2 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How can you determine whether two polynomials are equivalent? For example, suppose that $f(x) = (x - 2)^2$, $g(x) = (2 - x)^2$, and $h(x) = (x + 2)(x - 2)$.

A1: You can simplify both polynomials. If their simplified versions are the same, the polynomials are equivalent; otherwise, they are not.

Simplifying yields
$f(x) = x^2 - 4x + 4$, $g(x) = 4 - 4x + x^2$, and $h(x) = x^2 - 4$.

So $f(x)$ and $g(x)$ are equivalent, but $h(x)$ is not equivalent to either of them.

A2: If the domains of two functions differ in value for any number in both domains, then the functions are not equivalent.

For example, for the functions above, $f(0) = 4$ while $h(0) = -4$. So $f(x)$ and $h(x)$ are not equivalent.

A3: You can graph both functions. If the graphs are exactly the same, then the functions are equivalent; otherwise, they are not.

Q: How do you add, subtract, and multiply polynomials?

A: When the variables of a polynomial are replaced with numbers, the result is a number. The properties for adding, subtracting, and multiplying polynomials are the same as the properties for the numbers.

For any polynomials a, b, c:

Commutative Property

$a + b = b + a$; $ab = ba$

(*Note:* $a - b \neq b - a$, except in special cases.)

Associative Property

$(a + b) + c = a + (b + c)$; $(ab)c = a(bc)$

(*Note:* $(a - b) - c \neq a - (b - c)$, except in special cases.)

Distributive Property

$a(b + c) = ab + ac$; $a(b - c) = ab - ac$

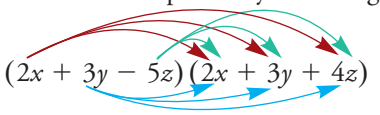
Study *Aid*

- See Lesson 2.1, Examples 1, 2, and 3 and Lesson 2.2, Example 3.
- Try Mid-Chapter Review Question 2.

Study *Aid*

- See Lesson 2.1, Example 1 Anita's Solution and Example 2.
- See Lesson 2.2, Examples 1, 2, and 3 Lee's Solution.
- Try Mid-Chapter Review Questions 1 and 3 to 6.

Because of the distributive property, the product of two polynomials can be found by multiplying each term in one polynomial by each term in the other and can be simplified by collecting like terms. For example,



$$(2x + 3y - 5z)(2x + 3y + 4z)$$
$$= 4x^2 + 6xy + 8xz + 6xy + 9y^2 + 12yz - 10xz - 15yz - 20z^2$$
$$= 4x^2 + 12xy - 2xz + 9y^2 - 3yz - 20z^2$$

Study Aid

- See Lesson 2.3, Examples 1 to 5.
- Try Mid-Chapter Review Questions 7 to 10.

Q: What strategies can you use to factor polynomials?

A: The strategies include:

Common factoring

EXAMPLE

$$9x^2 - 18x$$
$$= 9x(x - 2)$$

Decomposition

EXAMPLE

$$6x^2 + 5x - 4$$
$$= 6x^2 - 3x + 8x - 4$$
$$= 3x(2x - 1) + 4(2x - 1)$$
$$= (2x - 1)(3x + 4)$$

Factoring a difference of squares

EXAMPLE

$$9x^2 - 16$$
$$= (3x + 4)(3x - 4)$$

Factoring by grouping

EXAMPLE

$$5b + 2ab + 4a + 10$$
$$= (5b + 2ab) + (4a + 10)$$
$$= b(5 + 2a) + 2(2a + 5)$$
$$= (2a + 5)(b + 2)$$

PRACTICE Questions

Lesson 2.1

1. Simplify.

a) $(4a^2 - 3a + 2) - (-2a^2 - 3a + 9)$

b) $(2x^2 - 4xy + y^2) - (4x^2 + 7xy - 2y^2) + (3x^2 + 6y^2)$

c) $-(3d^2 - 2cd + d) + d(2c - 5d) - 3c(2c + d)$

d) $3x(2x + y) - 4x[5 - (3x + 2)]$

e) $2a(3a - 5b + 4) - 6(3 - 2a - b)$

f) $7x(2x^2 + 3y - 3) - 3x(9 - 2x + 4y)$

2. Determine whether each pair of functions is equivalent.

a) $g(t) = (t - 2)^5$ and $h(t) = (2 - t)^5$

b) $f(x) = (x^2 - 6x) - (x^2 + x - 4) + (2x^2 + 1)$ and $g(x) = (4x^2 - 7x - 3) - (2x^2 - 8)$

c) $h(x) = (x - 4)(x + 7)(x + 4)$ and $d(x) = (x + 7)(x^2 - 16)$

d) $b(t) = (3t + 1)^3$ and $c(t) = 27t^3 + 27t^2 + 9t - 1$

Lesson 2.2

3. If you multiply a linear polynomial by a quadratic one, what is the degree of the product polynomial? Justify your answer.

4. The sum of the ages of Pam, Dion, and their three children, in years, is $5x - 99$, where x is Dion's age. Pam is five years younger than Dion. What is the sum of the ages of their children?

5. Expand and simplify.

a) $2(x - 5)(3x - 4)$

b) $(3x - 1)^3$

c) $2(x^2 - 3x + 4)(-x^2 + 3x - 4)$

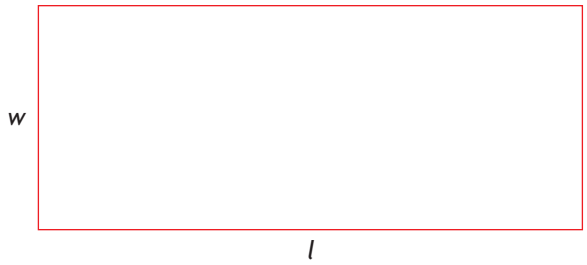
d) $(5x - 4)(3x - 5) - (2x - 3)^2$

e) $3(2x - 5) - 9(4x - 5)$

f) $-(x - y)^3$

6. If the length of the rectangle shown is increased by 2 and the width is decreased by 1, determine the change in

a) the perimeter **b)** the area



Lesson 2.3

7. Factor.

a) $x(x - 2) - 3(x - 2)$

b) $x^2 - 11x + 28$

c) $3a^2 - 10a - 8$

d) $30x^2 - 9x - 3$

e) $16 - 25x^2$

f) $4(2 - a)^2 - 81$

8. Factor.

a) $2n - 6m + 5n^2 - 15mn$

b) $y^2 + 9 - 6y - x^2$

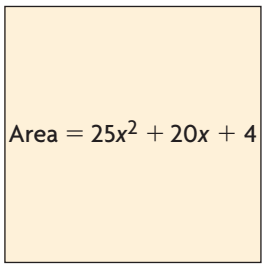
c) $y - b - (y - b)^2$

d) $2x^2 - 8y^2 + 8x + 8$

e) $w^2 + wb - aw - ab$

f) $ab + b^2 + 6a + 6b$

9. What is the perimeter of the square shown?



10. The expression $3n^2 - 11n + k$ can be factored into two linear polynomials with integer coefficients. Determine the possible values of k.