1 Chapter Review

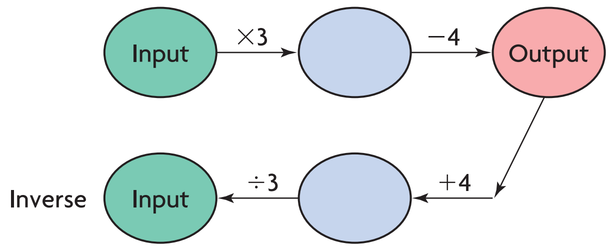
FREQUENTLY ASKED Questions

Study Aid

- See Lesson 1.5, Examples 1 and 2.
- Try Chapter Review Questions 10 and 11.

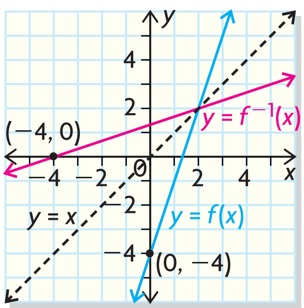
Q: How can you determine the inverse of a linear function?

A1: The inverse of a linear function is the reverse of the original function. It undoes what the original has done. This means that you can find the equation of the inverse by reversing the operations on x. For example, if $f(x) = 3x - 4$, the operations on x are as follows: Multiply by 3 and then subtract 4. To reverse these operations, you add 4 and divide by 3, so the inverse function is $f^{-1}(x) = \frac{x + 4}{3}$.



A2: If (x, y) is on the graph of $f(x)$, then (y, x) is on the inverse graph, so you can switch x and y in the equation to find the inverse equation. For example, if $f(x) = 3x - 4$, you can write this as $y = 3x - 4$. Then switch x and y and solve for y.

$$x = 3y - 4$$
$$x + 4 = 3y - 4 + 4$$
$$x + 4 = 3y$$
$$\frac{x + 4}{3} = y, \text{ so the inverse function is } f^{-1}(x) = \frac{x + 4}{3}.$$



A3: If you have the graph of a linear function, you can graph the inverse function by reflecting in the line $y = x$.

The inverse of a linear function is another linear function, unless the original function represents a horizontal line.

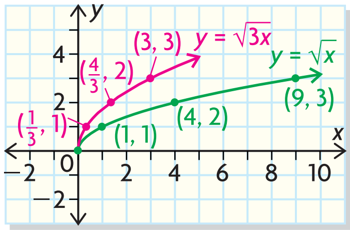
Study Aid

- See Lesson 1.7, Example 1.
- Try Chapter Review Questions 12 and 13.

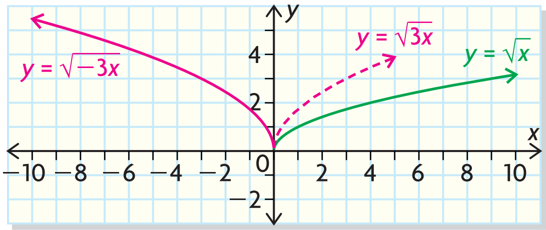
Q: How do you apply a horizontal stretch, compression, or reflection to the graph of a function?

A: The graph of $y = f(kx)$ is the graph of $y = f(x)$ after a horizontal stretch, compression, or reflection. When k is a number greater than 1 or less than -1, the graph is compressed horizontally by the factor $\frac{1}{k}$. When k is a number between -1 and 1, the graph is stretched horizontally by the factor $\frac{1}{k}$. Whenever k is negative, the graph is also reflected in the y-axis.

You apply a horizontal compression by dividing the x-coordinates of points on the original graph by k. For example, to graph $y = \sqrt{3x}$, graph $y = \sqrt{x}$ and then divide the x-coordinates of the points (0, 0), (1, 1), (4, 2), (9, 3) by 3 (or multiply by $\frac{1}{3}$) to get the points (0, 0), $(\frac{1}{3}, 1)$, $(\frac{4}{3}, 2)$, (3, 3) on the transformed graph.



When k is negative, you also reflect the graph in the y-axis. For example, $y = \sqrt{-3x}$.



Q: How do you sketch the graph of $y = af[k(x - d)] + c$ when you have the graph of $y = f(x)$?

A1: You can graph the parent function and then apply the transformations one by one, starting with the compressions, stretches, and reflections and leaving the translations until last. For example, to sketch the graph of $y = 3f(6 - 2x) - 5$ when $f(x) = \frac{1}{x}$, begin by putting the equation into the form $y = af[k(x - d)] + c$ by factoring. This gives $y = 3f[-2(x - 3)] - 5$. Then identify all the transformations you need to apply:

- $a = 3$ means a vertical stretch by the factor 3.
- $k = -2$ means a horizontal compression by the factor $\frac{1}{2}$ and a reflection in the y-axis.
- $d = 3$ means a horizontal translation 3 units right.
- $c = -5$ means a vertical translation 5 units down.

A2: You can graph the function in two steps: Apply both stretches or compressions and any reflections to the parent function first, and then both translations.

Study *Aid*

- See Lesson 1.8, Examples 1, 2, and 3.
- Try Chapter Review Questions 14 to 18.

PRACTICE Questions

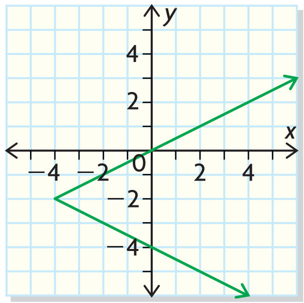
Lesson 1.1

1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) $\{(-3, 0), (-1, 1), (0, 1), (4, 5), (0, 6)\}$

b) $y = 4 - x$

c)



d) $x^2 + y^2 = 16$

2. What rule can you use to determine, from the graph of a relation, whether the relation is a function? Graph each relation and determine which are functions.

a) $\{(-2. 1), (1, 1), (0, 0), (1, -1), (1, -2), (2, -2)\}$

b) $y = 4 - 3x$

c) $y = (x - 2)^2 + 4$

d) $x^2 + y^2 = 1$

e) $y = \frac{1}{x}$

f) $y = \sqrt{x}$

3. Sketch the graph of a function whose domain is the set of real numbers and whose range is the set of real numbers less than or equal to 3.

Lesson 1.2

4. If $f(x) = x^2 + 3x - 5$ and $g(x) = 2x - 3$, determine each.

a) $f(-1)$

b) $f(0)$

c) $g\left(\frac{1}{2}\right)$

d) $f(2b)$

e) $g(1 - 4a)$

f) x when $f(x) = g(x)$

5. a) Graph the function $f(x) = -2(x - 3)^2 + 4$, and state its domain and range.

b) What does $f(1)$ represent on the graph? Indicate, on the graph, how you would find $f(1)$.

c) Use the equation to determine each of the following.

i) $f(3) - f(2)$

ii) $2f(5) + 7$

iii) $f(1 - x)$

6. If $f(x) = x^2 - 4x + 3$, determine the input(s) for x whose output is $f(x) = 8$.

Lesson 1.4

7. A ball is thrown upward from the roof of a building 60 m tall. The ball reaches a height of 80 m above the ground after 2 s and hits the ground 6 s after being thrown.

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

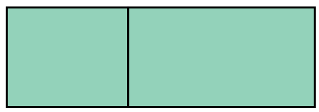
c) Determine an equation for the function.

8. State the domain and range of each function.

a) $f(x) = 2(x - 1)^2 + 3$

b) $f(x) = \sqrt{2x + 4}$

9. A farmer has 540 m of fencing to enclose a rectangular area and divide it into two sections as shown.



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.

Lesson 1.5

10. Using the functions listed as examples, describe three methods for determining the inverse of a linear function. Use a different method for each function.

a) $f(x) = 2x - 5$

b) $f(x) = \frac{x + 3}{7}$

c) $f(x) = 4 - \frac{1}{2}x$

11. For a fundraising event, a local charity organization expects to receive \$15 000 from corporate sponsorship, plus \$30 from each person who attends the event.

a) Use function notation to express the total income from the event as a function of the number of people who attend.

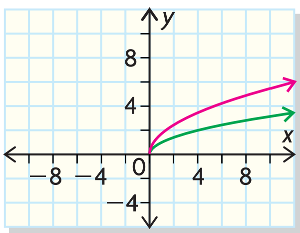
b) Suggest a reasonable domain and range for the function in part (a). Explain your reasoning.

c) The organizers want to know how many tickets they need to sell to reach their fundraising goal. Create a function to express the number of people as a function of expected income. State the domain of this new function.

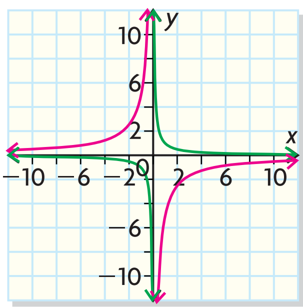
Lesson 1.7

12. In each graph, a parent function has undergone a transformation of the form $f(kx)$. Determine the equations of the transformed functions graphed in red. Explain your reasoning.

a)



b)



13. For each set of functions, transform the graph of $f(x)$ to sketch $g(x)$ and $h(x)$, and state the domain and range of each function.

a) $f(x) = x^2, g(x) = \left(\frac{1}{2}x\right)^2, h(x) = -(2x)^2$

b) $f(x) = |x|, g(x) = |-4x|, h(x) = \left|\frac{1}{4}x\right|$

Lesson 1.8

14. Three transformations are applied to $y = x^2$: a vertical stretch by the factor 2, a translation 3 units right, and a translation 4 units down.

a) Is the order of the transformations important?

b) Is there any other sequence of these transformations that could produce the same result?

15. The point $(1, 4)$ is on the graph of $y = f(x)$. Determine the coordinates of the image of this point on the graph of $y = 3f[-4(x + 1)] - 2$.

16. a) Explain what you would need to do to the graph of $y = f(x)$ to graph the function $y = -2f[(\frac{1}{3}x + 4)] - 1$.

b) Graph the function in part (a) for $f(x) = x^2$.

17. In each case, write the equation for the transformed function, sketch its graph, and state its domain and range.

a) The graph of $f(x) = \sqrt{x}$ is compressed horizontally by the factor $\frac{1}{2}$, reflected in the y-axis, and translated 3 units right and 2 units down.

b) The graph of $y = \frac{1}{x}$ is stretched vertically by the factor 3, reflected in the x-axis, and translated 4 units left and 1 unit up.

18. If $f(x) = (x - 4)(x + 3)$, determine the x-intercepts of each function.

a) $y = f(x)$

b) $y = -2f(x)$

c) $y = f\left(-\frac{1}{2}x\right)$

d) $y = f(-(x + 1))$

19. A function $f(x)$ has domain $\{x \in \mathbf{R} \mid x \geq -4\}$ and range $\{y \in \mathbf{R} \mid y < -1\}$. Determine the domain and range of each function.

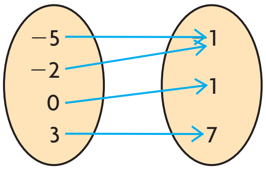
a) $y = 2f(x)$

b) $y = f(-x)$

c) $y = 3f(x + 1) + 4$

d) $y = -2f(-x + 5) + 1$

1 Chapter Self-Test



1. For each relation, determine the domain and range and whether the relation is a function. Explain your reasoning.

a) The function shown at the left.

b) $y = \sqrt{x + 2}$

2. An incandescent light bulb costs \$0.65 to buy and \$0.004/h for electricity to run. A fluorescent bulb costs \$3.50 to buy and \$0.001/h to run.

a) Use function notation to write a cost equation for each type of bulb.

b) State the domain and range of each function.

c) After how long is the fluorescent bulb cheaper than the regular bulb?

d) Determine the difference in costs after one year. Assume the light is on for an average of 6 h/day.

3. Determine the domain and range of each function. Show your steps.

a) $f(x) = \dfrac{1}{x - 2}$

b) $f(x) = \sqrt{3 - x} - 4$

c) $f(x) = -|x + 1| + 3$

4. Explain what the term *inverse* means in relation to a linear function. How are the domain and range of a linear function related to the domain and range of its inverse?

5. For each function, determine the inverse, sketch the graphs of the function and its inverse, and state the domain and range of both the function and its inverse.

a) $\{(-2, 3), (0, 5), (2, 6), (4, 8)\}$

b) $f(x) = 3 - 4x$

6. At Phoenix Fashions, Rebecca is paid a monthly salary of \$1500, plus 4% commission on her sales over \$2500.

a) Graph the relation between monthly earnings and sales.

b) Use function notation to write an equation of the relation.

c) Graph the inverse relation.

d) Use function notation to write an equation of the inverse.

e) Use the equation in part (d) to express Rebecca's sales if she earned \$1740 one month. Then evaluate.

7. The function $y = f(x)$ has been transformed to $y = f(kx)$. Determine the value of k for each transformation.

a) a horizontal stretch by the factor 5

b) a horizontal compression by the factor $\frac{1}{3}$ and a reflection in the y-axis

8. The function $y = f(x)$ has been transformed to $y = af[k(x - d)] + c$. Determine a, k, d, and c; write the equation; sketch the graph; and state the domain and range of each transformed function.

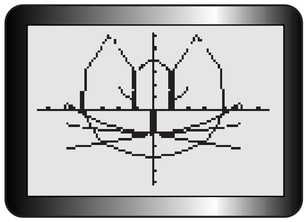
a) vertical compression by the factor $\frac{1}{2}$, reflection in the y-axis, and translation 2 units right, applied to $y = \sqrt{x}$

b) vertical stretch by the factor 4, reflection in the x-axis, translation 2 units left, and translation 3 units down, applied to $y = \dfrac{1}{x}$

c) horizontal compression by the factor $\frac{1}{4}$, vertical stretch by the factor $\frac{3}{2}$, reflection in the x-axis, translation 3 units right, and translation 2 units down, applied to $y = |x|$

Functional Art

Parts of transformed parent functions were used to make this cat's face on a graphing calculator. The functions used are listed in the table that follows.



$$-6.58 \le X \le 6.58, \text{Xscl } 1$$

$$-6.2 \le Y \le 6.2, \text{Yscl } 1$$

Feature	Function	Domain
ears	$y = -\|2x - 5\| + 6$ $y = -\|-2x - 5\| + 6$	$1 \le x \le 4$ $-4 \le x \le -1$
top of head	$y = -x^2 + 4$	$-1 \le x \le 1$
chin	$y = \frac{5}{x + 5} - 5$ $y = \frac{5}{-x + 5} - 5$	$-4 \le x \le 0$ $0 \le x \le 4$
eyes	$y = -\sqrt{4 - 2x} + 2$ $y = -\sqrt{4 - 2x} + 2$	$1 \le x \le 3$ $-3 \le x \le -1$
whiskers	$y = 0.1x^2 - 2$ $y = 0.03x^2 - 2$ $y = -0.25\|x\| - 2$	$-5 \le x \le 5$ $-5 \le x \le 5$ $-5 \le x \le 5$

? How can you use transformations of parent functions to create other pictures?

A. Re-create the cat's face on a graphing calculator. Begin by putting the calculator in DOT mode. Then enter each function listed in the table, along with its domain. The first three entries are shown.

B. Describe how transformations were used to create the cat's features. For each feature, describe
- which properties of the parent function were useful for that feature
- which transformations were used and why
- how symmetry was used and which transformations ensured symmetry

C. Create your own picture, using transformations of the parent functions $f(x) = x^2, f(x) = \sqrt{x}, y = \frac{1}{x}$, and $f(x) = |x|$. You must use each parent function at least once.

D. List the parts of your picture, the functions used, and the corresponding domains in a table. Explain why you chose each function and each transformation.

Task *Checklist*

- ✔ Did you use each parent function at least once?
- ✔ Did you list the transformed functions and the corresponding domains?
- ✔ Did you explain why you chose each function and each transformation?
- ✔ Did you describe the transformations in appropriate math language?