4.6 Transformations of Exponential Functions

Oct 29-10:17 AM

In chapter 1 we learned about the "parent" functions and how they moved.

We will now consider $y = 2^x$ as the parent function and its movements

Let's review the terminology:

$$f(x) = af\left(k\left(x - d\right)\right) + c$$

a

k

d

c

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Let's consider the general form of an exponential function and its transformations.

$$f(x) = ab^{k(x-d)} + c$$

a

b

k

d

c

(asymptote occurs at y = c)

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you might have to factor the exponent to see what the transformations are.

EX. $y = 2(3)^{2x+2}$

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Ex. Describe the transformations

a) $g(x)=\frac{1}{2}(3)^{4x}-5$

b) $g(x)=2\left(\frac{1}{3}\right)^{4x+2}+1$

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Let's consider the general form of an exponential function and its transformations.

$$f(x)=ab^{k(x-d)}+c$$

In order to sketch or graph these transformations, a formula we can use to transform all points of the function:

$$\left(\frac{x}{k}+d, ay+c\right)$$

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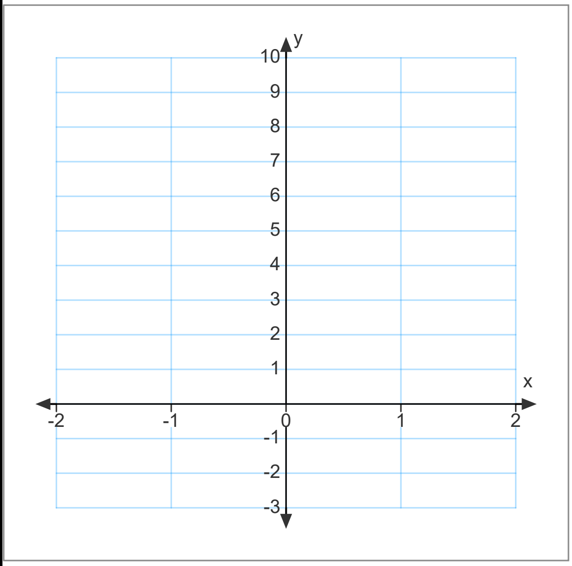
Ex. Sketch the following:

a) $y = 3\left(\frac{1}{2}\right)^{2x} - 3$

a = k = d = c =

Base Function



Key Points "New" Points

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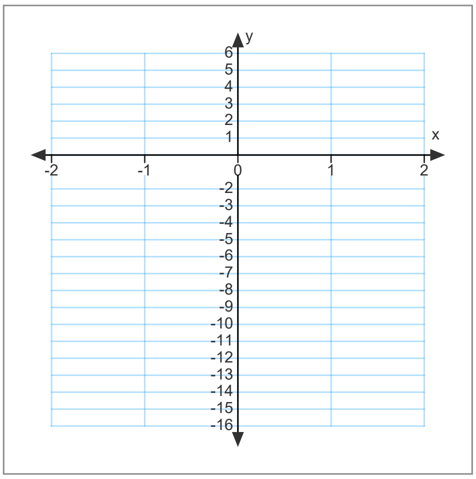
b) $y = -4(2)^{-x}$

a = k = d = c =

Base Function



Key Points "New" Points

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An exponential function with a base of 2 has been stretched vertically by a factor of 1.5 and reflected in the *y*-axis. Its asymptote is the line $y = 2$.
Its *y*-intercept is (0, 3.5).
Write an equation of the function and state its domain and range.

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In Summary

Key Ideas

- In functions of the form $g(x) = af(k(x - d)) + c$, the constants a, k, d, and c change the location or shape of the graph of $f(x)$. The shape is dependent on the value of the base function $f(x) = b^x$, as well as on the values of a and k.
- Functions of the form $g(x) = af(k(x - d)) + c$ can be graphed by applying the appropriate transformations to the key points of the base function $f(x) = b^x$, one at a time, following the order of operations. The horizontal asymptote changes when vertical translations are applied.

Need to Know

- In exponential functions of the form $g(x) = a\,b^{k(x-d)} + c$:
  - If $|a| > 1$, a vertical stretch by a factor of $|a|$ occurs. If $0 < |a| < 1$, a vertical compression by a factor of $|a|$ occurs. If a is also negative, then the function is reflected in the *x*-axis.
  - If $|k| > 1$, a horizontal compression by a factor of $|\frac{1}{k}|$ occurs. If $0 < |k| < 1$, a horizontal stretch by a factor of $|\frac{1}{k}|$ occurs. If k is also negative, then the function is reflected in the *y*-axis.
  - If $d > 0$, a horizontal translation of d units to the right occurs. If $d < 0$, a horizontal translation to the left occurs.
  - If $c > 0$, a vertical translation of c units up occurs. If $c < 0$, a vertical translation of c units down occurs.
  - You might have to factor the exponent to see what the transformations are. For example, if the exponent is $2x + 2$, it is easier to see that there was a horizontal stretch of 2 and a horizontal translation of 1 to the left if you factor to $2(x + 1)$.
  - When transforming functions, consider the order. You might perform stretches and reflections followed by translations, but if the stretch involves a different coordinate than the translation, the order doesn't matter.
  - The domain is always $\{x \in \mathbb{R}\}$. Transformations do not change the domain.
  - The range depends on the location of the horizontal asymptote and whether the function is above or below the asymptote. If it is above the asymptote, its range is $y > c$. If it is below, its range is $y < c$.

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Homework

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