2 Chapter Review

FREQUENTLY ASKED Questions

Q: What is a rational function and how do you determine its simplified form?

A: A rational function is a function that can be expressed as a quotient of two polynomials.

The domain of a rational function is the set of all real numbers, except the zeros of the denominator.

To simplify, divide out common factors of the numerator and denominator.

EXAMPLE

$$f(x) = \frac{4\overset{1}{\cancel{(x+1)}}(x+2)}{2\underset{1}{\cancel{(x+1)}}(x+3)} = \frac{2(x+2)}{(x+3)}; x \neq -1, -3$$

Study Aid

- See Lesson 2.4, Examples 1 to 5.
- Try Chapter Review Questions 9, 10, and 11.

Q: How do we add, subtract, multiply, and divide rational expressions?

A: Rules for adding, subtracting, multiplying, and dividing rational expressions are the same as those for rational numbers.

EXAMPLE

$$\frac{2x^2}{(x-1)^2} \div \frac{4x}{x^2-1} + \frac{7}{2x-2}$$

$$= \frac{2x^2}{(x-1)(x-1)} \div \frac{4x}{(x+1)(x-1)} + \frac{7}{2(x-1)}$$

$$= \frac{2x^2}{(x-1)(x-1)} \times \frac{(x-1)(x+1)}{4x} + \frac{7}{2(x-1)}$$

$$= \frac{x(x+1)}{2(x-1)} + \frac{7}{2(x-1)}$$

$$= \frac{x^2+x+7}{2(x-1)}; x \neq 1, -1, 0$$

Study Aid

- See Lesson 2.6, Examples 1 to 4 for multiplication and division.
- See Lesson 2.7, Examples 1 to 4 for addition and subtraction.
- Try Chapter Review Questions 12 to 17.

Q: Why are there sometimes restrictions on the variables in a rational expression, and how do you determine these restrictions?

A: The restrictions occur because division by zero is undefined. To determine restrictions, set all denominators equal to zero before simplifying and solve, usually by factoring.

In the preceding example, set

$(x-1)^2 = 0, \quad x^2 - 1 = 0, \quad 2x - 2 = 0, \quad$ and $\quad 4x = 0$

Solve by factoring:

$(x-1)^2 = 0, \quad (x-1)(x+1) = 0, \quad 2(x-1) = 0, \quad$ and $\quad 4x = 0$

Solving gives the restrictions $x \neq 1, -1, 0$.

Study Aid

- See Lessons 2.4, 2.6, and 2.7, all Examples.
- Try Chapter Review Questions 9 to 17.

PRACTICE Questions

Lesson 2.1

1. Simplify.

a) $(7x^2 - 2x + 1) + (9x^2 - 4x + 5) - (4x^2 + 6x - 7)$

b) $(7a^2 - 4ab + 9b^2) - (-a^2 + 2ab + 6b^2)$

2. Determine two non-equivalent polynomials $f(x)$ and $g(x)$, such that $f(0) = g(0)$ and $f(1) = g(1)$.

3. Ms. Flanagan has three daughters: Astrid, Beatrice, and Cassandra. Today, January 1, their ages are, respectively,

$$A(n) = -(n + 30) + (2n + 5)$$

$$B(n) = (7 - n) - (32 - 2n)$$

$$C(n) = (n - 26) - (n + 4) + (n - 3)$$

All ages are expressed in years, and n represents Ms. Flanagan's age.

a) Are the daughters triplets? Explain.

b) Are any of them twins? Explain.

c) How old was Ms. Flanagan when Cassandra was born?

Lesson 2.2

4. Expand and simplify.

a) $-3(7x - 5)(4x - 7)$

b) $-(y^2 - 4y + 7)(3y^2 - 5y - 3)$

c) $2(a + b)^3$

d) $3(x^2 - 2)^2(2x - 3)^2$

5. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. Determine the volume of the cone in simplified form if the radius is increased by x and the height is increased by $2x$.

Lesson 2.3

6. Simplify.

a) $(2x^4 - 3x^2 - 6) + (6x^4 - x^3 + 4x^2 + 5)$

b) $(x^2 - 4)(2x^2 + 5x - 2)$

c) $-7x(x^2 + x - 1) - 3x(2x^2 - 5x + 6)$

d) $-2x^2(3x^3 - 7x + 2) - x^3(5x^3 + 2x - 8)$

e) $-2x[5x - (2x - 7)] + 6x[3x - (1 + 2x)]$

f) $(x + 2)^2(x - 1)^2 - (x - 4)^2(x + 4)^2$

g) $(x^2 + 5x - 3)^2$

7. Factor.

a) $12m^2n^3 + 18m^3n^2$

b) $x^2 - 9x + 20$

c) $3x^2 + 24x + 45$

d) $50x^2 - 72$

e) $9x^2 - 6x + 1$

f) $10a^2 + a - 3$

8. Factor.

a) $2x^2y^4 - 6x^5y^3 + 8x^3y$

b) $2x(x + 4) + 3(x + 4)$

c) $x^2 - 3x - 10$

d) $15x^2 - 53x + 42$

e) $a^4 - 16$

f) $(m - n)^2 - (2m + 3n)^2$

Lesson 2.4

9. Simplify. State any restrictions on the variables.

a) $\dfrac{10a^2b + 15bc^2}{-5b}$

b) $\dfrac{30x^2y^3 - 20x^2z^2 + 50x^2}{10x^2}$

c) $\dfrac{xy - xyz}{xy}$

d) $\dfrac{16mnr - 24mnp + 40kmn}{8mn}$

10. Simplify. State any restrictions on the variables.

a) $8xy^2 + 12x^2y - \dfrac{6x^3}{2xy}$

b) $\dfrac{7a - 14b}{2(a - 2b)}$

c) $\dfrac{m + 3}{m^2 + 10m + 21}$

d) $\dfrac{4x^2 - 4x - 3}{4x^2 - 9}$

e) $\dfrac{3x^2 - 21x}{7x^2 - 28x + 21}$

f) $\dfrac{3x^2 - 2xy - y^2}{3x^2 + 4xy + y^2}$

11. If two rational functions have the same restrictions, are they equivalent? Explain and illustrate with an example.

Lessons 2.6 and 2.7

12. Simplify. State any restrictions on the variables.

a) $\frac{6x}{8y} \times \frac{2y^2}{3x}$

b) $\frac{10m^2}{3n} \times \frac{6mn}{20m^2}$

c) $\frac{2ab}{5bc} \div \frac{6ac}{10b}$

d) $\frac{5p}{8pq} \div \frac{3p}{12q}$

13. Simplify. State any restrictions on the variables.

a) $\frac{x^2}{2xy} \times \frac{x}{2y^2} \div \frac{(3x)^2}{xy^2}$

b) $\frac{x^2 - 5x + 6}{x^2 - 1} \times \frac{x^2 - 4x - 5}{x^2 - 4} \div \frac{x - 5}{x^2 + 3x + 2}$

c) $\frac{1 - x^2}{1 + y} \times \frac{1 - y^2}{x + x^2} \div \frac{y^3 - y}{x^2}$

d) $\frac{x^2 - y^2}{4x^2 - y^2} \times \frac{4x^2 + 8xy + 3y^2}{x + y} \div \frac{2x + 3y}{2x - y}$

14. Simplify. State any restrictions on the variables.

a) $\frac{4}{5x} - \frac{2}{3x}$

b) $\frac{5}{x + 1} - \frac{2}{x - 1}$

c) $\frac{1}{x^2 + 3x - 4} + \frac{1}{x^2 + x - 12}$

d) $\frac{1}{x^2 - 5x + 6} - \frac{1}{x^2 - 9}$

15. Simplify and state any restrictions on the variables.

a) $\frac{1}{2x} - \frac{7}{3x^2} + \frac{4}{x^3}$

b) $\frac{3x}{x + 2} + \frac{4x}{x - 6}$

c) $\frac{6x}{x^2 - 5x + 6} - \frac{3x}{x^2 + x - 12}$

d) $\frac{2(x - 2)^2}{x^2 + 6x + 5} \times \frac{3x + 15}{(2 - x)^2}$

e) $\frac{(x - 2y)^2}{x^2 - y^2} \div \frac{(x - 2y)(x + 3y)}{(x + y)^2}$

f) $\frac{2b - 5}{b^2 - 2b - 15} + \frac{3b}{b^2 + b - 30} \times \frac{b^2 + 8b + 12}{b + 3}$

16. Fred's final mark in an online course was determined entirely by two exams. The mid-term exam was out of x marks and was worth 25% of his final mark. The final exam was out of $2x$ marks and was worth 75% of his final mark. Fred scored 40 marks on the first exam and 60 marks on the second exam. Determine the value of x if Fred earned a final mark of 50% in the course.

17. Sam plays a game in which he selects three different numbers from 1 to n ($n > 3$). After he selects his numbers, four different winning numbers from 1 to n are chosen, one at a time. Sam wins if all three of his numbers are among the four winning numbers.

The first number chosen is one of Sam's! His probability of winning is now given by

$$P(n) = \frac{24}{n^3 - 3n^2 + 2n} \div \frac{3}{n}$$

a) Simplify $P(n)$ and state the restrictions on n.

b) What would Sam's probability of winning be if

i) $n = 5$?

ii) $n = 4$?

2 Chapter Self-Test

1. Simplify.
   a) $(-x^2 + 2x + 7) + (2x^2 - 7x - 7)$
   b) $(2m^2 - mn + 4n^2) - (5m^2 - n^2) + (7m^2 - 2mn)$

2. Expand and simplify.
   a) $2(12a - 5)(3a - 7)$
   b) $(2x^2y - 3xy^2)(4xy^2 + 5x^2y)$
   c) $(4x - 1)(5x + 2)(x - 3)$
   d) $(3p^2 + p - 2)^2$

3. Is there a value of a such that $f(x) = 9x^2 + 4$ and $g(x) = (3x - a)^2$ are equivalent? Explain.

4. If Bonnie is away from Clyde for n consecutive days, then the amount of heartache Clyde feels is given by $h(n) = (2n + 1)^3$.
   a) If Bonnie is absent, by how much does Clyde's pain increase between day n and day $n + 1$?
   b) How much more pain will he feel on day 6 than on day 5?

5. Factor.
   a) $3m(m - 1) + 2m(1 - m)$
   b) $x^2 - 27x + 72$
   c) $15x^2 - 7xy - 2y^2$
   d) $(2x - y + 1)^2 - (x - y - 2)^2$
   e) $5xy - 10x - 3y + 6$
   f) $p^2 - m^2 + 6m - 9$

6. Use factoring to determine the x-intercepts of the curve $y = x^3 - 4x^2 - x + 4$.

7. Simplify. State any restrictions on the variables.
   a) $\dfrac{4a^2b}{5ab^3} \div \dfrac{6a^2b}{35ab}$
   b) $\dfrac{x - 2}{x^2 - x - 12} \times \dfrac{2x - 8}{x^2 - 4x + 4}$
   c) $\dfrac{5}{t^2 - 7t - 18} + \dfrac{6}{t + 2}$
   d) $\dfrac{4x}{6x^2 + 13x + 6} - \dfrac{3x}{4x^2 - 9}$

8. Mauro found that two rational functions each simplified to $f(x) = \dfrac{2}{x + 1}$.

   Are Mauro's two rational functions equivalent? Explain.

9. Roman thinks that he has found a simple method for determining the sum of the reciprocals of any three consecutive natural numbers. He writes, for example,

   $$\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}, \quad \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{74}{120}, \quad \text{or} \quad \frac{37}{60}$$

   Roman conjectures that before simplification, the numerator of the sum is three times the product of the first and third denominators, plus 2. Also, the denominator of the sum is the product of the three denominators. Is Roman's conjecture true?

2 Chapter Task

The Algebraic Dominos Challenge

The game shown at the right consists of eight pairs of coloured squares called dominos.

1*R*	2*R*	2*P*	4*P*
1*P*	3*R*	5*R*	4*R*
6*P*	3*P*	5*P*	7*R*
6*R*	8*P*	8*R*	7*P*

Rules:

1. Write a polynomial in each square marked *P* and a rational function in each square marked *R*.
2. The expressions you write must satisfy each of these conditions:
   - Polynomials and numerators and denominators of each rational function must be quadratics without a constant common factor.
   - Restrictions on the variable of each rational function must be stated in its square.
   - When two polynomials are side by side, then one or both of the polynomials must be perfect squares.
   - When a polynomial and a *different-coloured* rational expression are side by side, their product must simplify.
   - When two rational expressions are side by side, their product must simplify.
   - When a polynomial is on top of another polynomial, their quotient must simplify.
   - When a polynomial is on top of a *different-coloured* rational expression (or vice versa), their quotient must simplify.
   - When a rational expression is on top of a rational expression, their quotient must simplify.
3. After you have completed the table, simplify the products and quotients wherever possible. You get one point for every *different* linear factor that remains in your table.
4. Count the linear factors and write your score next to your table.

? How can you maximize your score?

A. What form for the polynomials, including numerators and denominators, will make filling the table and counting your score as easy as possible?

B. Why should you avoid reusing a factor unless it is necessary?

C. Play the game by completing the table.

D. Tally your score.

E. Check your answers. What could you do to increase your score?

F. List some strategies you can use to maximize your score.

Task Checklist

- ✔ Does each square contain a polynomial and rational function of the right type?
- ✔ Are all of the rules satisfied?
- ✔ Did you check to see if you could make changes to improve your score?