5 Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 5.5, Examples 1 to 4.
- Try Chapter Review Questions 6 and 7.

Q: What steps would you follow to prove a trigonometric identity?

A: A trigonometric identity is an equation involving trigonometric ratios that is true for all values of the variable. You may rewrite the trigonometric ratios in terms of x, y, and r and then simplify, or you may rewrite each side of the equation in terms of sine and cosine and then use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$, where appropriate. If a trigonometric ratio is in the denominator of a fraction, there are restrictions on the variable because the denominator cannot equal zero.

For example, the solution below is one way to prove that $\tan^2 \theta + 1 = \sec^2 \theta$ is an identity.

EXAMPLE

$\tan^2 \theta + 1 = \sec^2 \theta$

$$\begin{aligned} \text{L.S.} &= \tan^2 \theta + 1 \\ &= \left(\frac{\sin \theta}{\cos \theta}\right)^2 + 1 \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} + 1 \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{1}{\cos^2 \theta} \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{R.S.} &= \sec^2 \theta \\ &= \left(\frac{1}{\cos \theta}\right)^2 \\ &= \frac{1}{\cos^2 \theta} \end{aligned}$$

First separate both sides of the equation.

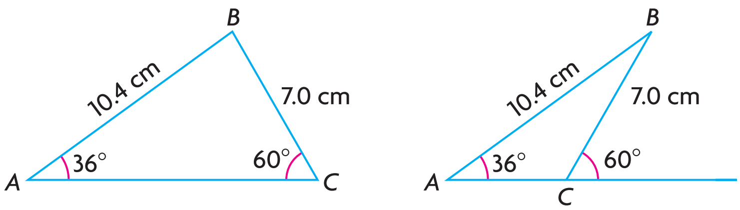
Write $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$. The left side of the equation is more complicated, so simplify it. Find a common denominator. Then use the Pythagorean identity. Since the denominator cannot equal 0, there is a restriction on θ, so $\cos \theta \neq 0$.

$\therefore \tan^2 \theta + 1 = \sec^2 \theta$ for all angles θ, where $\cos \theta \neq 0$.

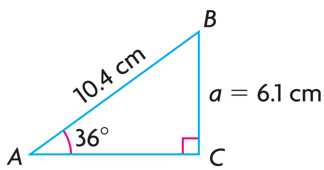
Q: **How do you know when you are dealing with the ambiguous case of the sine law?**

A: The ambiguous case of the sine law refers to the situation where 0, 1, or 2 triangles are possible given the information in a problem. This situation occurs when you know two side lengths and an angle (SSA).

For example, given $\triangle ABC$, where $\angle A = 36°$, $a = 7.0$ cm, and $c = 10.4$ cm, there are two possible triangles:



If $a = 6.1$ cm, then $\triangle ABC$ is a right triangle and 6.1 cm is the shortest possible length for a:



If $a < 6.1$ cm, a triangle cannot be drawn.

Study Aid

- See Lesson 5.6, Examples 1 and 2.
- Try Chapter Review Questions 8 and 9.

Q: **How do you decide when to use the sine law or the cosine law to solve a problem?**

A: Given any triangle, if you know two sides and the angle between those sides, or all three sides, use the cosine law. If you know an angle opposite a side, use the sine law.

Study Aid

- See Lesson 5.7, Examples 1 and 2.
- Try Chapter Review Questions 10 and 11.

Q: **What approaches are helpful in solving two- and three-dimensional trigonometric problems?**

A: Always start with a sketch of the given information because the sketch will help you determine whether the Pythagorean theorem, the sine law, or the cosine law is the best method to use. If you have right triangles, use the Pythagorean theorem and/or trigonometric ratios. If you know three sides or two sides and the contained angle in an oblique triangle, use the cosine law. For all other cases, use the sine law.

Study Aid

- See Lesson 5.8, Examples 1, 2, and 3.
- Try Chapter Review Questions 12 and 13.

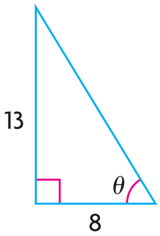
PRACTICE Questions

Lesson 5.1

1. i) For each triangle, state the reciprocal trigonometric ratios for angle θ.

ii) Calculate the value of θ to the nearest degree.

a)



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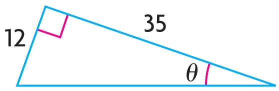

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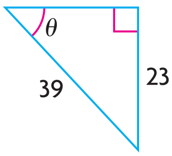


Lesson 5.2

2. Determine the exact value of each trigonometric expression. Express your answers in simplified radical form.

a) $(\sin 45°)(\cos 45°) + (\sin 30°)(\cos 60°)$

b) $(1 - \tan 45°)(\sin 30°)(\cos 30°)(\tan 60°)$

c) $\tan 30° + 2(\sin 45°)(\cos 60°)$

Lesson 5.3

3. i) State the sign of each trigonometric ratio. Use a calculator to determine the value of each ratio.

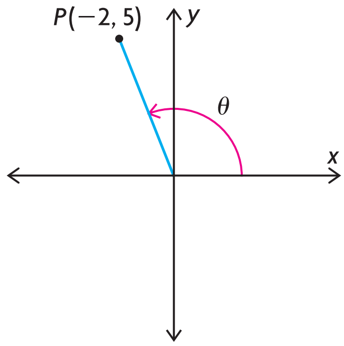
ii) For each trigonometric ratio, determine the principal angle and, where appropriate, the related acute angle. Then sketch another angle that has the equivalent ratio. Label the principal angle and the related acute angle on your sketch.

a) $\tan 18°$ **b)** $\sin 205°$ **c)** $\cos(-55°)$

Lesson 5.4

4. For each sketch, state the primary trigonometric ratios associated with angle θ. Express your answers in simplified radical form.

a)



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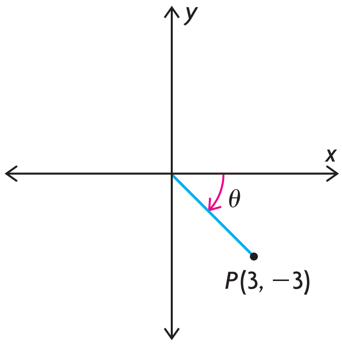

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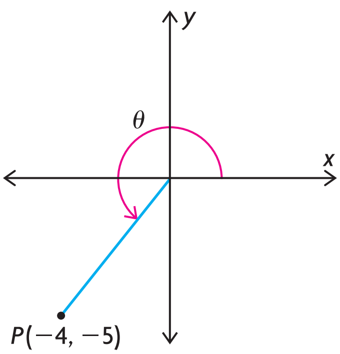


5. Given $\cos \phi = \frac{-7}{\sqrt{53}}$, where $0° \leq \phi \leq 360°$,

a) in which quadrant(s) does the terminal arm of angle ϕ lie? Justify your answer.

b) state the other five trigonometric ratios for angle ϕ.

c) calculate the value of the principal angle ϕ to the nearest degree.

Lesson 5.5

6. Determine whether the equation $\cos \beta \cot \beta = \frac{1}{\sin \beta} - \sin \beta$ is an identity. State any restrictions on angle β.

7. Prove each identity. State any restrictions on the variables if all angles vary from 0° to 360°.

a) $\tan \alpha \cos \alpha = \sin \alpha$

b) $\frac{1}{\cot \phi} = \sin \phi \sec \phi$

c) $1 - \cos^2 x = \frac{\sin x \cos x}{\cot x}$

d) $\sec \theta \cos \theta + \sec \theta \sin \theta = 1 + \tan \theta$

Lesson 5.6

8. Determine whether it is possible to draw a triangle given each set of information. Sketch all possible triangles where appropriate. Calculate, then label, all side lengths to the nearest tenth of a centimetre and all angles to the nearest degree.

a) $b = 3.0$ cm, $c = 5.5$ cm, $\angle B = 30°$

b) $b = 12.2$ cm, $c = 8.2$ cm, $\angle C = 34°$

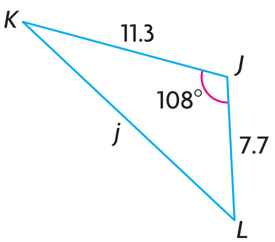
c) $a = 11.1$ cm, $c = 5.2$ cm, $\angle C = 33°$

9. Two forest fire stations, P and Q, are 20.0 km apart. A ranger at station Q sees a fire 15.0 km away. If the angle between the line PQ and the line from P to the fire is 25°, how far, to the nearest tenth of a kilometre, is station P from the fire?

Lesson 5.7

10. Determine each unknown side length to the nearest tenth.

a)



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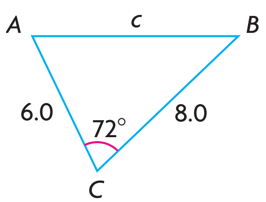

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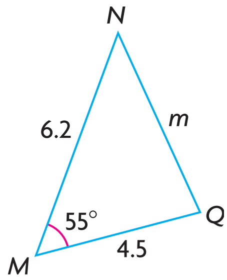
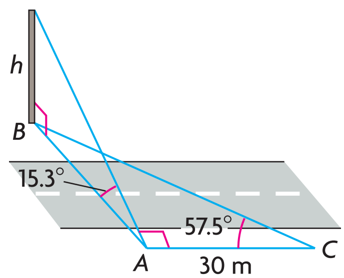


11. Two spotlights, one blue and the other white, are placed 6.0 m apart on a track on the ceiling of a ballroom. A stationary observer standing on the ballroom floor notices that the angle of elevation is 45° to the blue spotlight and 70° to the white one. How high, to the nearest tenth of a metre, is the ceiling of the ballroom?

Lesson 5.8

12. To determine the height of a pole across a road, Justin takes two measurements. He stands at point A directly across from the base of the pole and determines that the angle of elevation to the top of the pole is 15.3°. He then walks 30 m parallel to the freeway to point C, where he sees that the base of the pole and point A are 57.5° apart. From point A, the base of the pole and point C are 90.0° apart. Calculate the height of the pole to the nearest metre.



13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?

5 Chapter Self-Test

1. i) For each point, sketch the angle in standard position to determine all six trigonometric ratios.

ii) Determine the value of the principal angle and the related acute angle, where appropriate, to the nearest degree.

a) $P(-3, 0)$ **b)** $S(-8, -6)$

2. Given angle θ, where $0° \leq \theta \leq 360°$, determine all possible angles for θ.

a) $\sin \theta = -\frac{1}{2}$

b) $\cos \theta = \frac{\sqrt{3}}{2}$

c) $\cot \theta = -1$

d) $\sec \theta = -2$

3. Given $\cos \theta = -\frac{5}{13}$, where the terminal arm of angle θ lies in quadrant 2, evaluate each trigonometric expression.

a) $\sin \theta \cos \theta$ **b)** $\cot \theta \tan \theta$

4. i) Prove each identity. Use a different method for parts (a) and (b). State any restrictions on the variables.

ii) Explain why these identities are called Pythagorean identities.

a) $\tan^2 \phi + 1 = \sec^2 \phi$ **b)** $1 + \cot^2 \alpha = \csc^2 \alpha$

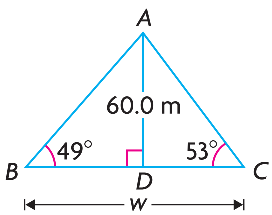
5. a) Sketch a triangle of your own choice and label the sides and angles.

b) State all forms of the cosine law that apply to your triangle.

c) State all forms of the sine law that apply to your triangle.

6. For each triangle, calculate the value of w to the nearest tenth of a metre.

a)



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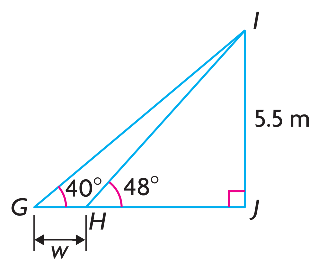


7. Given each set of information, determine how many triangles can be drawn. Calculate, then label, all side lengths to the nearest tenth and all interior angles to the nearest degree, where appropriate.

a) $a = 1.5$ cm, $b = 2.8$ cm, and $\angle A = 41°$

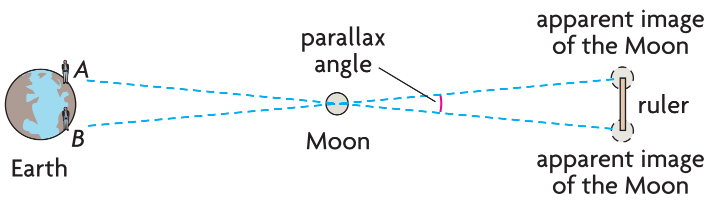
b) $a = 2.1$ cm, $c = 6.1$ cm, and $\angle A = 20°$

8. To estimate the amount of usable lumber in a tree, Chitra must first estimate the height of the tree. From points A and B on the ground, she determined that the angles of elevation for a certain tree were 41° and 52°, respectively. The angle formed at the base of the tree between points A and B is 90°, and A and B are 30 m apart. If the tree is perpendicular to the ground, what is its height to the nearest metre?

5 Chapter Task

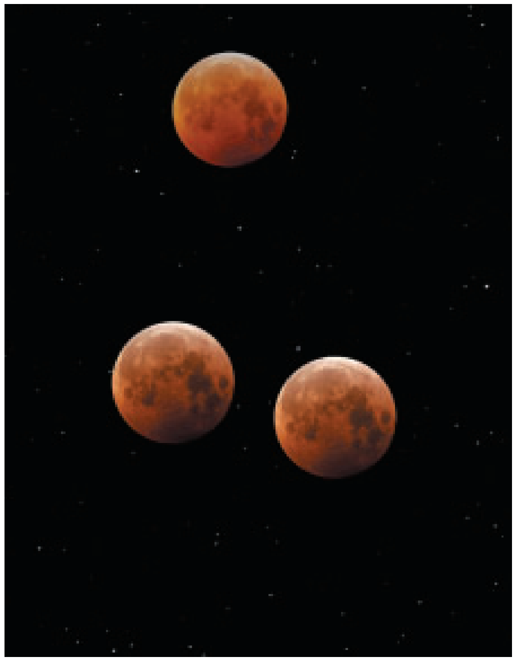
Parallax

Parallax is the apparent displacement of an object when it is viewed from two different positions.



Astronomers measure the parallax of celestial bodies to determine how far those bodies are from Earth.

On October 28, 2004, three astronomers (Peter Cleary, Pete Lawrence, and Gerardo Addiègo) each at a different location on Earth, took a digital photo of the Moon during a lunar eclipse at exactly the same time. The data related to these photos is shown.



	Shortest Distance on Earth's Surface Between Two Locations	Parallax Angle
AB (Montréal, Canada to Selsey, UK)	5 220 km	0.7153°
AC (Montréal, Canada to Montevideo, Uruguay)	9 121 km	1.189°
BC (Selsey, UK to Montevideo, Uruguay)	10 967 km	1.384°

? What is the most accurate method to determine the distance between the Moon and Earth, from the given data?

A. Sketch a triangle with the Moon and locations *A* and *B* as the vertices. Label all the given angles and distances. What kind of triangle do you have?

B. Determine all unknown sides to the nearest kilometre and angles to the nearest thousandth of a degree. How far, to the nearest kilometre, is the Moon from either Montréal or Selsey?

C. Repeat parts A and B for locations *B* and *C*, and for *A* and *C*.

D. On October 28, 2004, the Moon was about 391 811 km from Earth (surface to surface). Calculate the relative error, to the nearest tenth of a percent, for all three distances you calculated.

E. Which of your results is most accurate? What factors contribute most to the error in this experiment?

Task *Checklist*

- ✔ Did you draw the correct sketches?
- ✔ Did you show your work?
- ✔ Did you provide appropriate reasoning?
- ✔ Did you explain your thinking clearly?