

Chapter 7

Trig Identities and Equations

In this chapter we will:

- find equivalent trig relationships
- compound angle formulas
- prove trig identities using a variety of strategies (common denominator, factor, substitution, ...)
- solve trig equations

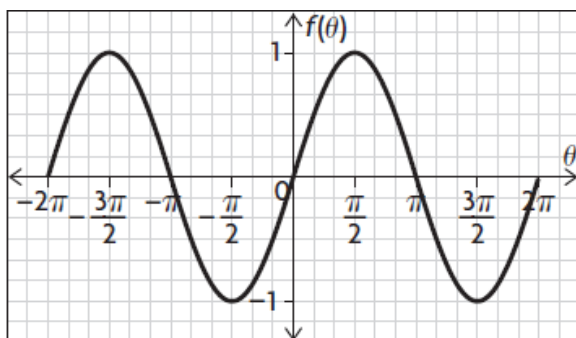
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Today...

- we are going to look at equivalent trig functions (some new and some review)
- review some of the trig identities

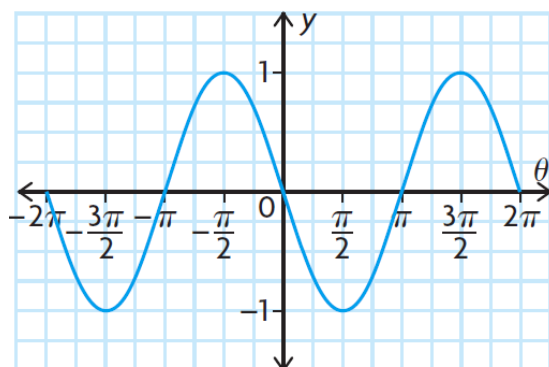
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What are two possible equations of this graph?



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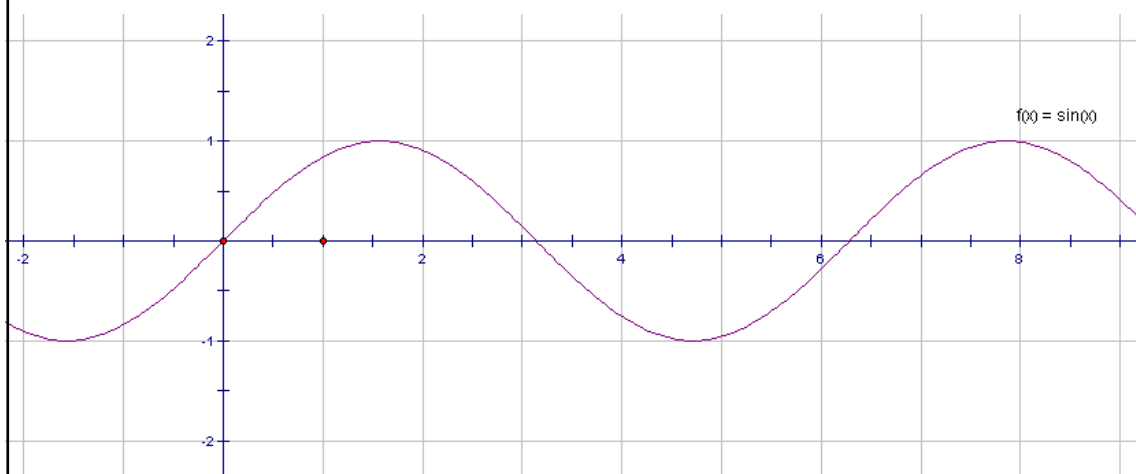


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We see that there are many ways to write the equation of the previous functions - these are called EQUIVALENT TRIG FUNCTIONS.

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if you shift $y=\sin x$ left $\frac{\pi}{2}$ you get $y=\cos x$



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What are the equivalent trig relationships that you need to know?

1/ If you shift $y = \sin x$ $\frac{\pi}{2}$ (90°) back you get $y = \cos x$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos \theta$$

$$\sin \theta = \cos\left(\theta - \frac{\pi}{2}\right)$$

Sin and Cos.gsp

2/ $y = \cos x$ is an even function so...

$$\cos \theta = \cos(-\theta)$$

3/ $y = \sin x$ and $y = \tan x$ are odd functions so...

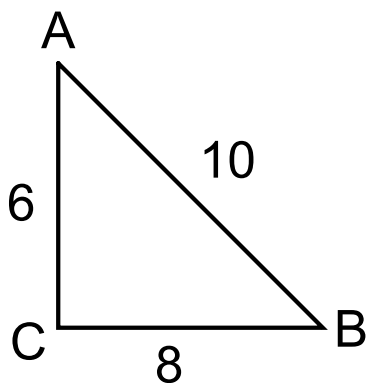
$$\sin(-\theta) = -\sin \theta$$

$$\tan(-\theta) = -\tan \theta$$

Sin and neg sin.gsp

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Proof



Sin A =

compare to

Cos B =

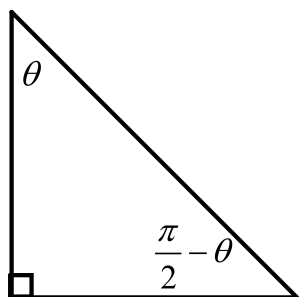
Tan A =

compare to

Cot B =

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CoFunction Identities



$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$$

$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$\tan \theta = \cot \left(\frac{\pi}{2} - \theta \right)$$

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Notice the following equivalent trigonometric relationships are just based on the **CAST** rule.

Principal Angle in Quadrant II	Principal Angle in Quadrant III	Principal Angle in Quadrant IV
$\sin (\pi - \theta) = \sin \theta$	$\sin (\pi + \theta) = -\sin \theta$	$\sin (2\pi - \theta) = -\sin \theta$
$\cos (\pi - \theta) = -\cos \theta$	$\cos (\pi + \theta) = -\cos \theta$	$\cos (2\pi - \theta) = \cos \theta$
$\tan (\pi - \theta) = -\tan \theta$	$\tan (\pi + \theta) = \tan \theta$	$\tan (2\pi - \theta) = -\tan \theta$

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Using equivalent relationships...

CoFunction

Equivalent - CAST

$$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\sin \frac{\pi}{6} =$$

$$\sin \frac{\pi}{6} = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$\sin \frac{\pi}{6} = \cos\left(\frac{3\pi}{6} - \frac{\pi}{6}\right)$$

$$\sin \frac{\pi}{6} = \cos\left(\frac{\pi}{3}\right)$$

$$\sin \frac{\pi}{6} =$$

$$\sin \frac{\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right)$$

$$\sin \frac{\pi}{6} = \sin\left(\frac{6}{6}\pi - \frac{\pi}{6}\right)$$

$$\sin \frac{\pi}{6} = \sin\left(\frac{5}{6}\pi\right)$$

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TRIG IDENTITIES!!!

Recall (from 3U):

RECIPROCAL IDENTITIES

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

QUOTIENT IDENTITIES

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

tan.gsp

PYTHAGOREAN IDENTITIES

$$\sin^2 \theta + \cos^2 \theta = 1$$

Find 2 more variations of the PT identity...



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Homework

p386 #7

p392 #3, 5, 7

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Attachments

Sin and Cos.gsp

tan.gsp

Sin and neg sin.gsp