7.6 Geometric Series

Learning Goal: By the end of today, I will be able to find the sum of a Geometric series.

A geometric series is created by adding the terms of a geometric sequence.

For the sequence 3, 6, 12, 24, ... , the related geometric series is 3 + 6 + 12 + 24 + ...

The formula for the geometric series is what we will learn today.

Proof $t_n = ar^{n-1}$ geometric sequence formula

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-2}, ar^{n-1}$$

similar to Gauss, but with a twist, we will create two equations, the original and a modified one (multiply both sides by r)

$$rS_n = \quad ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$$

(modified)

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-2}, ar^{n-1}$$

(original)

I can eliminate almost all of the terms by subtracting the two equations.

$$rS_n - S_n = -a + ar^n$$

Isolate S_n

$$S_n(r-1) = a(-1 + r^n) \quad \text{or} \quad ar^n - a$$

rearrange

$$S_n = \frac{a(r^n - 1)}{(r-1)} \quad \text{or} \quad \frac{ar^n - a}{(r-1)}$$

$$S_n = \frac{t_{n+1} - t_1}{(r-1)}$$

series
the sum of the terms of a sequence

geometric series
the sum of the terms of a geometric sequence

You can use either formula to find the sum of a geometric series...which one you choose is depends on the information given in the problem.

$$S_n = \frac{t_{n+1} - t_1}{r-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

PROBLEM 1

For the given geometric series, calculate the sixth term and the sum of the first six terms: $\frac{4}{5}+\frac{8}{15}+\frac{16}{45}+.....$

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PROBLEM 2

At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched on each of the first four days after fertilization was 2, 10, 50, and 250, respectively. If the pattern continues, calculate the total number of fish hatched during the first 10 days.

What do you know?

r =

a =

n =

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

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PROBLEM 3

Calculate the sum of the geometric series:

7 971 615 + 5 314 410 + 3 542 940 + ... + 92 160

What do you need to calculate first? $S_n = \frac{t_{n+1} - t_1}{r - 1}$

r =

n =

the term n + 1 =

	A	B
1	n	tn
2	1	7971615
3	2	5314410
4	3	3542940
5	4	2361960
6	5	1574640
7	6	1049760
8	7	699840
9	8	466560
10	9	311040
11	10	207360
12	11	138240
13	12	92160
14	13	61440

$$S_{12} = \frac{61\,440 - 7\,971\,615}{\frac{2}{3} - 1}$$

$$= 23\,730\,525$$

The sum of the series 7 971 615 + 5 314 410 + 3 542 940 + ... + 92 160 is 23 730 525.

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PROBLEM 4

A large company has a phone tree to contact its employees in case of an emergency factory shutdown. Each of the five senior managers calls three employees, who each call three other employees, and so on. If the tree consists of seven levels, how many employees does the company have?

In Summary

Key Idea

- A geometric series is created by adding the terms of a geometric sequence. For the sequence 3, 6, 12, 24, ... , the related geometric series is 3 + 6 + 12 + 24 + ...

Need to Know

- The sum of the first n terms of a geometric sequence can be calculated using
  - $S_n = \frac{a(r^n - 1)}{r - 1}$, where $r \neq 1$ or
  - $S_n = \frac{t_{n+1} - t_1}{r - 1}$, where $r \neq 1$.

  In both cases, $n \in \mathbf{N}$, a is the first term, and r is the common ratio.
- You can use either formula, but you need to know the common ratio and the first term. If you know the $(n + 1)$th term, use the formula in terms of t_1 and t_{n+1}. If you can calculate the number of terms, use the formula in terms of a, r, and n.

PRACTICE

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3acd, 4bc, 5ab, 6c, 7, 9, 10, 11

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