1.3 Exploring Properties of Parent Functions

GOAL

Explore and compare the graphs and equations of five basic functions.

EXPLORE the Math

As a child, you learned how to recognize different animal families.

In mathematics, every function can be classified as a member of a family. Each member of a family of functions is related to the simplest, or most basic, function sharing the same characteristics. This function is called the parent function. Here are some members of the linear and quadratic families. The parent functions are in green.

> **Communication | Tip**
>
> $\sqrt{x}$ always means the positive square root of x.

family

a collection of functions (or lines or curves) sharing common characteristics

parent function

the simplest, or base, function in a family

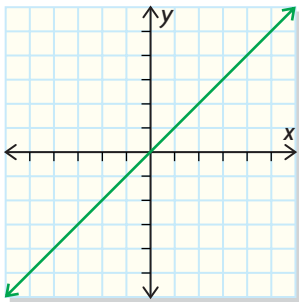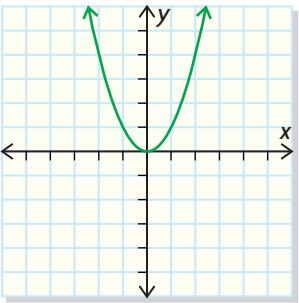| Linear Functions	Quadratic Functions
Parent function: $f(x) = x$	Parent function: $f(x) = x^2$
Family members: $f(x) = mx + b$	Family members: $f(x) = a(x - h)^2 + k$
Examples: $f(x) = 3x + 2$, $f(x) = -\frac{1}{2}x - 2$	Examples: $f(x) = 5(x - 3)^2 - 1$, $f(x) = -x^2 + 3$
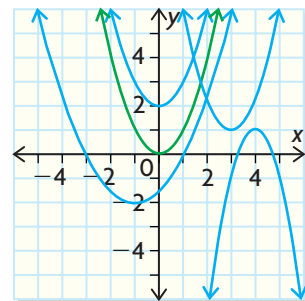
absolute value

written as |x|; describes the distance of x from 0; equals x when $x \geq 0$ or $-x$ when $x < 0$; for example, $|3| = 3$ and $|-3| = -(-3) = 3$

Three more parent functions are the square root function $f(x) = \sqrt{x}$, the reciprocal function $f(x) = \dfrac{1}{x}$, and the **absolute value** function $f(x) = |x|$.
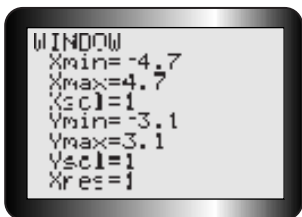
? **What are the characteristics of these parent functions that distinguish them from each other?**

A. Make a table like the one shown.

Equation of Function	Name of Function	Sketch of Graph	Special Features/ Symmetry	Domain	Range
$f(x) = x$	linear function		• straight line that goes through the origin • slope is 1 • divides the plane exactly in half diagonally • graph only in quadrants 1 and 3		
$f(x) = x^2$	quadratic function		• parabola that opens up • vertex at the origin • y has a minimum value • y-axis is axis of symmetry • graph only in quadrants 1 and 2		
$f(x) = \sqrt{x}$	square root function				
$f(x) = \dfrac{1}{x}$	reciprocal function				
$f(x) = |x|$	absolute value function				

B. Use your graphing calculator to check the sketches shown for $f(x) = x$ and $f(x) = x^2$ and add anything you think is missing from the descriptions. Explain how you know that these equations are both functions.

C. In your table, record the domain and range of each of $f(x) = x$ and $f(x) = x^2$.

D. Clear all equations from the equation editor. Graph the square root function, $f(x) = \sqrt{x}$. In your table, sketch the graph and describe its shape. Is it a function? Explain. How is it different from the graphs of linear and quadratic functions?

E. Go to the table of values and scroll up and down the table. Does ERR: appear in the Y column? Explain why this happens.

F. Using the table of values and the graph, determine and record the domain and range of the function.

G. Repeat parts D through F for the reciprocal function $f(x) = \dfrac{1}{x}$. Use the table of values to see what happens to y when x is close to 0 and when x is far from 0. Explain why the graph is in two parts with a break in the middle.

H. Where are the <mark>asymptotes</mark> of this graph?

I. Repeat parts D through F for the absolute value function $f(x) = |x|$. Which of the other functions is the resulting graph most like? Explain. When you have finished, make sure that your table contains enough information for you to recognize each of the five parent functions.

Reflecting

J. Explain how each of the following helped you determine the domain and range.
  a) the table of values
  b) the graph
  c) the function's equation

K. Which graphs lie in the listed quadrants?
  a) the first and second quadrants
  b) the first and third quadrants

L. Which graph has asymptotes? Why?

M. You have used the slope and y-intercept to sketch lines, vertices, and directions of opening to sketch parabolas. What characteristics of the new parent functions $f(x) = \sqrt{x}, f(x) = \dfrac{1}{x}$, and $f(x) = |x|$ could you use to sketch their graphs?
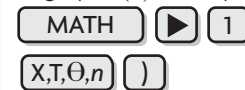

In Summary

Key Idea

- Certain basic functions, called parent functions, form the building blocks for families of more complicated functions. Parent functions include, but are not limited to, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \dfrac{1}{x}$, and $f(x) = |x|$.
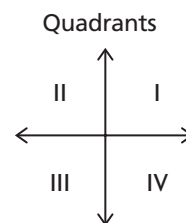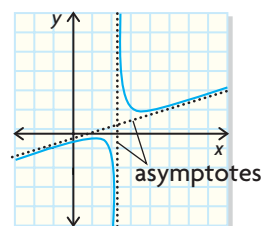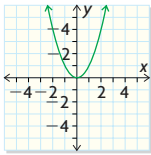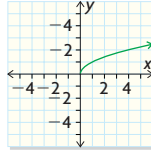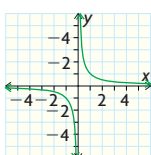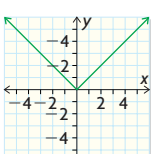
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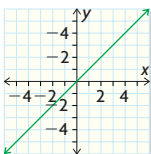
<mark>asymptote</mark>

a line that the graph of a relation or function gets closer and closer to, but never meets, on some portion of its domain

asymptotes

Quadrants

Need to Know

- Each of the parent functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \dfrac{1}{x}$, and $f(x) = |x|$ has unique characteristics that define the shape of its graph.

Equation of Function	Name of Function	Sketch of Graph
$f(x) = x$	linear function	
$f(x) = x^2$	quadratic function	
$f(x) = \sqrt{x}$	square root function	
$f(x) = \dfrac{1}{x}$	reciprocal function	
$f(x) = |x|$	absolute value function	

FURTHER *Your Understanding*

1. Sketch the graphs of $f(x) = x$ and $g(x) = \dfrac{1}{x}$ on the same axes. What do the graphs have in common? What is different about the graphs? Write equations of the asymptotes for the reciprocal function.

2. Sketch the graphs of $f(x) = x^2$ and $g(x) = |x|$ on the same axes. Describe how these graphs are alike and how they are different.

3. Sketch the graph of $f(x) = x^2$ for values of $x \geq 0$. On the same axes, sketch the graphs of $g(x) = \sqrt{x}$ and $h(x) = x$. Describe how the three graphs are related.