Sec. 8.6 - Regular Annuities (payments) - Determining Present Value

Learning Goal:

By the end of today, I will be able to calculate the present value required to create a regular set of payments that experience compound interest.

Annuity - a series of equal deposits or payments made at regular intervals;

A simple annuity is an annuity in which the payments coincide with the compounding period, or *conversion* period; (ie. monthly payments will be matched with monthly compounding)

An ordinary annuity is an annuity in which the payments are made at the end of each interval;

Unless otherwise stated, each annuity in this chapter is a simple, ordinary annuity

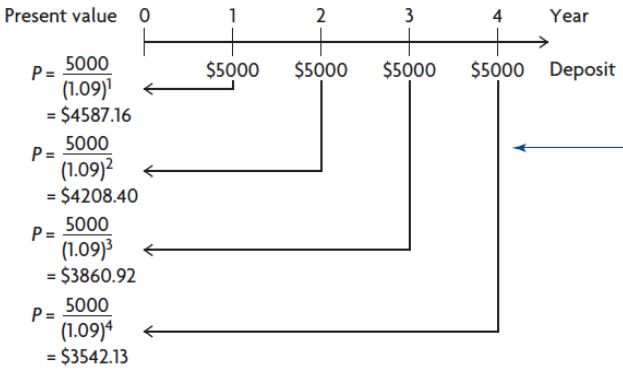
Harry wants to put aside a large sum of money at the start of his university career so that his tuition is covered and he does not need to worry about that expense year to year.

Harry has determined that he will be in university for 4 years, and his tuition is going to be $5,000 a year.

If Harry put all of his savings in a coffee can, in his closet, where it did not earn interest, how much would he need to pay for his tuition for 4 years?

If Harry wisely invested his savings in an investment that generated 9%/a compounded annually, how much would he need at the start?

Present Value of Annuity at 9%/a Compounded Annually



I used a timeline to organize the solution.

The annual interest rate is 9%, or 0.09. At the end of 1 year, the first payment's present value will have earned interest for 1 year: $n = 1$.

The second payment's present value will have earned interest for 2 years: $n = 2$.

The third payment's present value will have earned interest for 3 years: $n = 3$.

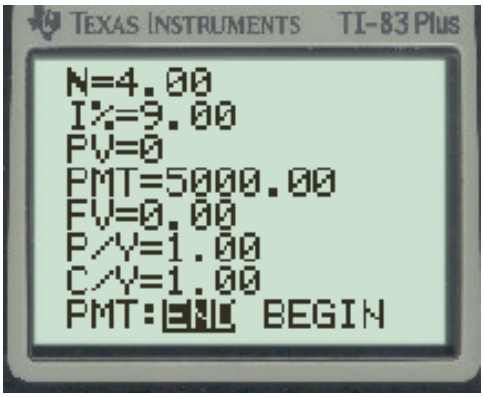
The fourth payment's present value will have earned interest for 4 years: $n = 4$.

$$PV = \frac{5000}{(1.09)^1} + \frac{5000}{(1.09)^2} + \frac{5000}{(1.09)^3} + \frac{5000}{(1.09)^4}$$
$$= 4587.16 + 4208.40 + 3860.92 + 3542.13$$
$$= 16\,198.61$$

The present value is the total of the present values of all the payments.

To have four equal annual payments of \$5000 starting 1 year from now, Harry needs \$16 198.61 in his account now.

Using the TVM Solver



Present Value Formula for Simple, Ordinary Annuities

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

Where "R" is the payment,

"A" is the future amount

"i" is the annual interest rate,

"n" is the total number of payments

Harry's investment: 4 years, 9%/a, compounded annually, $5,000 payments.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

Example

Robin bought a bicycle for $1500. She arranged to make a payment to the store at the end of every month for 1 year. The store is charging 11%/a interest compounded monthly.

a) How much is each monthly payment?

b) How much interest is Robin paying?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

TVM Solver

Homework

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