R–5 Slope and Rate of Change of a Linear Function

The slope of a line is a ratio that compares the change in the dependent variable, y, with the change in the independent variable, x.

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The equation of a linear relation can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Values of the Slope

- The slope of a line that rises to the right is positive.
- The slope of a line that drops to the right is negative.
- The slope of a horizontal line is zero. The equation of the line can be written in the form $y = b$.
- The slope of a vertical line is undefined. The equation of the line can be written in the form $x = a$.

Equations of Straight Lines

- point-slope equation of a line: $y - y_1 = m(x - x_1)$
- general form of the equation of a line: $Ax + By + C = 0$
- slope-intercept equation of a line: $y = mx + b$

Parallel and Perpendicular Lines

Two lines, with slopes m_1 and m_2, are

- parallel if and only if $m_1 = m_2$
- perpendicular if and only if $m_1 m_2 = -1$; that is, if their slopes are negative reciprocals: $m_2 = -\frac{1}{m_1}$

EXAMPLE

Find the slope and equation of a line that passes through points (5, 6) and (15, 2). Explain how the slope is a rate of change.

Solution

The slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{15 - 5} = -\frac{4}{10} = -\frac{2}{5}$.

Substituting $m = -\frac{2}{5}$ and $(x_1, y_1) = (5, 6)$ into $y - y_1 = m(x - x_1)$,

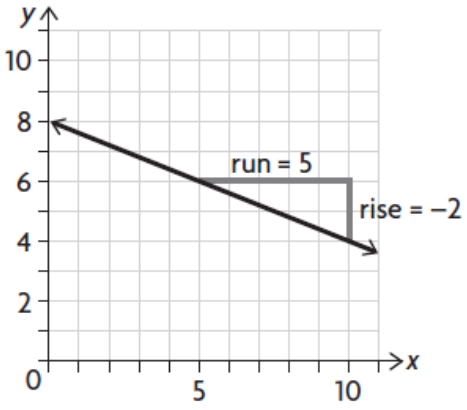
$$y - 6 = -\frac{2}{5}(x - 5)$$

$$y = -\frac{2}{5}x + 2 + 6$$

$$y = -\frac{2}{5}x + 8$$

The slope of the line is $-\frac{2}{5}$, and the equation is $y = -\frac{2}{5}x + 8$.

The slope is a rate of change because y will decrease by 2 units for each 5 unit increase in x.



Practising

1. Determine the slope of a line that passes through each pair of points.
   a) $(1, -5)$ and $(-4, -9)$
   b) $(-1, 4)$ and $(7, 4)$
   c) $(5, -2)$ and $(5, -4)$
   d) $(-3, 5)$ and $(-2, 9)$
2. Describe the graph of
   a) $x = -3$ b) $y = 6$
3. Suppose that you buy a plant. The height of the plant t weeks after you buy it is $h(t) = 26 + 1.2t$, where h is the height in centimetres. What is the slope of the height function, and what does the slope mean in the context of this situation?
4. Determine the slope and y-intercept of each line.
   a) $3x + 5y + 10 = 0$ b) $Ax + By + C = 0$