Sec 1.4 Sketching Graphs of Functions

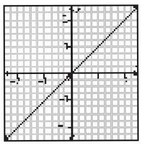
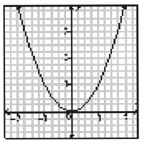
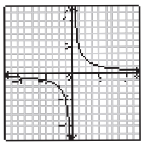
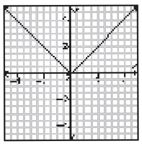
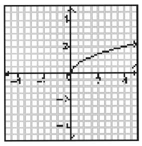
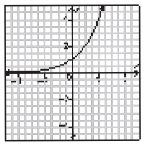
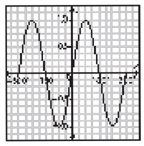
Learning Goal: Graph sketching
and transformations

p615 of text

Answers to Investigate the Math

A–G.

Learn the shapes, deduce the rest.

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$
Sketch							
Domain	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$
Range	$\{f(x) \in \mathbf{R}\}$	$\{g(x) \in \mathbf{R} \mid g(x) \geq 0\}$	$\{h(x) \in \mathbf{R} \mid h(x) \neq 0\}$	$\{k(x) \in \mathbf{R} \mid k(x) \geq 0\}$	$\{m(x) \in \mathbf{R} \mid m(x) \geq 0\}$	$\{p(x) \in \mathbf{R} \mid p(x) > 0\}$	$\{r(x) \in \mathbf{R} \mid -1 \leq r(x) \leq 1\}$
Intervals of Increase	$(-\infty, \infty)$	$(0, \infty)$	None	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$[90°(4k - 1), 90°(4k + 1)], k \in \mathbf{Z}$
Intervals of Decrease	None	$(-\infty, 0)$	$(-\infty, 0)$ $(0, \infty)$	$(-\infty, 0)$	None	None	$[90°(4k + 1), 90°(4k + 3)], k \in \mathbf{Z}$
Location of Discontinuities and Asymptotes	None/ None	None/ None	Discontinuous at $x = 0$, $y = 0$, $x = 0$	None/ None	None/ None	None/$y = 0$	None/ None
Zeros	0	0	None	0	0	None	$180°k$, $k \in \mathbf{Z}$
y-intercepts	0	0	None	0	0	1	0
Symmetry	Odd	Even	Odd	Even	Neither	Neither	Odd
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0$ $x \to -\infty, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to \infty$; $x \to 0, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0$	Oscillating between 1 and -1

When all of the transformations are applied to the parent curve, the result can be summarized as follows:

$$y = a\ f\left(b(x-h)\right) + k$$

Describe the transformations of the following: $f(x) = \dfrac{-3}{(2x-8)} + 5$

Graphing Strategies

Steps to graph:

1/ Graph the parent function f(x)

2/ Apply the stretch/compression transformation.

3/ Apply the reflection transformation.

4/ Translate left/right and up/down.

(each transformed graph is essentially a "new" function)

$$z(x)=af(b(x-h))^2+k$$

move up or down

vertical stretch/
compression
reflection

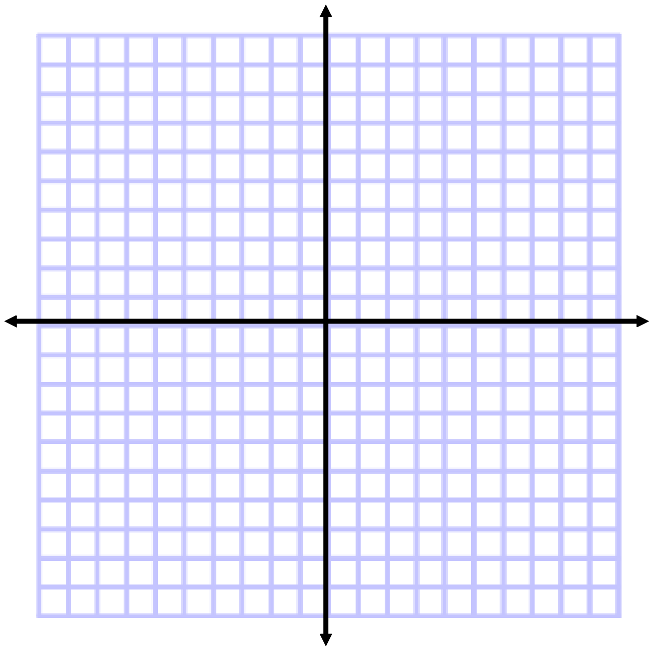
move left or right

vertical stretch/
compression
reflection

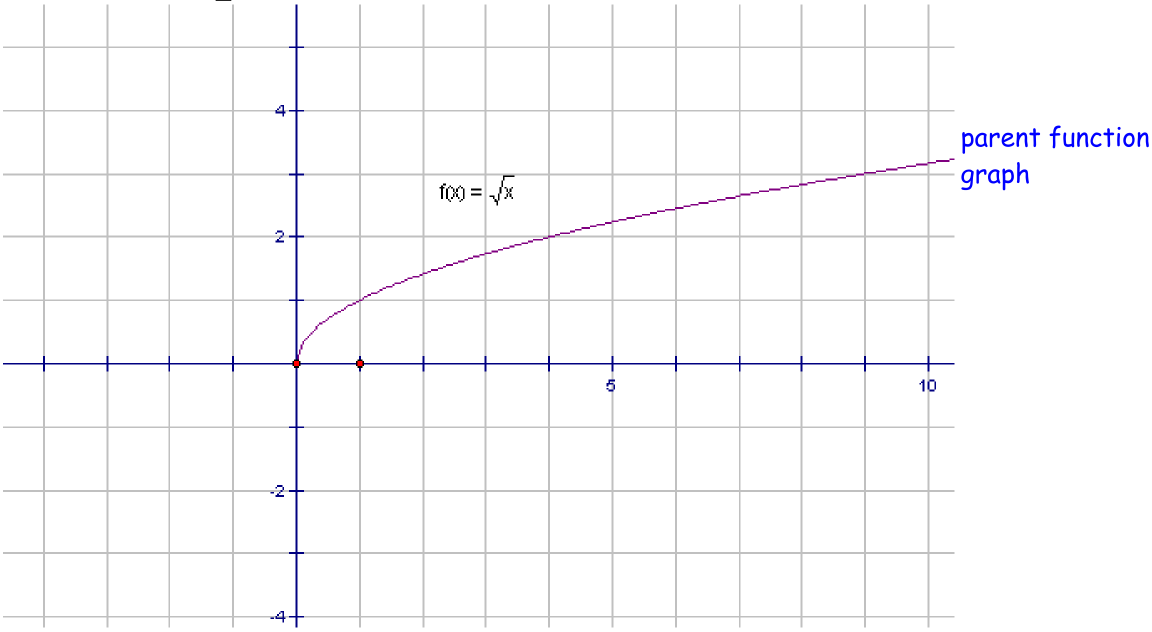
In this section we will review the use of transformations of parent functions to graph a complicated function.

Ex: Graph $y = -3 \cdot 2^{(x+1)}$

Not easy from a table of values. But... as a series of transformations, not too bad.

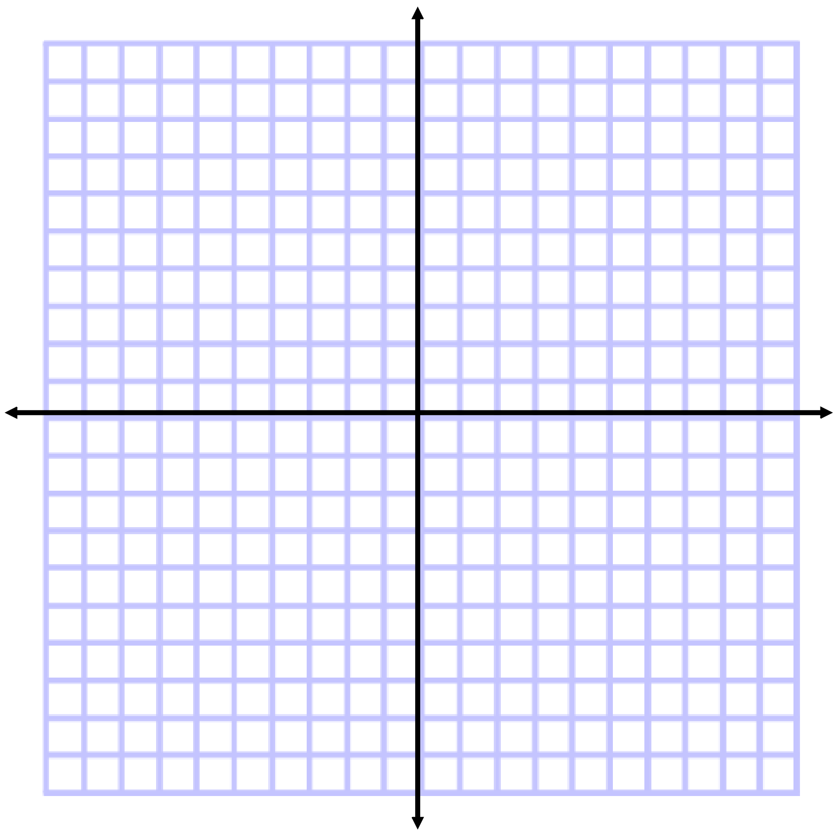


Ex: Graph $y = \frac{1}{2}\sqrt{x+3} - 2$



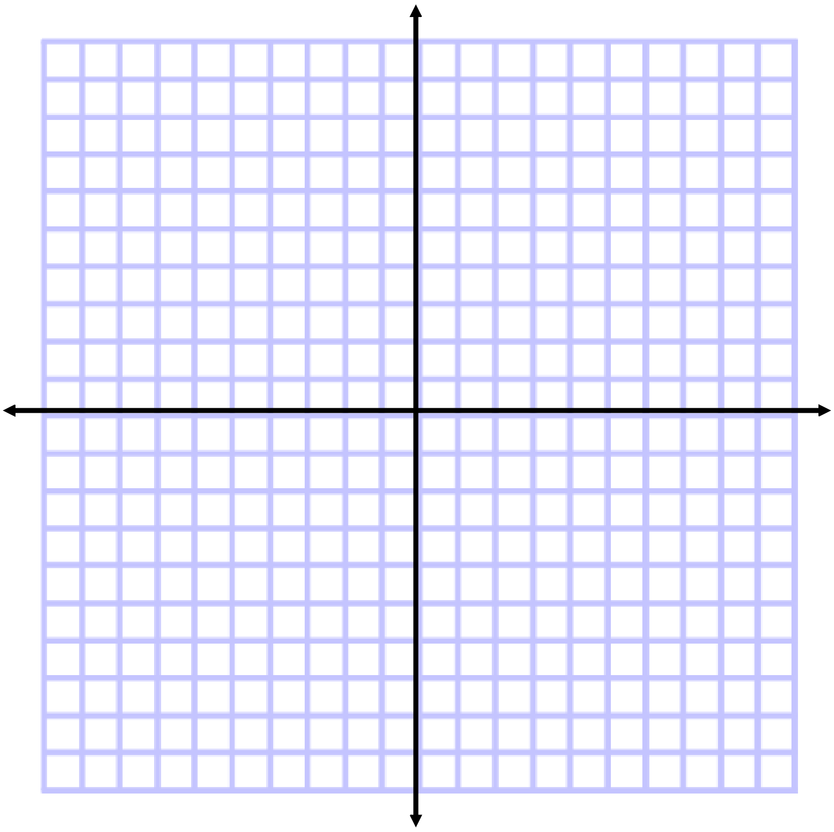
Graph

$$y = \frac{-0.5}{x-6} + 4$$



Graph

$$y = 2\left|4x - 12\right| - 5$$



In general:

Parent function: $y = f(x)$

Note: ***k*** *is factored first*

Transformed function: $y = af(k(x-d)) + c$

parameters affecting *y*-values	parameters affecting *x*-values
a = vertical stretch or compression and reflection if negative	k= horizontal stretch or compression and reflection if negative
c= vertical translation c>0 move up c<0 move down	d= horizontal translation x-d moves to the right x+d moves to the left

- Remember the order in which to apply transformations!

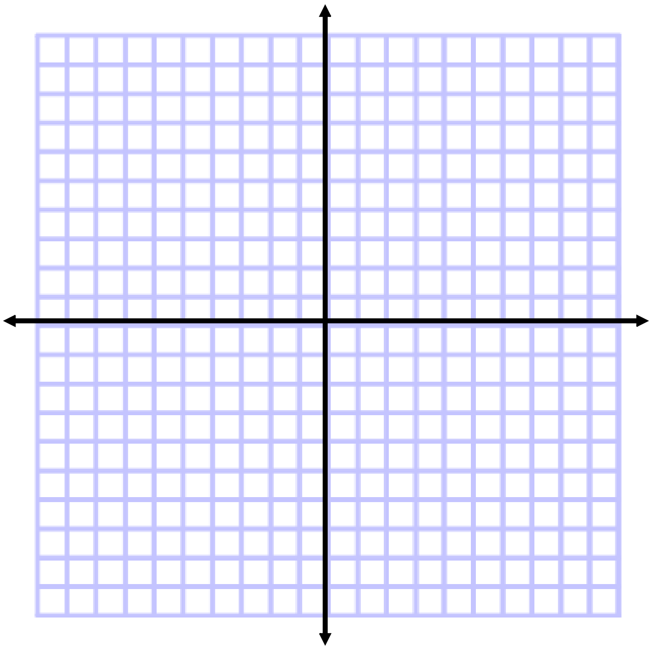
- Remember that the transformations also change the domain, range, asymptotes, areas of increase/decrease, and end behaviours.

- Interval notation can be used in place of greater than or less than signs.

$3 \leq x < 4$ becomes [3,4)

In grade 12 math we place a bigger emphasis on domain and range. The new parts of the transformation section include:

- -areas of increase and decrease of function
- -end behaviours of functions



$y=x^2$

areas of inc:

areas of dec:

Add: TURNING POINT definition from p30

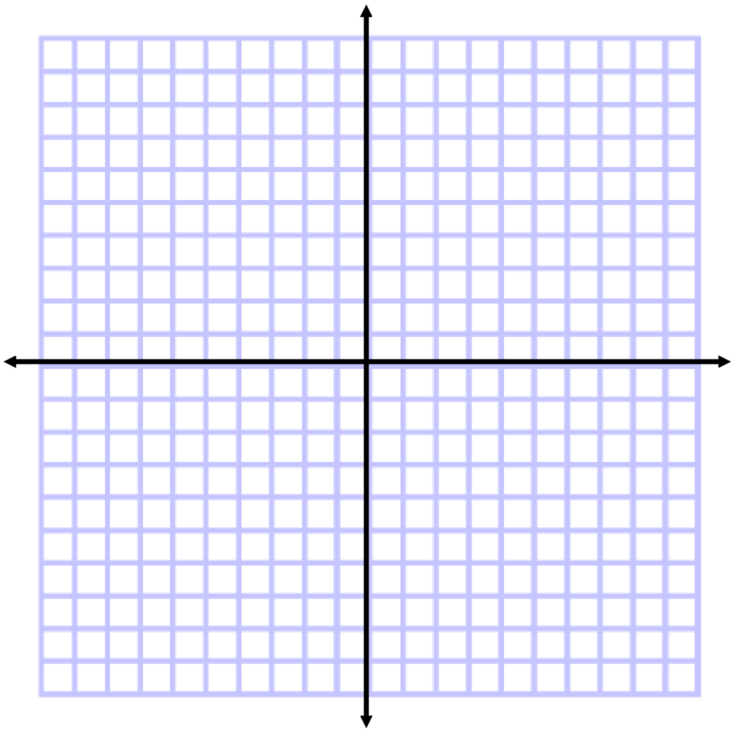
Creating New Function Expressions

Apply the following transformations to $f(x) = \sqrt{x}$

(i) vertically stretch by 3

(ii) left 4 units

(iii) down 2 units



$$f(x) = \sqrt{x}$$

$$g(x) = 3f(x)$$
$$g(x) = 3\sqrt{x}$$

$$h(x) = g(x+4)$$
$$h(x) = 3\sqrt{x+4}$$

$$z(x) = h(x) - 2$$
$$z(x) = 3\sqrt{x+4} - 2$$

Homework

Read Ex 4 p34
Note the "In Summary" Box
Try questions:
p35 #1ace, 5ace, 6, 9ace