# 2.5 Exploring Graphs of Rational Functions

**GOAL**

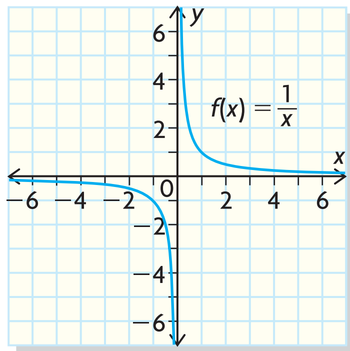
Explore some features of rational functions.

**YOU WILL NEED**

- graphing calculator

## *EXPLORE the Math*

The graph of the rational function $f(x) = \frac{1}{x}$ is shown at the right. Its domain is $\{x \in \mathbf{R} \mid x \neq 0\}$, and it has a vertical **asymptote** at $x = 0$ and a horizontal asymptote at $y = 0$.




**?** What are some features of the graphs of rational functions, at or near numbers that are not in their domain?

**A.** Some rational functions simplify to polynomials. For example, $f(x) = \frac{x^2 - 4}{x - 2}$ can be simplified by factoring from $f(x) = \frac{(x + 2)(x - 2)}{x - 2}$ to $f(x) = x + 2$, where $x \neq 2$. Graph $f(x)$ prior to simplifying it, and zoom in and trace near $x = 2$. Describe what happens to the graph at $x = 2$.

**B.** Determine another rational function that simplifies to a polynomial with domain $\{x \in \mathbf{R} \mid x \neq 1\}$. Describe what happens to the graph at $x = 1$.

**C.** Some rational functions cannot be simplified; for example, $g(x) = \frac{1}{x - 3}$. Graph $g(x)$ and zoom in near $x = 3$. Describe what happens to the graph near $x = 3$.

**D.** Determine another rational function with domain $\{x \in \mathbf{R} \mid x \neq 2\}$ that can't be simplified. Graph your function and describe what happens to the graph at $x = 2$.

**E.** Determine the equation of a simplified rational function that has two vertical asymptotes: $x = -1$ and $x = 2$. Graph your function.

**F.** Determine the equation of a rational function that has both a vertical asymptote and a "hole." Graph your function.

**G.** The rational function $h(x) = \frac{1}{x}$ has a horizontal asymptote $y = 0$. Apply a transformation to $h(x)$ that will result in a rational function that has the horizontal asymptote $y = 2$. Determine the equation of this function and graph it.

**H.** Determine the equation of a rational function without any "holes," vertical asymptotes, or horizontal asymptotes. Graph your function.

**I.** Review what you have discovered and summarize your findings.

## Reflecting

**J.** What determines where a rational function has a hole? A vertical asymptote?

**K.** When does a rational function have the horizontal asymptote $y = 0$? When does a rational function have another horizontal line as a horizontal asymptote?

**L.** Some rational functions have asymptotes, others have holes, and some have both. Explain how you can identify, without graphing, which graphical features a rational function will have.

### In Summary

#### Key Idea

- The restricted values of rational functions correspond to two different kinds of graphical features: holes and vertical asymptotes.

#### Need to Know

- Holes occur at restricted values that result from a factor of the denominator that is also a factor of the numerator. For example,

$$g(x) = \frac{x^2 + 7x + 12}{x + 3}$$

has a hole at $x = -3$, since $g(x)$ can be simplified to the polynomial

$$g(x) = \frac{(x + 3)(x + 4)}{(x + 3)} = x + 4$$

- Vertical asymptotes occur at restricted values that are still zeros of the denominator after simplification. For example,

$$h(x) = \frac{5}{x - 8}$$

has a vertical asymptote at $x = 8$.

## ***FURTHER*** *Your Understanding*

**1.** Identify a rational function whose graph is a horizontal line except for two holes. Graph the function.

**2.** Identify a rational function whose graph lies entirely above the $x$-axis and has a single vertical asymptote. Graph the function.

**3.** Identify a rational function whose graph has the horizontal asymptote $y = 2$ and two vertical asymptotes. Graph the function.