)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Period, *P*(s)	0.64	0.90	1.10	1.27	1.53	1.55	1.70	1.75	1.90	2.01

? What is the period when the length is 0.38 m?

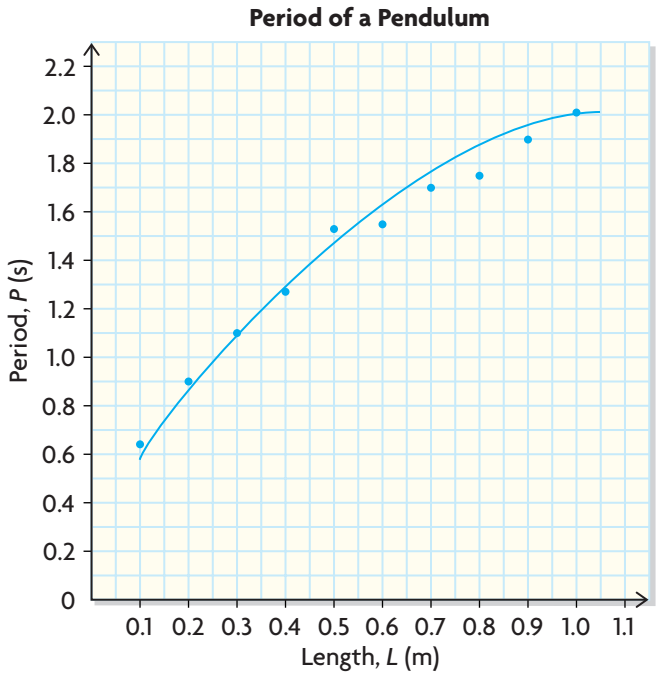
EXAMPLE 1 Solving a problem using a curve of best fit

Estimate the period of a pendulum that has a length of 0.38 m.

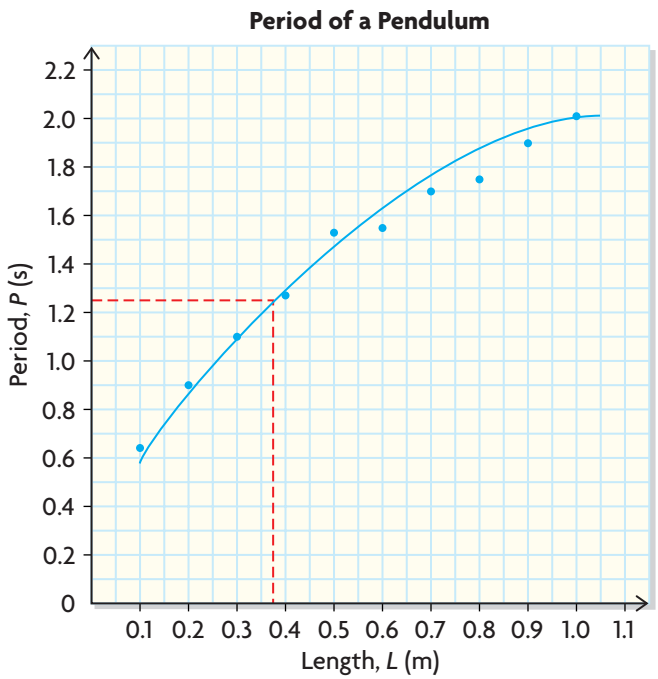
Walid's Solution



Since I want to know if the period depends on the length of a pendulum, I chose length as the independent variable and period as the dependent variable. Then, I created a scatter plot of the data.



The plotted points seem to follow a curve, so I used a smooth curve of best fit instead of a line of best fit. I made the curve solid because the variables are continuous.



I used the curve to estimate the period for a pendulum 0.38 m long.

My estimate for the period is 1.24 s when the length is 0.38 m.

Reflecting

A. How did Walid decide that a curve of best fit is a more appropriate representation than a line of best fit?

B. How did Walid use the curve of best fit to estimate the period when the length is 0.38 m?

C. If you were to draw a curve of best fit, would it be exactly the same as Walid's? If not, how would this change the estimate for the period when the length is 0.38 m?

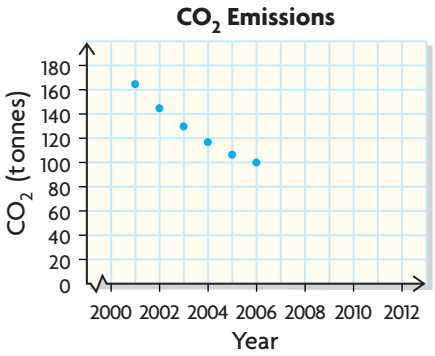
APPLY the Math

EXAMPLE 2 Using a curve of best fit to represent a trend

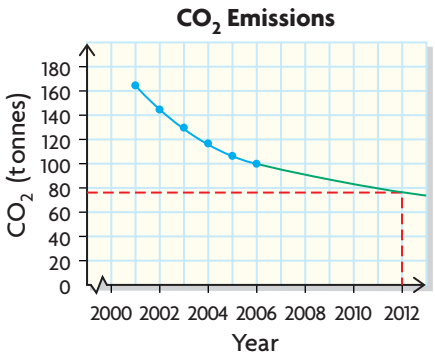
To help protect the environment, a steel factory is thinking about setting new standards for its carbon dioxide (CO_2) emissions. By 2012, the factory wants to emit less than 70 tonnes of CO_2 per year. If it does not change its practices, is the goal realistic?

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

Shannon's Solution



I plotted the data on a scatter plot.



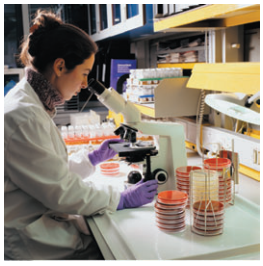
The emissions level for 2012 is estimated to be greater than 70 tonnes. This means the goal will probably not be met by 2012.

I sketched a curve of best fit. I could have used a line of best fit, but since the data seemed to follow a smooth curve, I thought using a curve of best fit would make my estimate more accurate. I used a solid curve because the variables are continuous.

I extended the curve out to the year 2012 assuming that this trend will continue.

I used the extrapolated graph to estimate the amount of CO_2 in 2012.

EXAMPLE 3 Using curves of best fit to reason about a trend



These tables show the population (in thousands) of two different bacterial colonies growing in separate Petri dishes.

Colony 1:

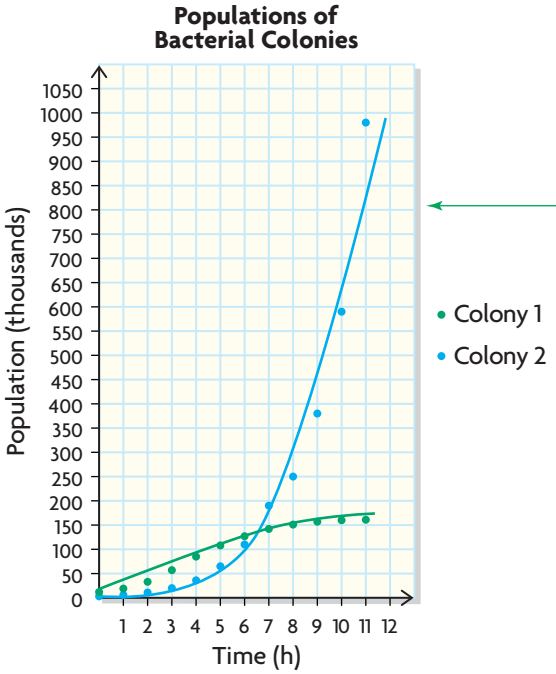
Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Population (thousands)	12	19	33	57	85	108	127	142	151	157	160	161

Colony 2:

Time (h)	0	1	2	3	4	5	6	7	8	9	10	11
Population (thousands)	3	6	11	20	36	65	110	190	250	380	590	980

Compare the growth patterns of the two colonies.

Alexis's Solution



I plotted both sets of data on a single scatter plot so that I could compare them more easily. I sketched each curve so that it was close to most of the plotted points.

Colony 2 keeps growing more and more quickly, but Colony 1 is growing more and more slowly.

Both curves are always going up, so the populations of both colonies are increasing.

Colony 1 grows rapidly at first. But later the curve almost levels off. The colony still grows, but very slowly.
Colony 2 starts off with a lower population than Colony 1 but, because it grows more quickly, at about 6.5 h it catches up and passes Colony 2.

Where the curves are steep, it means that the population is increasing rapidly. Where the curves are less steep, the population is growing slower.

In Summary

Key Idea

- Sometimes a curve represents the trend or pattern in a scatter plot better than a line.

Need to Know

- You can use a curve of best fit to extrapolate and interpolate values.
- Extending a curve involves more guesswork than extending a line, so you can't be as sure of your predictions.
- Sometimes it's not clear whether a curve or line of best fit can be drawn. This could be because there is no relationship between the variables, or it could mean that more data need to be collected.

CHECK Your Understanding

1. Computers use code numbers that are made up of only the digits 0 and 1. The data in the chart represent the number of possible code numbers of each length. (For example, the code numbers of length 1 are 0 and 1, the code numbers of length 2 are 00, 01, 10, and 11, etc.)

Length of Code Number	1	2	3	4	5	6
Number of Possible Code Numbers	2	4	8	16	32	64

a) Plot the data on a scatter plot.
b) Sketch a line of best fit.
c) Sketch a curve of best fit.
d) Which approximates the data better: the line in part b) or the curve in part c)?
e) Is it reasonable to use the line or curve of best fit to estimate the value of y when $x = 3.7$? Explain.

PRACTISING

2. Weekly earnings for the movie *Dude, Where's My Math Book?* in the
K weeks following its opening are listed in the table.

Earnings ($ millions)	9.2	21.4	34.0	25.7	19.6	14.1	13.3	11.2	6.6	3.1
Weeks Since Opening	1	2	3	4	5	6	7	8	9	10

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Should you use a solid curve or a dashed curve? Explain.
c) Does it make sense to use the curve of best fit to estimate the earnings after 3.5 weeks? Explain.
d) Does it make sense to use the curve of best fit to estimate the earnings 20 weeks after opening? Explain.

3. A basketball is dropped from a height of 200 cm. The table shows how high it bounces on each bounce.

Maximum Height (cm)	200	120	72	44	26	16	10	6	4
Bounce Number	0	1	2	3	4	5	6	7	8

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Should you use a solid curve or a dashed curve? Explain.
c) Does it make sense to use the curve of best fit for interpolation? Explain.
d) Does it make sense to use the curve of best fit for extrapolation? Explain.

4. C In order to obtain a medical image of a patient's thyroid gland, a chemical is injected into the patient's bloodstream. The chemical's concentration in the blood gradually decreases with time.

Concentration (mg/L)	29.0	15.0	7.7	3.9	2.1	1.3	0.7	0.5	0.4
Time (h)	0	1	2	3	4	5	6	7	8

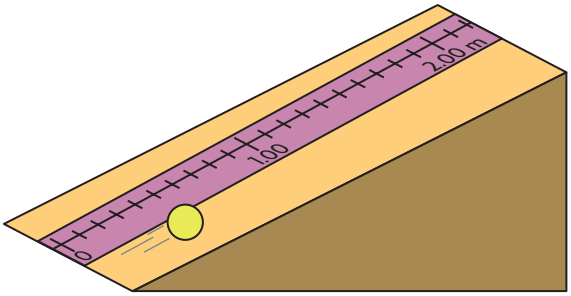
a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Did you use a solid curve or a dashed curve? Explain.
c) Describe the relationship between the variables.
d) Use your curve of best fit to estimate when the concentration of the chemical will be 6.1 mg/L.
e) Use your curve of best fit to estimate the concentration of the chemical after 12 h.

5. The ages and resting heart rates for some people are listed in the table.

Age (years)	21	24	26	29	31	35	39
Resting Heart Rate (beats per minute)	60	61	63	65	68	73	78

a) Plot the data on a scatter plot.
b) Sketch a curve of best fit. Did you use a solid curve or a dashed curve? Explain why.
c) Describe the relationship between the variables.
d) Does it make sense to use the curve of best fit for interpolation? Explain.
e) Does it make sense to use the curve of best fit to estimate the resting heart rate for an 85-year-old person? Explain.

6. In his experiments to study the Earth's gravity, Galileo rolled objects
A on inclined planes. In one such experiment, a ball is rolled up a plane, and then, rolls back down. The data are in the following table.



Time (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Position (m)	0	1.13	1.50	1.88	2.00	1.88	1.50	1.13

a) Use a graph to estimate the position of the ball after 0.3 s.
b) Use a graph to estimate when the ball will return to the bottom of the inclined plane.

7. The table shows the population of a bacterial colony growing in a test tube at various times.

Time (h)	0	1	2	3	4	5	6	7	8
Population (thousands)	1	1.4	2.0	2.7	3.8	5.4	7.5	10.5	14.8

a) Use a graph to describe the growth of the colony.
b) Use a graph to estimate the population of the colony after 7.5 h.

8. A herd of caribou is moved to a small, remote island where they have no predators. Data on the population of the herd were collected for 6 years.

Time (years)	0	1	2	3	4	5	6
Population	24	35	51	74	104	151	225

a) Sketch the data on a scatter plot.
b) Draw a line or curve of best fit through the plotted points. Explain which is more appropriate.
c) Describe the growth of the herd.
d) Predict the population of the herd after seven years.

9. In the Kingdom of Petrodalla, natural gas is the primary resource. The table shows the amount of natural gas produced each year.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Natural Gas Produced (millions of m^3)	1.6	2.1	3.0	4.1	4.3	4.4	3.6	2.1	0.5

a) Sketch the data on a scatter plot.
b) Draw a line or curve of best fit through the plotted points. Explain which is more appropriate.
c) Describe how the production of natural gas changes over time.
d) Predict when natural gas production will decrease to zero.

10. T Consider the carbon dioxide emission data from Example 2, which are repeated here:

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

a) Instead of using a curve of best fit, as is done in the example, use a line of best fit to estimate carbon dioxide emissions in 2012.
b) Compare your estimate in part a) with the result of Example 2. Which result do you think is more reliable? Justify your choice.

11. a) How do you know when to use a curve of best fit, and when to use a line of best fit?
b) How can you use a curve of best fit for interpolation or extrapolation if you don't know the equation for the curve?
c) How do you know how far you can reasonably extrapolate?

Extending

12. Consider the carbon dioxide emission data in Example 2, which are repeated here.

CO_2 (tonnes)	165	145	130	117	107	100
Year	2001	2002	2003	2004	2005	2006

a) Suppose that the carbon dioxide emissions are reduced in each year after 2006 by a constant rate of 10% per year. Will the emission target of no more than 70 tonnes be reached by 2012?
b) Repeat part a) if the reduction is a constant 5% per year.
c) What is the minimum yearly percentage reduction after 2006 that will guarantee that the emission target will be reached by 2012?

13. The data given in Example 1 are shown in the following table.

Length, *L* (m)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Period, *P* (s)	0.64	0.90	1.10	1.27	1.42	1.55	1.70	1.80	1.90	2.01
Squared Period, P^2 (s^2)	0.41	0.81	1.21							

a) Complete the new table, in which the periods are squared.
b) Plot the squared periods versus length.
c) Draw a line of best fit.
d) Calculate the slope of the line of best fit.
e) It's possible to show that the slope of the line of best fit should be equal to $\frac{4\pi^2}{g}$, where g is the acceleration due to the Earth's gravity. Use this formula and the results of part d) to calculate g.
f) Collect your own data on the period of a pendulum. Then, follow the steps in this exercise to obtain your own experimental value for the acceleration due to earth's gravity. How does your experimental value compare with values obtained in professional experiments?

14. Dorothy has 20 m of fencing to make a rectangular enclosure for her dog.
a) Draw all possible rectangular enclosures using whole numbers from 1 to 9 for the dimensions.
b) Organize the data in table of values with headings Length, Width, and Area.
c) Graph the relation area vs. length. Draw the line or curve of best fit.
d) What dimensions give the maximum area for a perimeter of 20 m.
e) Write an equation that relates the area of the enclosure to its length.

6.4 Reasoning About Data

YOU WILL NEED

- ruler
- grid paper

GOAL

Determine whether a conjecture about the relationship between two variables is valid.

LEARN ABOUT the Math

Adrian and Maya are researching global climate change. Because of what they have heard in the news, they think that the concentration of carbon dioxide (CO_2) in the atmosphere is increasing. They read in a report that, by 2015, the atmospheric concentration of CO_2 will be above 385 parts per million (ppm).

They both found data for the atmospheric concentration of CO_2 in the Northern Hemisphere.

? Do Adrian's and Maya's data sets support the report's conjecture?

EXAMPLE 1 Verifying or rejecting a conjecture

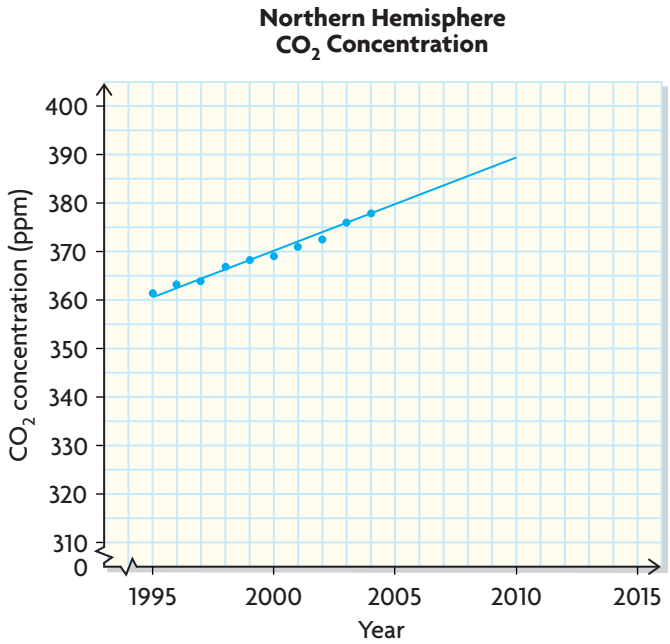
Adrian found yearly data, but Maya found monthly data. Use their data to verify or reject the report's conjecture.

Adrian's Solution: Using a line of best fit to represent yearly data

Adrian finds these data.

Yearly Mean Atmospheric CO_2 Concentration (ppm)

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Atmospheric CO_2 Concentration (ppm)	361	363	364	367	368	369	371	373	376	377



I plotted the data on a scatter plot.

I saw that the data seem to lie almost perfectly on a line, so I sketched a line of best fit.

The line of best fit clearly shows the trend of the data: The carbon dioxide concentration steadily increases.

An equation for the line of best fit is

$$\begin{aligned} m &= \frac{377 - 368}{2004 - 1999} \\ &= 1.8 \end{aligned}$$

I picked two points on the line of best fit. I read their coordinates off the graph. Then, I used the two points to calculate the slope of the line of best fit.

$$\begin{aligned} y &= 1.8x + b \\ 368 &= 1.8(1999) + b \\ b &= -3230.2 \end{aligned}$$

I used the coordinates of one of the points on the line of best fit to calculate the value of b.

$$\begin{aligned} y &= 1.8x - 3230.2 \\ y &= 1.8(2015) - 3230.2 \\ &= 3627 - 3230.2 \\ &= 396.8 \text{ ppm} \end{aligned}$$

I substituted 2015 for *x* so that I could estimate the CO_2 concentration in 2015.

If the pattern continues, the report that Maya read is correct: The atmospheric concentration of CO_2 in the Northern Hemisphere will be above 385 ppm by 2015.

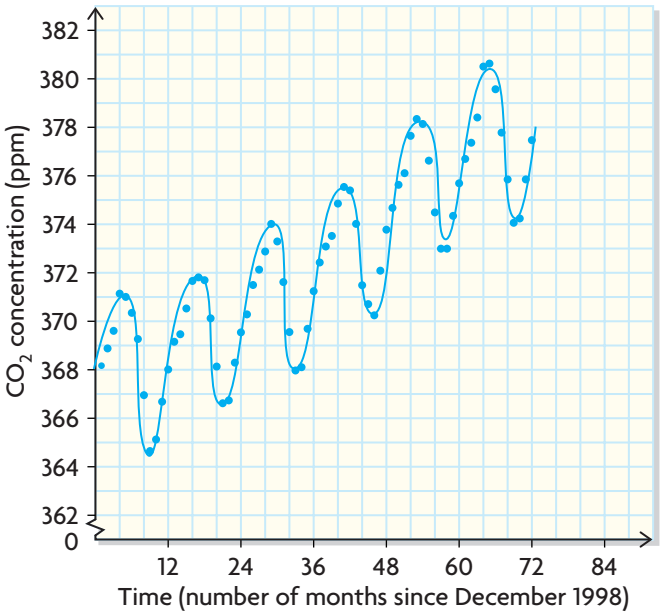
Maya's Solution: Using a curve of best fit to represent monthly data

Maya obtains more detailed data. These data show the monthly variation for several years.

Monthly Mean Atmospheric CO_2 Concentration (ppm)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
1999	368.2	368.9	369.6	371.1	371.0	370.4	369.3	366.9	364.6	365.1	366.7	368.1
2000	369.1	369.5	370.5	371.7	371.8	371.7	370.1	368.1	366.6	366.7	368.3	369.5
2001	370.3	371.5	372.1	372.9	374.0	373.3	371.6	369.6	368.0	368.1	369.7	371.2
2002	372.4	373.1	373.5	374.9	375.6	375.4	374.0	371.5	370.7	370.2	372.1	373.8
2003	374.7	375.6	376.1	377.7	378.4	378.1	376.6	374.5	373.0	373.0	374.4	375.7
2004	376.7	377.4	378.4	380.5	380.6	379.6	377.8	375.9	374.1	374.2	375.9	377.5

Monthly Mean Atmospheric CO_2 Concentration



I plotted the data. I used $x = 1$ for January 1999, $x = 2$ for February 1999, and so on.

Looking at the graph, I didn't think a line of best fit would help me to approximate monthly data.

I drew a curve of best fit instead. It shows a general increasing trend, but there were short-term ups and downs in each year.

There are two trends here.
Year to year, atmospheric CO_2 concentration is increasing.

These data also supported the conjecture. But, since they are more detailed, these data also help show the short-term changes in carbon dioxide concentration.

Within each year, the CO_2 concentration decreases each spring and summer, and then, increases each fall and winter.

Looking at more detailed data gave me new insights that I wouldn't have had from the yearly data alone.

I can't be certain that the report's conjecture is correct.

There are 132 months from December 2004 to December 2015. Making a prediction about that many months in the future is not reasonable.

Reflecting

A. Why do you think Adrian used a line of best fit instead of a curve of best fit?

B. Why do you think Adrian calculated the slope of the line of best fit?

C. Could Maya have used a line of best fit instead of a curve of best fit to show the trend in her data? Explain.

APPLY the Math

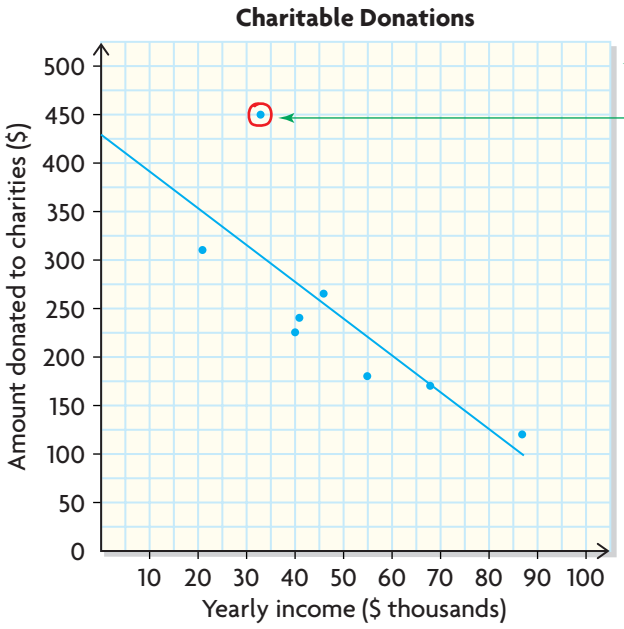
EXAMPLE 2 Reasoning about outliers

Cristina thinks that people with higher incomes donate more money to charities. She collects the following data.

Yearly Income ($ thousands)	21	33	40	41	46	55	68	87
Amount Donated to Charities ($)	310	450	225	240	265	180	170	120

Do the data support Cristina's conjecture?

Carla's Solution



I plotted the data on a scatter plot and sketched a line of best fit.

The circled point is an **outlier** because it is far from the line of best fit.

The line of best fit shows that the pattern of the data is decreasing. For this sample of people, the amount of money donated to charities generally decreases as income increases.

Although there is an outlier, it does not contradict the pattern that the line of best fit shows. This means that the outlier does not support Cristina's conjecture either.

This conclusion is valid for this set of data, and does not support Cristina's conjecture.

These data cannot be used to draw a conclusion about people in general, since the quantity of data is small. We have no way of knowing whether this small set of people is typical or unusual in some way.

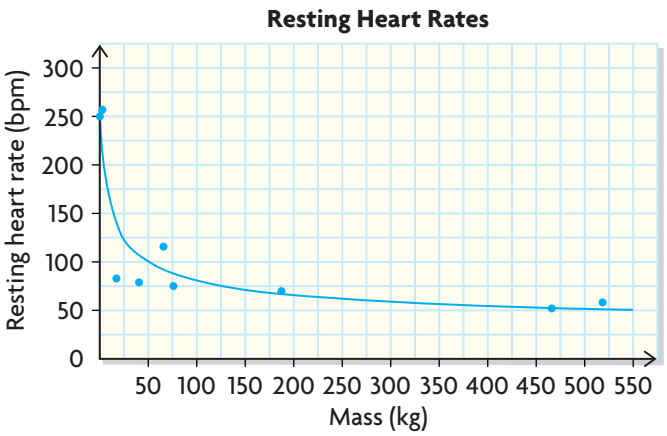
So, in this case, it's not valid to use the line of best fit to make predictions about the general population. More data are needed.

EXAMPLE 3 Supporting a conjecture using a curve of best fit

Alison hypothesizes that adult mammals' resting heart rates, measured in beats per minute (bpm), are lower for larger species. Do the following data support Alison's conjecture?

Animal	Cow	Horse	Pig	Sheep	Sea Lion	Goat	Dog	Rabbit	Rat
Mean Resting Heart Rate (bpm)	58	52	70	75	116	79	83	257	250
Mean Mass (kg)	518	466	188	77.1	66.7	41.8	18.4	3.7	0.5

Chris's Solution



I plotted the data and drew a curve of best fit. I can see from the shape of the curve what the general trend in the data is. The curve is really steep for smaller masses and more gradual for larger masses.

If there were only one data point that showed a heart rate much higher than the rest, I might not have included it in my line or curve. But there are two points like that, so maybe they are part of a pattern and are not outliers.

The data supports Alison's conjecture.

For small masses, as the mass of the species of animal increases, the heart rate decreases by quite a bit. Once the mass reaches a certain value (about 20 kg), as the mass increases, the decrease in heart rate is fairly small.

Since there is a small quantity of data, more data would be helpful in deciding whether the conjecture is valid for a wider range of mammals. It's difficult to tell whether there is a strong pattern because there isn't a lot of data.

EXAMPLE 4 Using a scatter plot to reject a conjecture

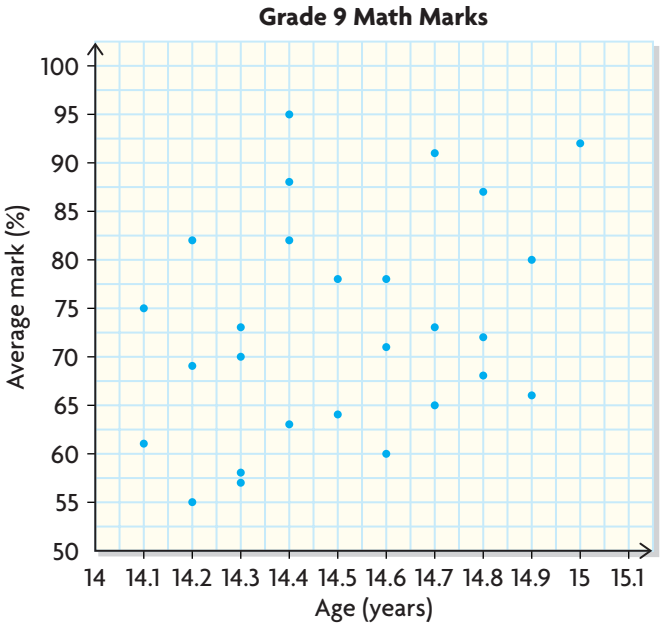
Derek hypothesizes that a student's average mark in Grade 9 is related to his or her age. Analyze the sample of data to see if it supports his conjecture.

Age (years)	14.1	14.2	14.3	14.4	14.5	14.7	14.7	14.8	14.8
Mark (%)	61	82	73	88	64	91	65	87	68

Age (years)	14.1	14.2	14.3	14.4	14.4	14.6	14.6	14.8	14.9
Mark (%)	75	69	58	63	95	71	78	72	66

Age (years)	14.2	14.3	14.3	14.4	14.5	14.6	14.7	14.9	15.0
Mark (%)	55	57	70	82	78	60	73	80	92

Sonia's Solution



I plotted the data.

I can't see how to sketch a line or curve of best fit.

The data are so spread out that there appears to be no pattern. This means that a relationship between age and Grade 9 math marks is unlikely.

The data do not support Derek's conjecture.

In Summary

Key Ideas

- You can support or reject a conjecture about a relationship by examining trends or patterns in data.

Need to Know

- You may see a trend or pattern in plotted data; however, more detailed data might not support your original conjecture.
- If a trend or pattern in the data agrees with a conjecture, then the data support the conjecture.
- If there is no apparent trend or pattern in the data, or if there is a trend or pattern and it contradicts the conjecture, then the conjecture is not supported by the data.
- Conjectures made from data sets that contain a small number of observations may not be valid. The larger the number of observations made, the more likely the trend you see is valid.

CHECK Your Understanding

1. Consider the data for a sample of Grade 9 students.

Science Mark (%)	61	67	69	74	77	81	84	90	91	94
Math Mark (%)	62	63	60	75	70	78	89	95	84	82

a) Make a conjecture about whether final marks in math and science are related.
b) Plot the data on a scatter plot.
c) Use a graph to support or reject your conjecture.

PRACTISING

2. K Natalie collected this data for the heights of 14-year-old girls and their mothers.

Mother's Height (m)	1.62	1.53	1.51	1.58	1.76	1.59	1.53	1.49	1.51	1.71
Girl's Height (m)	1.55	1.55	1.57	1.61	1.68	1.75	1.62	1.56	1.54	1.61

a) Plot the data on a scatter plot.
b) If possible, sketch a line or curve of best fit.
c) Is there a relationship between the variables? If so, describe it.

3. Research suggests that there is a relationship between age and vocabulary size for young children. C

Explain how the following data can be used to support that statement.

Age (years)	1	2	2	3	3	4	4	5	5	6	6
Vocabulary Size (number of words)	10	450	500	1000	1150	1400	1600	2000	2150	2400	2750

4. Consider a car travelling on a dry asphalt road.
   a) Suppose the driver slams on the brakes. Make a conjecture about the relationship between the travelling speed and the braking distance.
   b) Consider the following data. Plot the data on a scatter plot.

Travelling Speed (km/h)	10	20	30	40	50	60	70	80	90	100
Braking Distance (m)	0.5	1.9	4.3	7.7	12.1	17.4	23.6	30.9	39.1	48.2

   c) Do the data support your conjecture? Explain.

5. Suppose you are analyzing a set of data to see if it supports your conjecture.
   a) Under what circumstances would you accept the conjecture?
   b) Under what circumstances would you reject the conjecture?
   c) Is it possible for two people to analyze the same scatter plot and reach different conclusions about the validity of a conjecture? Explain.

Extending

6. Suppose that you collected data on the total lengths of roads in Canada for the past 100 years, and the incidence of cancer for the past 100 years.
   a) If you plotted the data on a scatter plot, what would you expect a line or curve of best fit to look like?
   b) If two variables are related by a line or curve of best fit on a scatter plot, does that mean that one of the variables influences the other? Explain.

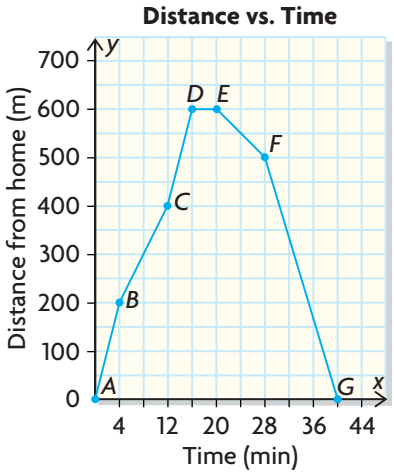
6.5 Describing Situations From Graphs

GOAL

Use a given graph to describe the situation it represents.

LEARN ABOUT the Math

Bill's mother sends him to the corner store for milk and tells him to be back in 30 min. The graph shown shows the relationship between his distance from home and time.



? How can the graph be used to describe Bill's trip?

EXAMPLE 1 Connecting the situation to a given graph

Use the graph to describe Bill's trip.

Marilyn's Solution

The line segments from points A to D represent his trip from his house to the store.

It took him 16 min to walk to the store.

The store is about 600 m from his home.

The slopes of each of these lines are positive and represent the rate at which he can walk per minute. This means that as time increases his distance from home increases. This can only happen if he is walking away from home toward the store.

On his way to the store he walked at the same speed between 0 and 4 min and 12 and 16 min. He walked at a slower speed between 4 and 12 min.

The line segments AB and CD are parallel. They have the same slope so he is walking at the same rate during these periods of time.

The line segment BC is not as steep as AB and CD so its slope is less.

The line segment from D to E represents the time he spent at the store. He was at the store for about 4 min.

DE is a horizontal line and its slope is 0. This means for this period of time between 16 and 20 minutes he wasn't walking from or to his home.

The line segments from points *E* to *G* represent his trip from the store to his house.

It took him 20 min to walk from the store to his house.

The slopes of each of these lines are negative and represent the rate at which he can walk per minute. This means that as time increases his distance from home decreases. This can only happen if he is walking away from the store toward his home.

On his trip back home he walked the fastest between 28 and 40 min. He walked at a slower rate between 20 and 28 min.

The line segment *FG* is steeper than *EF* which means between points *F* and *G* he is walking a greater distance each minute compared to the rate he walks between *E* and *F*.

Reflecting

A. In this graph, the slopes of each line segment represent a rate of change. What is another name for this rate of change?

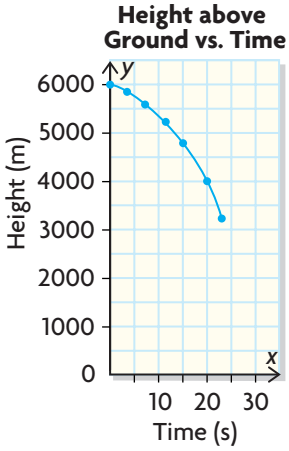
B. Calculate the slopes of *AB*, *BC*, *DE*, *EF* and *FG*. Do these values correspond with Marilyn's analysis of the situation? Explain.

C. Did Bill make it home in 30 min? Justify your answer.

Apply the Math

EXAMPLE 2 Describing the situation from a graphical representation

A skydiver jumps from an airplane. The graph shows her height above the ground during the first 24 s of her free fall, prior to the parachute opening. Use the graph to describe her motion during this part of her jump.



Daniel 's Solution

The graph is a curve so this is a nonlinear relationship.

The points of the scatter plot do not lie along a straight line. The points of the scatter plot have been joined by a smooth curve since the data is continuous.

Since this is a nonlinear relationship the skydiver can't be falling at a constant rate or speed. I created a table of values by estimating the coordinates of the points on the graph and calculated the first differences.

If the relationship was linear then the rate of change between each pair of points would be the same. This would indicate that the rate of change or speed would be constant.

Since height is on the vertical axis, it is the dependent variable and time, on the *x*-axis, is the independent variable. This makes sense since the skydiver's height above the ground depends on the length of time since she jumped.

Time (s)	Height (m)	First Differences
0	6000	
4	5920	−80
8	5680	−240
12	5280	−400
16	4720	−560
20	4000	−720
24	3120	−880

During the first 4 seconds she fell 80 m. Between 4 and 8 s she fell 240 m. Between 8 and 12 s she fell 400 m.

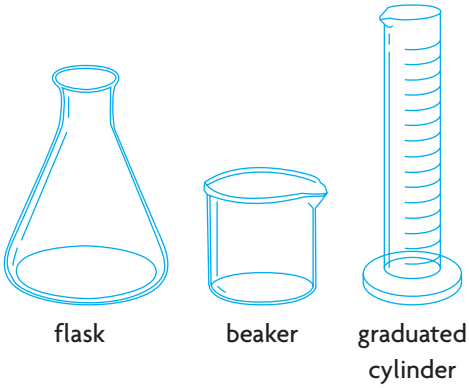
As time increases by 4 s intervals in the table, the change in height increases.

The first differences are not constant, which confirms my conjecture.

The skydiver is accelerating.

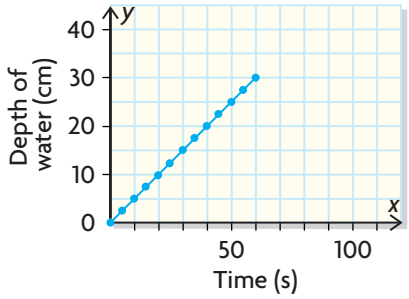
EXAMPLE 3 Using reasoning to connect a graph to its situation

Suppose tap water, flowing from a faucet at a constant rate, is used to fill each of these containers.

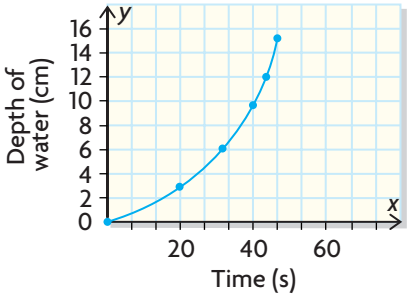


Match each of the following graphs with the appropriate container. Justify your choice.

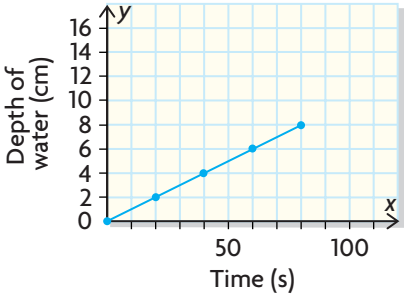
A.



B.



C.



Jose's Solution

I think the flask matches with graph B.

I reasoned that as the water pours into the flask the depth of the water can't increase at a constant rate due to the flasks shape. This means the graph of water depth versus time must be nonlinear. Since the shape of the flask is wide at the bottom and narrow at the top, the water depth will increase slowly as the flask begins to fill. As the water level rises the water depth will start to increase faster as the water level gets closer to the top.

I think the graduated cylinder matches with graph A.

I reasoned that as the water pours into the graduated cylinder the depth of the water increases at a constant rate since its shape does not change. This means the graph of water depth versus time must be linear. I chose graph A over graph C because it has greater slope and it reaches a greater depth. The graph with the greater slope makes sense since the graduated cylinder is narrow the water depth will rise faster than the beaker. The graph that reaches the greater depth makes sense because the graduated cylinder is taller than the beaker.

I think the beaker matches with graph C.

I reasoned that as the water pours into the beaker the depth of the water increases at a constant rate since its shape does not change. This means the graph of water depth versus time must be linear. I chose graph C over graph A because it has less slope and it reaches a lesser depth. The graph with less slope makes sense since the beaker is wide the water depth will rise slower than the graduated cylinder. The graph that reaches the smaller depth makes sense because the beaker is not as tall as the graduated cylinder.

In Summary

Key Idea

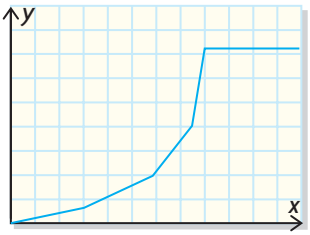
- In a problem involving movement, the graph shows **displacement** (distance, height, or depth) versus time. Distance, height, or depth is the dependent variable and time is the independent variable.
  The rate of change in these relationships is speed or **velocity;**
  $$velocity = \frac{change\ in\ displacement}{change\ in\ time} \text{ or } v = \frac{\Delta d}{\Delta t}$$

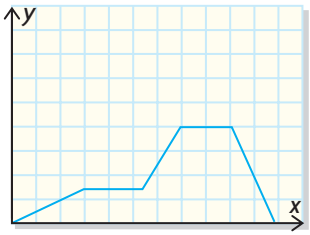
Need to Know

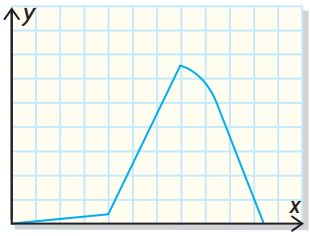
- Determining the first differences in a table of values can be used to confirm if the graph represents a linear or nonlinear relationship.
- In linear relationships,
  - the velocity is constant.
  - a rising line indicates displacement increases as time increases.
  - a falling line indicates displacement decreases as time increases.
  - a horizontal line indicates no change in displacement as time increases.
- In nonlinear relationships, the velocity changes with time.

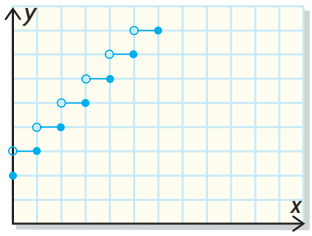
CHECK Your Understanding

1. Match each story to a graph on the next page that best describes the story.
   a) Michael walks to school at a steady pace. He waits once for a stop light and continues to school at a faster pace. After being at school, he returns home without stopping or slowing down.
   b) A log floating in a slow, steadily moving river goes through two sets of rapids before going over a waterfall into a lake.
   c) A taxi driver charges a passenger to get in the cab plus a fixed amount for every 100 m.
   d) A skydiver enters a plane that takes off and climbs at a steady rate. He jumps out and free-falls until the parachute opens. He descends the rest of the way at a constant speed.

i)


iii)


ii)


iv)


2. This graph shows how an all terrain vehicle (ATV) travels over time.

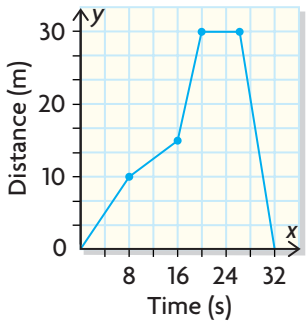
a) Over what interval of time is the ATV travelling the slowest? the fastest?

b) When does the ATV start to return to its starting point? When does it get there?

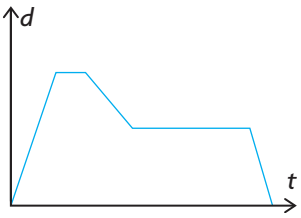
c) Determine the slope of the graph between 20 s and 26 s.

d) What does a zero slope mean in the context of this graph?

Distance vs. Time



3. Describe a situation that could match this graph.



PRACTISING

4. A drag racer begins from a stopped position and drives the length of a race track. The graph shows the distance from his starting position after each second of his run.

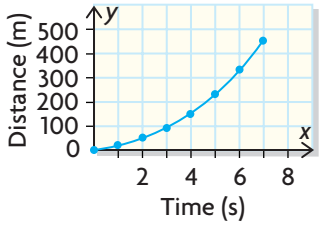
a) Does the data represent a linear or nonlinear relationship?

b) Verify your answer to part a by creating a table of values and calculate the first differences.

c) Describe the motion of the drag racer as it moves down the drag strip to the finish line.

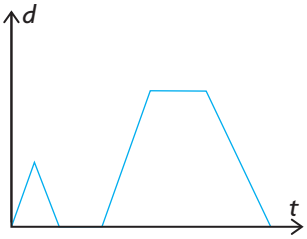
d) Once the drag racer crosses the finish line a parachute opens to slow it down. Redraw the graph to show what this might look like as the drag racer slows to a stop.

Distance from Start vs. Time

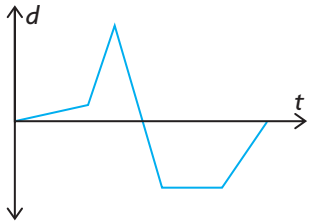


5. Write a story for each graph.

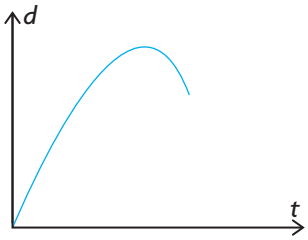
a)



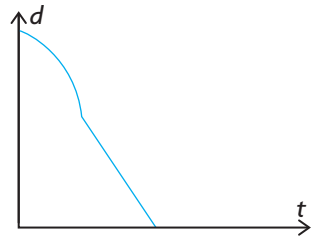
c)



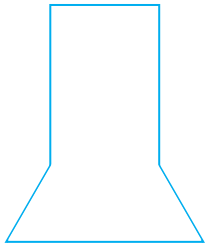
b)



d)



6. Water is poured into this container at a constant rate. Draw a graph that represents this situation where *d* is depth and *t* is time.



7. a) Plot each set of data, and then, determine if it represents a linear or nonlinear relationship. Justify your decision.

b) In each set of data, determine if the object is accelerating, decelerating, moving at a constant velocity, or a combination of all three.

i)

Time (s)	Distance (m)
7	19
8	22
9	25
10	28
11	31

ii)

Time (s)	Distance (m)
10	35
11	37
12	38
13	38.5
14	38.75

iii)

Time (s)	Distance (m)
0	0
1	7
2	16
3	29

8. A driver leaves at 8:00 a.m. and drives 120 km by 9:30 a.m. From then
A until 10:30 a.m., she travels another 50 km. She drives an additional 200 km by 12:30 p.m. She travels at a constant speed during each period of time.

a) Draw the distance-time graph that represents the trip.
b) When is the car travelling the fastest? the slowest?
c) What type of relationship is this?
d) What is the average speed for the entire trip?

9. A baseball is hit straight up into the air. The table shows the height of the ball after various time intervals.

a) Does the data represent a linear or nonlinear relationship?
b) Graph the data.
c) What is the height of the ball after 1.5 s?
d) When will the baseball be at 44 m?
e) When is the ball accelerating? Decelerating?
f) When is the ball stopped?
g) What is the maximum height the ball reaches?
h) What effect does gravity have on the velocity of the ball?

Time (s)	Height (m)
0	1
1	26
2	41
3	46
4	41
5	26
6	1

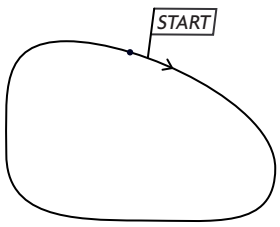
10. The table shows the length of time needed to drive 100 km at various speeds.

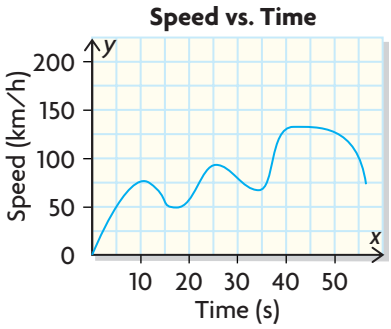
a) Copy and complete the table by determining the missing times.
$\left(\text{time} = \dfrac{\text{distance}}{\text{speed}}\right)$
b) Graph the data. What is the dependent variable? the independent variable?
c) Select three points on the graph that are separated by equal intervals of speed. Calculate the rate triangles between the first and second points and between the second and third points. Are the values the same or different?
d) Is the relationship linear or nonlinear?

Speed (km/h)	Time (h)
100	1
75	
50	
25	
12.5	
10	
1	

11. A race car travels around a race track. The graph of speed vs. time for the first lap by the car is shown.

a) What do the increasing parts of the graph represent? the decreasing parts?
b) What do the horizontal parts of the graph represent?
c) Using the graph, tell a story about the car as it makes its first lap around the track.

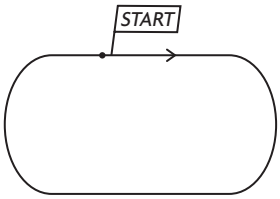




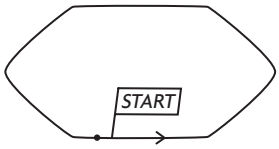
12. Draw the side profile of two different shaped containers that you could fill with water. For each shape you drew, draw the graph that shows the relationship between water depth and time as the container fills with water. Explain why you drew the graphs you did. **T**

13. Draw a graph of speed versus time as a car makes one circuit of each track. The arrow shows the direction the car is moving. **C**

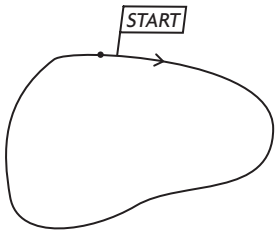
a)



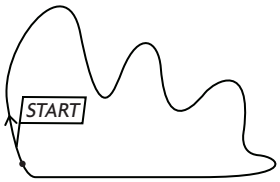
c)



b)



d)



Extending

14. Sheila takes a driver education course before applying for her driver's licence. One of the topics studied is the relationship between speed and stopping distance. The instructor gives Sheila the table on the right.

a) Graph the data using stopping distance as the dependent variable.

b) Determine the rate triangle from 60 km/h to 70 km/h and from 70 km/h to 80 km/h.

c) What do you notice about the rate of change in stopping distance as the speed increases by 10 km/h in part b)?

d) Sheila says, "If I drive twice as fast, I just need to leave twice as much distance between my car and the car in front of me." Explain whether this is accurate.

Speed (km/h)	Stopping Distance (m)
10	3.7
20	7.6
30	12.0
40	17.1
50	22.9
60	29.8
70	37.9
80	47.5
90	58.6
100	71.6
110	86.5
120	103.5
130	122.8
140	144.7

FREQUENTLY ASKED Questions

Q: How can you determine a curve of best fit?

A: Plot the data on a scatter plot. If you notice that the plotted points do not seem to lie on a line, but seem to lie close to a fairly simple curve, then draw a curve of best fit to approximate the points.

Q: How can you use a curve of best fit to predict values not recorded in the data?

A: Use the curve of best fit to estimate the coordinates between recorded data points. Extend the curve of best fit to estimate the coordinates of points beyond the recorded data points.

Q: How can you test a conjecture about a pattern in data or a relationship between variables?

A: Plot the data on a scatter plot. Then, examine the pattern of the data points and the line or curve of best fit (if one exists) to see if it matches the conjecture.

EXAMPLE

Consider the conjecture that the period of a pendulum increases if its length increases. The scatter plot supports this conjecture, since the curve of best fit matches the conjecture.

Q: How can you interpret and describe a situation from a displacement or distance vs. time graph?

A: If the graph contains a series of different line segments, then use your knowledge of slope and constant rates of change to interpret what the graph represents. Line segments that rise indicate that displacement increases as time increases. Line segments that fall indicate that displacement decreases as time increases. Horizontal line segments indicate that there is no change in displacement as time increases.

If the graph is nonlinear and curved, then discuss how the rates of change differ between consecutive points on the graph. This will help you decide if an object is speeding up or slowing down.

Study *Aid*

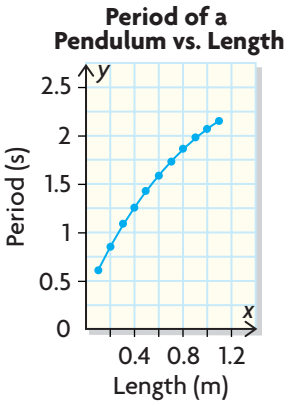
- See Lesson 6.3, Example 1.
- Try Chapter Review Questions 5 and 6.

Study *Aid*

- See Lesson 6.3, Examples 1, 2, and 3.
- Try Chapter Review Question 7.

Study *Aid*

- See Lesson 6.4, Examples 1 and 2.
- Try Chapter Review Questions 12, 13, and 14.



Study *Aid*

- See Lesson 6.5, Examples 1, 2, and 3.
- Try Chapter Review Question 10.

PRACTICE Questions

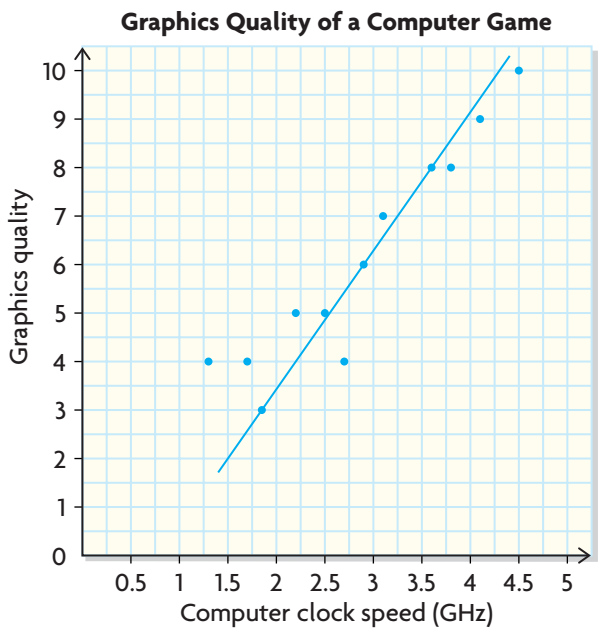
Lesson 6.1

1. What might a scatter plot look like, if it shows a relationship between allergies and the cleanliness of the air we breathe? Explain the expected relationship.

2. Suppose you are studying the possible relationship between the variables "Available Money" and "Number of Days After Payday."
   a) When plotting the data, which variable would you choose for the independent variable? Which would be the dependent variable? Explain.
   b) Draw a possible scatter plot.
   c) Describe the relationship in your scatter plot.

Lesson 6.2

3. The scatter plot below shows the graphics quality (on a scale of 1 to 10) for a video game when it is played on various computers.
   a) Is the line of best fit appropriate? If so, explain why. If not, sketch a more appropriate one.
   b) Determine the equation of the most appropriate line of best fit.



4. Students in Ryan's social science class collected data to study whether marks scored on a test were related to the number of hours of TV watched the night before the test. Data were collected for 10 students.

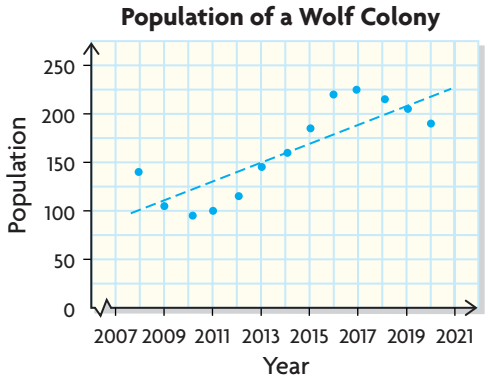
Test Mark (%)	Hours of TV Watched
82	2
64	4
84	0
70	3
74	2
76	2
85	1
73	3
94	1
90	2

   a) Draw a scatter plot of the data.
   b) Do the data show a pattern? If so, describe it.
   c) Draw a line of best fit for the data.
   d) Determine an equation for your line of best fit.
   e) Using your line of best fit, predict the test score for a student who watched 2.5 h of TV.
   f) Do you expect your prediction to be the same as all of your classmates' predictions? Explain.

5. Sometimes a solid line is used to draw a line of best fit, while at other times a dashed line is used.
   a) Explain why these two different types of lines of best fit are used on scatter plots that show linear relationships.
   b) Provide an example of two variables whose scatter plot of data could result in a solid line of best fit.
   c) Provide an example of two variables whose scatter plot of data could result in a dashed line of best fit.

Lesson 6.3

6. The scatter plot shows the population of a colony of wolves in a wilderness region of Northern Ontario.



a) Explain whether the line of best fit is appropriate.
b) Would a curve of best fit be more appropriate? If so, sketch one. If not, explain why not.
c) Might both a line of best fit and a curve of best fit be appropriate? Explain.

7. Is it necessary for a line or curve of best fit to pass through any of the plotted points? Explain.

8. Data are collected on shoe sizes and heights for men.

Shoe Size	8.5	9.0	9.0	10.0	10.5
Height (cm)	166	174	169	178	175

Shoe Size	10.5	11.0	11.5	12.0	12.5
Height (cm)	183	187	182	190	184

a) Draw a scatter plot of the data.
b) Do the data show a pattern? If so, describe it.
c) Draw a line or curve of best fit for the data.
d) If you have drawn a line of best fit, determine an equation for it.
e) Using your line or curve of best fit, predict the shoe size of a man who is 180 cm tall.
f) Using your line or curve of best fit, predict the height of a man whose shoe size is 17.5.

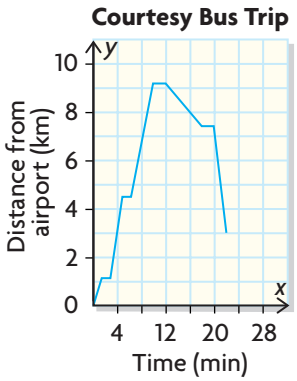
Lesson 6.4

9. Do you think that the number of chess grandmasters in a country is related to the size of its population?
a) Formulate a conjecture about the relationship.
b) Consider the data for a sample of countries. Plot the data on a scatter plot.

Country	Population (millions)	Number of Grandmasters
Canada	33	6
U.S.A.	300	61
Russia	142	159
Ukraine	46	58
Iceland	0.3	10
China	1317	19
Netherlands	16	21
Hungary	10	37
Serbia	10	49
Germany	82	61

c) If possible, sketch a line or curve of best fit.
d) Is there a relationship between the variables? If so, describe it.
e) Suggest other influences on the number of grandmasters in a country.

10. A hotel courtesy bus takes David from the airport to his hotel. Use the Distance versus Time graph to create a story that traces the route of the bus.



Up In Arms

Ananda and Justin watched a TV program on how archaeologists estimate the height of people who lived long ago. If scientists are able to find certain bones from a skeleton, they can use these to estimate the height of that person. One of these bones is the humerus, the upper arm bone that connects the shoulder to the elbow. The students wonder whether the relationship between the length of the humerus and a person's height is also valid for their classmates.

? Is the height of a person related to the length of his or her humerus?

A. Measure the height and length of humerus for 15 people in your class. Organize the data into a table.

B. Use your table from part A to create a scatter plot of the data.

C. Is there a relationship between the two variables? If so, describe it.

D. Sketch a line or curve of best fit, if appropriate.

E. Measure the humerus length of your teacher, or some other person not already in your data set. Use your line or curve of best fit to predict the height of the person. Then, measure the height of the person and comment on how accurate your prediction was.

F. Repeat part E, but this time measure the height and predict the humerus length.

G. Summarize your findings.

Task *Checklist*

- ✔ Did you construct the table of values accurately?
- ✔ Did you plot the data correctly?
- ✔ Did you select the dependent and independent variables appropriately?
- ✔ Did you use the appropriate type of line or curve of best fit (solid or dashed)?
- ✔ Did you discuss whether it is appropriate to use your line or curve of best fit to extrapolate?