7.1 Arithmetic Sequences

7.2 Geometric Sequences

Learning Goal: By the end of today, (i) I will be able to recognize an arithmetic and geometric sequence, (II) I will be able to use the equations to solve for missing elements in the sequence.

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Terms to know:

sequence
an ordered list of numbers

term
a number in a sequence. Subscripts are usually used to identify the positions of the terms.

arithmetic sequence
a sequence that has the same difference, the **common difference**, between any pair of consecutive terms

recursive sequence
a sequence for which one term (or more) is given and each successive term is determined from the previous term(s)

general term
a formula, labelled t_n, that expresses each term of a sequence as a function of its position. For example, if the general term is $t_n = 2n$, then to calculate the 12th term (t_{12}), substitute $n = 12$.

$$t_{12} = 2(12) = 24$$

ecursive formula
ı formula relating the general erm of a sequence to the revious term(s)

geometric sequence
a sequence that has the same ratio, the **common ratio**, between any pair of consecutive terms

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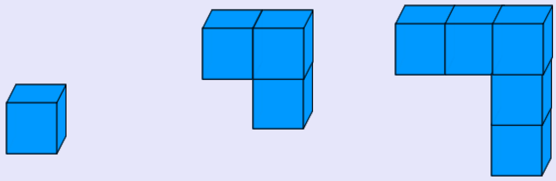
CONSIDER THIS...

Julien used linking cubes to create a sequence of shapes. The first three shapes are shown below.



How many linking cubes are there in the 100th figure?

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While pondering the 100th figure question let's answer the following...

a. What are the next 3 terms of this arithmetic sequence?

b. How is each term of this recursive sequence related to the previous term?

c. Construct a graph of term (number of cubes) vs figure number. What type of relationship is this?

d. Determine a formula for the general term of the sequence

e. Use the general term to calculate the 100th term.

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SOLUTIONS TO OUR INVESTIGATION

A. The next three terms in the sequence are 7, 9, and 11.
B. Each term contains two more cubes than the previous term.
C. Each term is two more than the previous term, which means the relation has a constant rate of change. So it is a linear relation.
D. The rate of change is 2 and the relation is linear, so $t_n = 2n + b$. Since $t_1 = 1$, I substituted this value into the equation for t_n and solved for b.

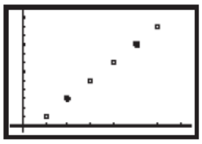
$$1 = 2(1) + b$$
$$1 = 2 + b$$
$$-1 = b$$

So $t_n = 2n - 1$.
E. I substituted $n = 100$ into the equation $t_n = 2n - 1$ from part D.

$$t_n = 2(100) - 1$$
$$= 199$$

The 100th term is 199.

C.



The linear relationship is evident by graphing.

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TO SUMMARIZE...

- An arithmetic sequence is created when the same value is added each time. The value that is being added is called the common difference.
- The terms can be determined by using the general formula $t_n = a + (n - 1)d$, where a is the initial term, d is the common difference, and n is the number of terms.

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PRACTICE ...ARITHMETIC SEQUENCES

What is the 30th term of the sequence: 18, 11, 4,...

a =

d =

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The 7th term of an arithmetic sequence is 53 and the 11th term is 97. Determine the 100th term.

List what you know...

$t_7 = 53$

$t_n = a + (n - 1)d$

$53 = a + (7 - 1)d$

$53 = a + 6d$

$t_{11} = 97$

$t_n = a + (n - 1)d$

$97 = a + (11 - 1)d$

$97 = a + 10d$

Now, solve the system, using elimination.

$97 = a + 10d$

$-\ 53 = a + 6d$

$44 = 4d$

$d = 11$

Use d = 11 to find a (substitute in either equation)

$53 = a + 6(11)$

$a = -13$

SUBSTITUTE d and a to get the general formula:

$t_n = a + (n - 1)d$

$t_n = -13 + (n - 1)11$

$t_n = 11n - 24$

THEREFORE, the 100th term is... $t_{100} = 11(100) - 24 = 1076$

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Find the number of terms given the following arithmetic sequence:

7, 9, 11, 13, ..., 63

List what you know...

Can you rewrite the formula in terms of n?

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FURTHER EXAMPLES FOR ARITHMETIC SEQUENCES

In an arithmetic sequence, the 50th term is 238 and the 93rd term is 539. Find the value of the 100th term.

Remember...solve the "a" and "d" by setting up the equations.

$t_{50}=238$ $t_{93}=539$

Jan 3-8:02 PM

There are also sequences that grow by multiplication...

1,2,4,8,16,....

These are called geometric sequences.

We use "r" as the ratio instead of "d" as difference. And now it is multiplied rather than added to get to the next term.

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For Example

1,2,4,8,16,....

a=1

r=2

term 1 = a =

term 2 = ar =

term 3 = arr =

term 4 = arrr =

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GEOMETRIC SEQUENCES

The general formula for a geometric sequence is:

$$t_n = ar^{n-1}$$

a is the initial term
r is the common ratio
n is the term number

1 Determine the following, given the geometric sequence 4, 20, 100,...

a) the general term (formula)

b) t_6 (using algebra)

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2 In a geometric sequence, term 2 is -4 and term 7 is 128. Determine the value of term 10.

$t_2 = -4$

$t_7 = 128$

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In Summary

Key Ideas

- Every sequence is a discrete function. Since each term is identified by its position in the list (1st, 2nd, and so on), the domain is the set of natural numbers, **N** = {1, 2, 3, ...}. The range is the set of all the terms of the sequence.
  For example, 4, 12, 20, 28, ... (4 is the 1st term, 12 is the 2nd term)

  Domain: {1, 2, 3, 4, ...}
  Range: {4, 12, 20, 28, ...}

- An arithmetic sequence is a recursive sequence in which new terms are created by adding the same value (the common difference) each time.
  For example, 2, 6, 10, 14, ... is increasing with a common difference of 4, (+4 +4 +4)

  $t_2 - t_1 = 6 - 2 = 4$
  $t_3 - t_2 = 10 - 6 = 4$
  $t_4 - t_3 = 14 - 10 = 4$
  ⋮

  and 9, 6, 3, 0, ... is decreasing with a common difference of −3. (−3 −3 −3)

  $t_2 - t_1 = 6 - 9 = -3$
  $t_3 - t_2 = 3 - 6 = -3$
  $t_4 - t_3 = 0 - 3 = -3$
  ⋮

Need to Know

- An arithmetic sequence can be defined
  - by the general term $t_n = a + (n - 1)d$,
  - recursively by $t_1 = a$, $t_n = t_{n-1} + d$, where $n > 1$, or
  - by a discrete linear function $f(n) = dn + b$, where $b = t_0 = a - d$.

  In all cases, $n \in \mathbf{N}$, a is the first term, and d is the common difference.

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In Summary

Key Idea

- A geometric sequence is a recursive sequence in which new terms are created by multiplying the previous term by the same value (the common ratio) each time.
  For example, 2, 6, 18, 54, ... is increasing with a common ratio of 3, (×3 ×3 ×3)

  $\frac{t_2}{t_1} = \frac{6}{2} = 3$
  $\frac{t_3}{t_2} = \frac{18}{6} = 3$
  $\frac{t_4}{t_3} = \frac{54}{18} = 3$
  ⋮

  and 144, 72, 36, 18, ... is decreasing with a common ratio of $\frac{1}{2}$. ($\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$)

  $\frac{t_2}{t_1} = \frac{72}{144} = \frac{1}{2}$
  $\frac{t_3}{t_2} = \frac{36}{72} = \frac{1}{2}$
  $\frac{t_4}{t_3} = \frac{18}{36} = \frac{1}{2}$
  ⋮

  If the common ratio is negative, the sequence has terms that alternate from positive to negative. For example, 5, −20, 80, −320, ... has a common ratio of −4. (× (−4) × (−4) × (−4))

Need to Know

- A geometric sequence can be defined
  - by the general term $t_n = ar^{n-1}$,
  - recursively by $t_1 = a$, $t_n = rt_{n-1}$, where $n > 1$, or
  - by a discrete exponential function $f(n) = ar^{n-1}$.

  In all cases, $n \in \mathbf{N}$, a is the first term, and r is the common ratio.

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PRACTICE

A.S. pg. 424 #1-2ac, 4, 5-6a, 8ac,10

G.S. pg. 430 #1-2ac, 4, 5-6a, 8ac, 11

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