FREQUENTLY ASKED Questions

Q: How do you determine the future value of an annuity?

A1: An annuity is a series of payments or investments made at regular intervals. The future value of an annuity is the sum of all regular payments and interest earned. You can determine the future value of each payment or investment by using the formula $A = P(1 + i)^n$.

Since an annuity consists of regular payments, the future values of the investments, starting from the last, will be P, $P(1 + i)$, $P(1 + i)^2$, ... These form a geometric sequence with common ratio $1 + i$. So the future value of all of the investments is the geometric series $P + P(1 + i) + P(1 + i)^2 + ...$, which can be calculated with the formula for the sum of a geometric series.

A2: You can use technology such as a spreadsheet or the TVM Solver on a graphing calculator to calculate the future value of an annuity.

EXAMPLE

The spreadsheet below is set up for an annuity in which 40 regular investments of $250 are made at the end of each compounding period. The annuity earns 2% interest per compounding period.

Since the last $250 investment was deposited at the end of the term, it earned no interest. The first $250 investment earned interest 39 times, but didn't earn interest during the first compounding period because it was deposited at the end of that period.

	A Number of Compounding Periods Invested	B Amount Invested	C Future Value
1			
2	0	$250.00	"=B2"
3	"=A2+1"	$250.00	"=B3*(1+0.02)"
4	"=A3+1"	$250.00	"=B4*(1+0.02)*2"

	A Number of Compounding Periods Invested	B Amount Invested	C Future Value
1			
2	0	$250.00	$250.00
3	1	$250.00	$255.00
4	2	$250.00	$260.10
5	3	$250.00	$265.30
6	4	$250.00	$270.61
7	5	$250.00	$276.02
8	6	$250.00	$281.54
37	35	$250.00	$499.97
38	36	$250.00	$509.97
39	37	$250.00	$520.17
40	38	$250.00	$530.57
41	39	$250.00	$541.19
42			$15 100.50

The future value of this annuity is $15 100.50

A3: Use the formula for the future value of an annuity, $FV = R \times \left(\dfrac{(1 + i)^n - 1}{i} \right)$,

where FV is the future value; R is the regular payment each compounding period; i is the interest rate per compounding period, expressed as a decimal; and n is the number of compounding periods.

Q: How can you determine the present value of an annuity?

A1: The present value of an annuity is the amount of money you have to invest to get a specific amount some time in the future. You can determine the present value of each investment by using the formula $PV = A(1 + i)^{-n}$.

Since an annuity consists of regular payments, the present values of the investments, starting from the first, will be $A(1 + i)^{-1}$, $A(1 + i)^{-2}$, $A(1 + i)^{-3}$, ... These form a geometric sequence with common ratio $(1 + i)^{-1}$. So the present value of all of the investments is the geometric series $A(1 + i)^{-1} + A(1 + i)^{-2} + A(1 + i)^{-3} + \ldots$, which can be calculated with the formula for the sum of a geometric series.

A2: You can use technology such as a spreadsheet or the TVM Solver on a graphing calculator to calculate the present value of an annuity.

EXAMPLE

The spreadsheet below is set up for an annuity earning 0.5% interest per compounding period and providing 20 regular payments of $50.

	A Number of Compounding Periods Invested	B Payment	C Present Value
1			
2	1	$50.00	"=B2/1.005"
3	"=A2+1"	$50.00	"=B3/(1.005)^A3"
4	"=A3+1"	$50.00	"=B4/(1.005)^A4"

	A Number of Compounding Periods Invested	B Payment	C Present Value
1			
2	1	$50.00	$49.75
3	2	$50.00	$49.50
4	3	$50.00	$49.26
5	4	$50.00	$49.01
6	5	$50.00	$48.77
7	6	$50.00	$48.53
17	16	$50.00	$46.17
18	17	$50.00	$45.94
19	18	$50.00	$45.71
20	19	$50.00	$45.48
21	20	$50.00	$45.25
22			$949.37

The present value of all of the investements in this annuity is $949.37.

A3: Use the formula for the present value of an annuity,

$PV = R \times \left(\dfrac{1 - (1 + i)^{-n}}{i} \right)$, where PV is the present value; R is the regular payment each compounding period; i is the interest rate per compounding period, expressed as a decimal; and n is the number of compounding periods.

PRACTICE Questions

Lesson 8.1

1. Each situation represents an investment earning simple interest. Calculate the interest earned and the total amount.

	Principal	Rate of Simple Interest per Year	Time
a)	$3 500	6%	10 years
b)	$15 000	11%	3 years
c)	$280	3.2%	34 months
d)	$850	29%	100 weeks
e)	$21 000	7.3%	42 days

2. Pia invests $2500 in an account that earns simple interest. At the end of each month, she earns $11.25 in interest.
   a) What annual rate of simple interest is Pia earning? Round your answer to two decimal places.
   b) How much money will be in her account after 7 years?
   c) How long will it take for her money to double?

3. Karl borrows some money and is charged simple interest. The graph below shows how the amount he owes grows over time.

Karl's Loan being Charged Simple Interest
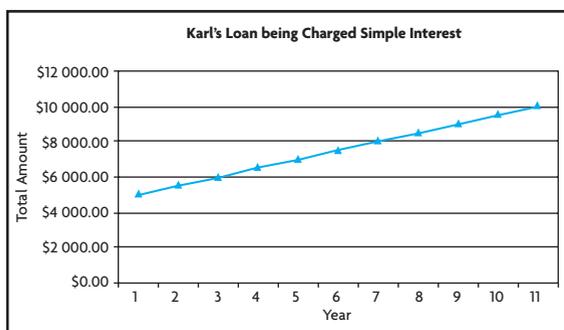
   a) How much did Karl borrow?
   b) What annual interest rate is he being charged?
   c) How long will it take before he owes $20 000?

Lesson 8.2

4. Isabelle invests $4350 at 7.6%/a compounded quarterly. How long will it take for her investment to grow to $10 000?

5. Each situation represents a loan being charged compound interest. Calculate the total amount and the interest being charged.

	Principal	Rate of Compound Interest per Year	Compounding Period	Time
a)	$4 300	9.1%	annually	8 years
b)	$500	10.4%	semi-annually	11.5 years
c)	$25 000	6.4%	quarterly	3 years
d)	$307	27.6%	monthly	2.5 years

6. Deana invests some money that earns interest compounded annually. At the end of the first year, she earns $400 in interest. At the end of the second year, she earns $432 in interest.
   a) What interest rate, compounded annually, is Deana earning? Round your answer to two decimal places.
   b) How much did she invest?

7. Vlad purchased some furniture for his apartment. The total cost was $2942.37. He paid $850 down and financed the rest for 18 months. At the end of the finance period, Vlad owed $2147.48. What annual interest rate, compounded monthly, was he being charged? Round your answer to two decimal places.

Lesson 8.3

8. Calculate the present value of each investment.

	Rate of Compound Interest per Year	Compounding Period	Time	Future Value
a)	6.7%	annually	5 years	$8 000
b)	8.8%	semi-annually	2.5 years	$1 280
c)	5.6%	quarterly	8 years	$100 000
d)	24.6%	monthly	1.5 years	$850

9. Roberto financed a purchase at 9.6%/a compounded monthly for 2.5 years. At the end of the financing period, he still owed $847.53. How much money did Roberto borrow?

10. Marisa invests $1650 for 3 years, at which time her investment is worth $2262.70. What interest rate, compounded annually, would yield the same results? Round your answer to two decimal places.

Lesson 8.4

11. For each annuity, calculate the future value and the interest earned.

	Regular Payment	Rate of Compound Interest per Year	Compounding Period	Time
a)	$2500 per year	7.6%	annually	12 years
b)	$500 every 6 months	7.2%	semi-annually	9.5 years
c)	$2500 per quarter	4.3%	quarterly	3 years

12. Naomi wants to save $100 000, so she makes quarterly payments of $1500 into an account that earns 4.4%/a compounded quarterly. How long will it take her to reach her goal?

Lesson 8.5

13. Ernie wants to invest some money each month at 9%/a compounded monthly for 6 years. At the end of that time, he would like to have $25 000. How much money does he have to put away each month?

14. For each loan, calculate the amount of the loan and the interest being charged.

	Regular Payment	Rate of Compound Interest per Year	Compounding Period	Time
a)	$450 per year	5.1%	annually	6 years
b)	$2375 every 6 months	9.2%	semi-annually	4.5 years
c)	$185.73 per quarter	12.8%	quarterly	3.5 years
d)	$105.27 per month	19.2%	monthly	1.5 years

15. Paul borrows $136 000. He agrees to make monthly payments for the next 20 years. The interest rate being charged is 6.6%/a compounded monthly.
a) How much will Paul have to pay each month?
b) How much interest is he being charged over the term of the loan?

16. Eden finances a purchase of $611.03 by making monthly payments of $26.17 for $2\frac{1}{2}$ years. What annual interest rate, compounded monthly, is she being charged? Round your answer to two decimal places.

17. Chantal purchases a moped for $1875.47 with $650 down. She finances the balance at 6.6%/a compounded monthly over 4 years. How much will Chantal have to pay each month?

Lesson 8.6

18. Starting at age 20, Ken invests $100 per month in an account that earns 5.4%/a compounded monthly. Starting at age 37, his twin brother, Adam, starts saving money in an account that pays 7.2%/a compounded monthly. How much more money will Adam need to invest each month if he wants his investment to be worth the same as Ken's by the time they are 55 years old?

19. Jenny starts a business and borrows $100 000 at 4.2%/a compounded monthly. She can afford to make payments between $1000 and $1500 per month. How much sooner can she pay off the loan if she pays the maximum amount compared with the minimum amount?

20. Kevin purchases a guitar on a payment plan of $17.85 per week for $2\frac{1}{2}$ years at 13%/a compounded weekly. What was the selling price of the guitar?

1. For each investment, determine the total amount and the interest earned.

Principal	Rate of Interest per Year	Time
a)	$850	9% simple interest	6 years
b)	$5460	8.4% compounded semi-annually	13 years
c)	$230 per month	4.8% compounded monthly	$6\frac{1}{2}$ years

Loan #1	
Year	Amount Owed
1	$3796
2	$3942
3	$4088

Loan #2	
Year	Amount Owed
2	$977.53
3	$1036.18
4	$1098.35

2. The amounts owed for two different loans are shown at the left.
   a) For each loan, determine whether simple interest or compound interest is being charged. Justify your answer.
   b) What annual interest rate is each loan being charged? Round your answer to two decimal places.
   c) How much was each loan originally?
   d) Determine the future value of each loan after 10 years.

3. Betsy inherits $15 000 and would like to put some of it away for a down payment on a house in 8 years. If she would like to have $25 000 for the down payment, how much of her inheritance must she invest at 9.2%/a compounded quarterly?

4. Derek invests $250 per month for $6\frac{1}{2}$ years at 4.8%/a compounded monthly. How much will his investment be worth at the end of the $6\frac{1}{2}$ years?

5. Simone wants to save money for her retirement. Her two best options are 5.88%/a compounded monthly or 6%/a compounded annually. Which option should she choose? Why?

6. Anand's parents have been paying $450 per month into a retirement fund for the last 30 years. The fund is now worth $450 000. What annual interest rate, compounded monthly, are Anand's parents earning? Round your answer to two decimal places.

7. Yvette wants to invest some money under these conditions:
   • Each quarter for the next 17 years, she wants to earn 8.4%/a compounded quarterly.
   • After 17 years, she plans to reinvest the money at 7.2%/a compounded monthly.
   • She wants to withdraw $5000 per month for the 10 years after the initial 17 years.
   How much more would she have to invest per quarter if she earned 7.2%/a compounded quarterly for the first 17 years and 8.4%/a compounded monthly for the next 10 years?

Saving for Retirement

Steve, Carol, and Lisa get their first full-time jobs and talk about saving for retirement. They are each 22 years old and plan to work until they are 55.

Steve starts investing immediately and puts aside $150 per month. Carol wants to enjoy life a bit and decides to start contributing when she is 30. Lisa thinks that they are both starting too early and decides to wait until she is 42 before starting to save.

Assume that Steve, Carol, and Lisa are each earning 9%/a compounded monthly. Carol and Lisa want to accumulate the same amount as Steve upon retirement. When they retire, Steve wants his investment to last 10 years, Carol wants hers to last 15 years, and Lisa wants hers to last 20 years.

? **How much will Steve, Carol, and Lisa be able to withdraw monthly upon retirement?**

A. What strategies will you use to solve this problem? Justify your strategies.

B. How much money will Steve have accumulated by the time he is 55?

C. For how many months will Carol and Lisa be making payments?

D. How much will Carol and Lisa have to put away each month to meet their goals?

E. For how many months will each person withdraw money?

F. How much will each person be able to withdraw from his or her nest egg each month?

Task | Checklist

✔ Did you explain and justify your strategies?

✔ Did you show your work?

✔ Did you support your calculations with appropriate reasoning?

✔ Did you explain your thinking clearly?