

## 7.5 Arithmetic Series

Learning Goal: By the end of today, I will be able to solve/sum and arithmetic series.

Dec 16-10:40 AM

**Recall:**

An arithmetic sequence is a set of numbers that have the same difference

$a, a+d, a+2d, a+3d, \dots$

$$t_n = a + (n-1)d$$

Dec 16-12:06 PM

# ARITHMETIC SERIES

## MATH BACKGROUND



An arithmetic series is created by adding the terms of an arithmetic sequence.



A partial sum is the sum of a finite number of terms of a sequence.

Jan 5-9:22 AM

Ex:

1, 3, 5, 7, .... is an arithmetic *sequence*



1 + 3 + 5 + 7 + .... is an arithmetic *series*



Dec 16-12:08 PM

Gauss...

Add 1 to 100 ...

1, 2, 3, 4, .....98, 99, 100

100, 99, 98, 97, .....3, 2, 1



100 terms

Multiply  $100 \times 101$ , divide by 2, voila

Dec 16-12:10 PM

Let's find the formula...

This is the SEQUENCE

$a, a+d, a+2d, a+3d, \dots, a+(n-2)d, a+(n-1)d$   
last term

This is the SERIES

$a + a+d + a+2d + a+3d + \dots + a+(n-2)d, a+(n-1)d$

let  $t_n = a + (n-1)d$

So... adding the first "n" terms

$S_n = a + a+d + a+2d + a+3d + \dots + a+(n-1)d$

So

$S_n = a + a+d + \dots + a+(n-2)d + a+(n-1)d$

$S_n = a+(n-1)d + a+(n-2)d + \dots + (a+d) + a$

---

$2S_n =$

Dec 16-12:26 PM

The sum of the first  $n$  terms of an arithmetic series is

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

HOW?

Can you figure out how to manipulate the first formula so that it becomes the second?

$$= \frac{n(t_1 + t_n)}{2}$$

**NOTE**

Either formula will provide you with the sum of terms. You choose the formula that works with the information given.

Jan 4-10:59 PM

Determine whether each series is arithmetic.

If it is Arithmetic, calculate the sum of the first 20 terms.

a)  $2 + 10 + 50 + 250 + \dots$

b)  $18 + 22 + 26 + 30 + \dots$

May 27-3:22 PM

For each series, calculate the 12th term and the sum of the 12 first terms.

$$t_{12}$$

$$S_{12}$$

a)  $-18 - 12 - 6 + 0 + \dots$

b)  $-13 - 24 - 35 - 46 - \dots$

May 27-3:22 PM

In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?



What do you know?

$$S_{50} = \frac{(50)[2(23) + (50 - 1)(4)]}{2}$$

$$= 6050$$

There are 6050 seats in the amphitheatre.

Jan 4-10:59 PM

Determine the sum of -31, -35, -39, ... -403.

**Step 1** - Determine the type of series it is.

$$t_2 - t_1 = -35 - (-31) = -4$$

$$t_3 - t_2 = -39 - (-35) = -4$$

I checked to see if the series was arithmetic. So I calculated a few 1st differences. The differences were the same, so the series is arithmetic.

**Step 2**

Use the general formula to find n.

$$t_n = a + (n - 1)d$$

$$-403 = -31 + (n - 1)(-4)$$

$$-403 + 31 = (n - 1)(-4)$$

$$-372 = (n - 1)(-4)$$

$$\frac{-372}{-4} = \frac{(n - 1)(-4)}{-4}$$

$$93 = n - 1$$

$$93 + 1 = n$$

$$94 = n$$

**Step 3**

Use the sum of a series formula.

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{94} = \frac{94[-31 + (-403)]}{2}$$

$$= -20\,398$$

Jan 4-10:59 PM

**PRACTICE**

PAGE 452

2, 3, 4acf, 5adf, 6ab, 7e, 9, 11, 13, 16

Jan 5-10:14 AM