

8.1

Simple Interest

YOU WILL NEED

- graphing calculator
- spreadsheet software



principal

a sum of money that is borrowed or invested

simple interest

interest earned or paid only on the original sum of money that was invested or borrowed

interest

the money earned from an investment or the cost of borrowing money

amount

the total value of an investment or loan. The amount is given by $A = P + I$, where A is the amount, P is the principal, and I is the interest.

Tech Support

For help using a graphing calculator to enter lists and to create scatter plots, see Technical Appendix, B-11.

GOAL

Calculate simple interest.

INVESTIGATE the Math

Amanda wants to invest \$2000. Her bank will pay 6% of the **principal** per year each year the money is kept in a savings account that earns **simple interest**.

- ?** What function can be used to model the growth of Amanda's money?
- A.** Calculate the **interest** earned and the **amount** of the investment at the end of the first year. Record your results in a table as shown.

Year	Interest Earned	Amount
0	—	\$2000
1		
2		
3		

- B.** Calculate the interest earned and the amount of the investment at the end of the second and third years. Record your results in your table.
- C.** Enter your data for Year and Amount into either lists on a graphing calculator or columns in a spreadsheet.
- D.** Create a scatter plot from your two lists or columns, using Year as the independent variable.
- E.** What type of function best models the growth of Amanda's money? You may need to calculate more data points before you decide. Explain your reasoning.
- F.** Determine the equation of the function that models the amount of Amanda's investment over time.

Reflecting

- G. What type of sequence could you use to represent the amount of Amanda's investment for successive years? How do you know?
- H. How does the recursive formula for this sequence relate to her investment?
- I. How do the principal, interest, and amount of Amanda's investment relate to
- the sequence from part G that represents the amount of the investment over time?
 - the function from part F that represents the amount of the investment over time?

APPLY the Math

EXAMPLE 1

Representing any situation earning simple interest as a function

Allen invests \$3240 at 2.4%/a simple interest.

- Calculate the interest earned each year.
- Calculate the amount and the total interest earned after 20 years.
- Determine the total amount, A , and the interest, I , earned if he invested a principal, $\$P$, for t years at $r\%/a$ simple interest.



Communication | Tip

Interest rates are often advertised as a certain percent per year. So a rate of 5% means that 5% interest is earned each year. These rates are sometimes abbreviated to 5%/a, which means 5% per annum, or year.

Jasmine's Solution

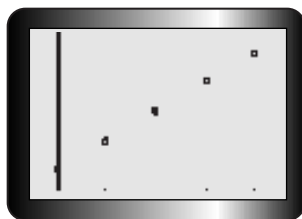
a) $I = 0.024 \times \$3240$
 $= \$77.76$

Each year, Allen earns 2.4% of the principal as interest. 2.4% as a decimal is 0.024. To calculate the interest earned each year, I multiplied the principal by the interest rate.

b)

Year	Interest (\$)	Amount (\$)
0	0	3240
1	77.76	$3240 + 77.76 = 3317.76$
2	77.76	$3317.76 + 77.76 = 3395.52$
3	77.76	$3395.52 + 77.76 = 3473.28$
4	77.76	$3473.28 + 77.76 = 3551.04$

I set up a table to calculate the amount at the end of each year. I added the interest earned each year to the previous amount. Then I entered the year and amount into separate lists on a graphing calculator.



I used the calculator to create a scatter plot of amount versus time. The graph is linear. I used two points to calculate the slope and I found that it was \$77.76. That is the rate of change of the amount. The y -intercept is Allen's principal of \$3240.

$$f(t) = 77.76t + 3240$$

I used the slope and y -intercept to create a linear function in terms of t , the time in years, and $f(t)$, the amount.

$$\begin{aligned} f(20) &= 77.76(20) + 3240 \\ &= 4795.20 \end{aligned}$$

To determine the amount after 20 years, I substituted $t = 20$.

$$\begin{aligned} I &= 4795.20 - 3240 \\ &= 1555.20 \end{aligned}$$

To determine the total interest earned, I subtracted the principal from the amount.

After 20 years, Allen will have \$4795.20 and will have earned \$1555.20 in interest.

c) end of 1st year:

$$I_1 = Pr$$

After one year, Allen would earn $r\%$ of his original investment of $\$P$.

end of 2nd year:

$$I_2 = Pr + Pr = 2Pr$$

Each year, the total interest earned would go up by the same amount. The increase would be the interest earned in one year, $P \times r$.

end of 3rd year:

$$I_3 = Pr + Pr + Pr = 3Pr$$

Since the interest earned at the end of each year depends on time, I wrote a formula for interest in terms of t , the time in years.

end of t th year:

$$I = Prt$$

The amount is the sum of the principal and the interest. Since interest depends on time, the amount must also depend on time.

$$A = P + I$$

$$A = P + Prt$$

$$A = P(1 + rt)$$

I wrote the formula for the amount by factoring out the P .

The total amount of an investment of $\$P$ for t years at $r\%$ /a simple interest is $A = P(1 + rt)$, and the total interest earned is $I = Prt$.

EXAMPLE 2

Using a spreadsheet to represent the amount owed

Tina borrows \$15 000 at 6.8%/a simple interest. She plans to pay back the loan in 10 years. Calculate how much she will owe at the end of each year during this period.

Tom's Solution

	A	B	C
1	Time (Years)	Total Interest Charged	Total Amount of Loan
2			\$15 000.00
3	1	"= C2* (6.8/100)"	"= C2 + B3"
4	2	"= C2* (6.8/100)"	"= C3 + B4"

	A	B	C
1	Time (Years)	Total Interest Charged	Total Amount of Loan
2			\$15 000.00
3	1	\$1 020.00	\$16 020.00
4	2	\$1 020.00	\$17 040.00
5	3	\$1 020.00	\$18 060.00
6	4	\$1 020.00	\$19 080.00
7	5	\$1 020.00	\$20 100.00
8	6	\$1 020.00	\$21 120.00
9	7	\$1 020.00	\$22 140.00
10	8	\$1 020.00	\$23 160.00
11	9	\$1 020.00	\$24 180.00
12	10	\$1 020.00	\$25 200.00

I set up a spreadsheet to calculate the interest charged every year and the loan amount. Each year, 6.8% of \$15 000, or \$1020, will be charged in interest.

I used the spreadsheet to calculate the amount Tina would need to pay back at the end of each year during the 10-year period.

Tech Support

For help using a spreadsheet to enter values and formulas, and to fill down, see Technical Appendix, B-21.

EXAMPLE 3

Selecting a strategy to calculate the amount owed after less than a year

Philip borrows \$540 for 85 days by taking a cash advance on his credit card. The interest rate is 26%/a simple interest. How much will he need to pay back at the end of the loan period, and how much interest will he have paid?

Lara's Solution

$$t = \frac{85}{365}$$

$$P = \$540$$

$$r = 26\% = 0.26$$

$$A = P(1 + rt)$$

$$= (\$540) \left(1 + 0.26 \times \frac{85}{365} \right)$$

$$\doteq (\$540)(1.061)$$

$$= \$572.70$$

Of the \$572.70 that Philip has to pay back, \$32.70 is interest.

Philip isn't borrowing the money for a full year, so I expressed the time as a fraction of 365 days in a year. I knew the principal, P , and I wrote the interest rate, r , as a decimal.

To calculate the amount at the end of the loan period, I substituted the values of P , r , and t into the formula.

I rounded to the nearest cent.

I subtracted the principal to get the interest.

EXAMPLE 4**Calculating the time needed to earn a specific amount on an investment**

Tanya invests \$4850 at 7.6%/a simple interest. If she wants the money to increase to \$8000, how long will she need to invest her money?

Josh's Solution

$$P = \$4850$$

$$r = 7.6\% = 0.076$$

$$A = \$8000$$

$$A = P + Prt$$

$$8000 = 4850 + (4850)(0.076)t$$

$$8000 = 4850 + 368.6t$$

$$3150 = 368.6t$$

$$t = \frac{3150}{368.6}$$

$$\doteq 8.546$$

$$0.546 \times 365 \text{ days} \doteq 199.2 \text{ days}$$

Tanya would have to invest her money for 8 years and 200 days to get \$8000.

I knew the principal, the interest rate, and the total amount.

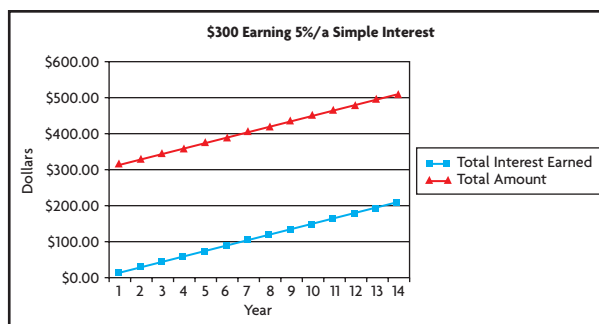
I substituted the values of P , r , and A into the formula and solved for t .

When I solved for t , I got a value greater than 8.

The 8 meant 8 years, so I had to figure out what 0.546 of a year was.

In Summary**Key Ideas**

- Simple interest is calculated only on the principal.
- The total amount, A , and interest earned, I , are linear functions in terms of time, so their graphs are straight lines (see graph below). The values of A and I at the end of each interest period form the terms of two arithmetic sequences.



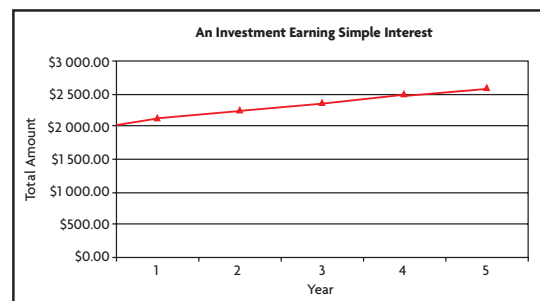
(continued)

Need to Know

- Simple interest can be calculated using the formula $I = Prt$, where I is the interest; P is the principal; r is the interest rate, expressed as a decimal; and t is time, expressed in the same period as the interest rate, usually per year.
- The total amount, A , of an investment earning simple interest can be calculated using the formula $A = P + Prt$ or $A = P(1 + rt)$.
- Unless otherwise stated, an interest rate is assumed to be per year.

CHECK Your Understanding

- Each situation represents an investment earning simple interest. Calculate the total amount at the end of each period.
 - 1st, 2nd, and 3rd years ii) 15 years
 - \$500 at 6.4%/a c) \$25 000 at 5%/a
 - \$1250 at 4.1%/a d) \$1700 at 2.3%/a
- The graph at the right represents the total amount of an investment of principal P earning a fixed rate of simple interest over a period of 5 years.
 - What is the principal?
 - How much interest is earned in 5 years?
 - What interest rate is being applied?
 - State the equation that represents the amount as a function of time.
- Michel invests \$850 at 7%/a simple interest. How long will he have to leave his investment in the bank before earning \$200 in interest?
- Sally has a balance of \$2845 on her credit card. What rate of simple interest is she being charged if she must pay \$26.19 interest for the 12 days her payment is late?



PRACTISING

- For each investment, calculate the interest earned and the total amount.

K

	Principal	Rate of Simple Interest per Year	Time
a)	\$500	4.8%	8 years
b)	\$3 200	9.8%	12 years
c)	\$5 000	3.9%	16 months
d)	\$128	18%	5 months
e)	\$50 000	24%	17 weeks
f)	\$4 500	12%	100 days

6. Mario borrows \$4800 for 8.5 years at a fixed rate of simple interest. At the end of that time, he owes \$8000. What interest rate is he being charged?
7. How much money must be invested at 6.3%/a simple interest to earn \$250 in interest each month?
8. Nina deposits \$3500 into a savings account. The rate of simple interest is 5.5%/a.
 - a) By how much does the amount in her account increase each year?
 - b) Determine the amount in her account at the end of each of the first 5 years.
 - c) State the total amount as the general term of a sequence.
 - d) Graph this sequence.
9. Ahmad deposits an amount on September 1, 2005, into an account that earns simple interest quarterly. His bank sends him statements after each quarter. The amounts for the first four quarters are shown.
 - a) How much did Ahmad invest?
 - b) What rate of simple interest is he earning?

Statement	Date	Balance
1	Dec. 1, 2005	\$3994.32
2	Mar. 1, 2006	\$4248.64
3	Jun. 1, 2006	\$4502.96
4	Sept. 1, 2006	\$4757.28

Year	Amount Owed
1	\$2081.25
2	\$2312.50
3	\$2543.75
4	\$2775.00
5	\$3006.25

10. Anita borrows some money at a fixed rate of simple interest. The amount she **A** owes at the end of each of the first five years is shown at the left.
 - a) How much did Anita borrow?
 - b) State the total amount as the general term of a sequence.
 - c) How much time will have passed before Anita owes \$7500?
11. Len invests \$5200 at 3%/a simple interest, while his friend Dave invests **T** \$3600 at 5%/a simple interest. How long will it take for Dave's investment to be worth more than Len's?
12. Lotti invests some money at a fixed rate of simple interest. She uses the **C** function $A(t) = 750 + 27.75t$ to calculate how much her investment will be worth after t years. How much did she invest and what interest rate is she earning? Explain your reasoning.

Extending

13. The doubling time of an investment is the length of time it takes for the total amount invested to become double the original amount invested. Determine a formula for the doubling time, D , of an investment of principal $\$P$ earning a rate of simple interest of $r\%/a$.
14. Sara's parents decide to invest \$500 on each of her birthdays from the day she is born until she becomes 25. Each investment earns 6.4%/a simple interest. What will be the total amount of the investments when Sara is 25?

