Section 1.4 - Transformations Using Quadratic Functions

Transformations

Mathematical operations performed on functions to change the position or shape of the associated curves or lines (reflection, up, down, left, right, compression, stretch)

Learning Goal:

To understand how the parameters a, h, and k in $f(x) = a(x - h)^2 + k$ affect and change the graph of $f(x) = x^2$

Working in pairs, use a graphing calculator to

- sketch the following graphs
- complete the tables
- report your findings

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PART A

Use a graphing calculator or graphing software to graph the quadratic functions shown in the table on the same set of axes. Sketch and label the graph of each function on the same set of axes. Then copy and complete the table.

	Function	Value of k in $f(x) = x^2 + k$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = x^2 + 2$	2				
c)	$f(x) = x^2 + 4$					
d)	$f(x) = x^2 - 1$					
e)	$f(x) = x^2 - 3$					

PART B

Use the graphs and the table from part A to answer the following:

i) What information about the graph does the value of k provide in functions of the form $f(x) = x^2 + k$?

ii) What happens to the graph of the function $f(x) = x^2 + k$ when the value of k is changed? Consider both positive and negative values of k.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = x^2 + k$? What happens to the y-coordinates?

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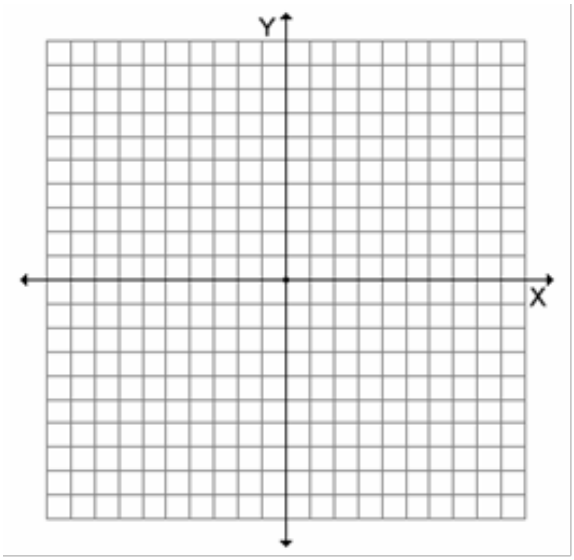
PART A & B SOLUTIONS

Function
$f(x) = x^2$
$f(x) = x^2 + 2$
$f(x) = x^2 + 4$
$f(x) = x^2 - 1$
$f(x) = x^2 - 3$



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SOLUTIONS TO THE INVESTIGATION

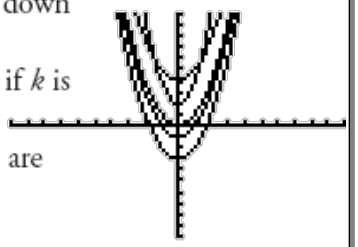
A.

	Function	Value of k in $f(x)=x^2+k$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x)=x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = x^2 + 2$	2	up	(0, 2)	$x = 0$	yes
c)	$f(x) = x^2 + 4$	4	up	(0, 4)	$x = 0$	yes
d)	$f(x) = x^2 - 1$	−1	up	(0, −1)	$x = 0$	yes
e)	$f(x) = x^2 - 3$	−3	up	(0, −3)	$x = 0$	yes

B. i) The value of k determines whether the graph is moved up or down compared to the graph of $f(x) = x^2$.

ii) The graph moves up k units if k is positive and down k units if k is negative.

iii) The x-coordinates do not change, but all of the y-coordinates are changed by adding k to them.



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PART C

Clear all previous equations from your calculator or graphing program. Repeat part A for the quadratic functions shown in the table.

	Function	Value of h in $f(x) = (x - h)^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = (x - 2)^2$	2				
c)	$f(x) = (x - 4)^2$					
d)	$f(x) = (x + 2)^2$					
e)	$f(x) = (x + 4)^2$					

PART D

Use the graphs and the table from part C to answer the following:

i) What information about the graph does the value of h provide in functions of the form $f(x) = (x - h)^2$?

ii) What happens to the graph of the function $f(x) = (x - h)^2$ when the value of h is changed? Consider both positive and negative values of h.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = (x - h)^2$? What happens to the y-coordinates?

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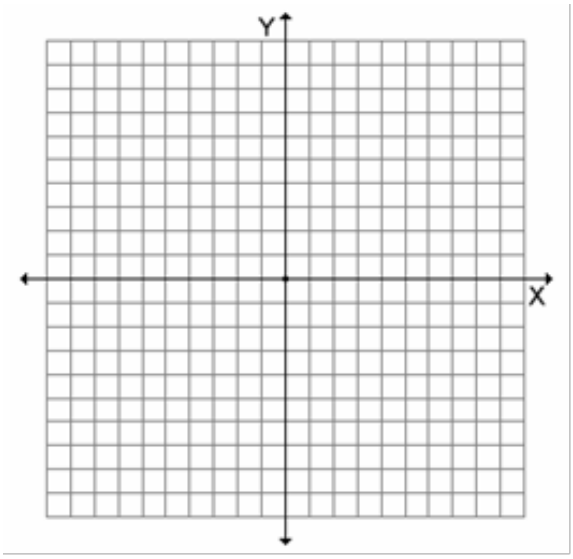
PART C & D SOLUTIONS

Function
$f(x) = x^2$
$f(x) = (x - 2)^2$
$f(x) = (x - 4)^2$
$f(x) = (x + 2)^2$
$f(x) = (x + 4)^2$



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SOLUTIONS TO THE INVESTIGATION

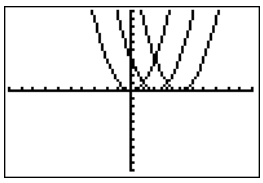
C.

	Function	Value of h in $f(x)=(x-h)^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x)=x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = (x - 2)^2$	2	up	(2, 0)	$x = 2$	yes
c)	$f(x) = (x - 4)^2$	4	up	(4, 0)	$x = 4$	yes
d)	$f(x) = (x + 2)^2$	−2	up	(−2, 0)	$x = -2$	yes
e)	$f(x) = (x + 4)^2$	−4	up	(−4, 0)	$x = -4$	yes

D. i) The value of h tells the x-coordinate of the vertex.

ii) If h is positive, the graph moves h units to the right. If h is negative, the graph moves h units to the left.

iii) The y-coordinates do not change, but all of the x-coordinates are changed by adding h to them.



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PART E

Clear all previous equations from your calculator or graphing program.
Repeat part A for the quadratic functions shown in the table.

	Function	Value of a in $f(x) = ax^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	1	up	(0, 0)	$x = 0$	yes
b)	$f(x) = 2x^2$					
c)	$f(x) = 0.5x^2$					
d)	$f(x) = -2x^2$					
e)	$f(x) = -0.5x^2$					

PART F

Use the graphs and the table from part E to answer the following:

i) What information about the graph does the value of a provide in functions of the form $f(x) = ax^2$? Consider values of a that are greater than 1 and values of a between 0 and 1.

ii) What happens to the graph of the function $f(x) = ax^2$ when the value of a is changed? Consider both positive and negative values of a.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = ax^2$? What happens to the y-coordinates?

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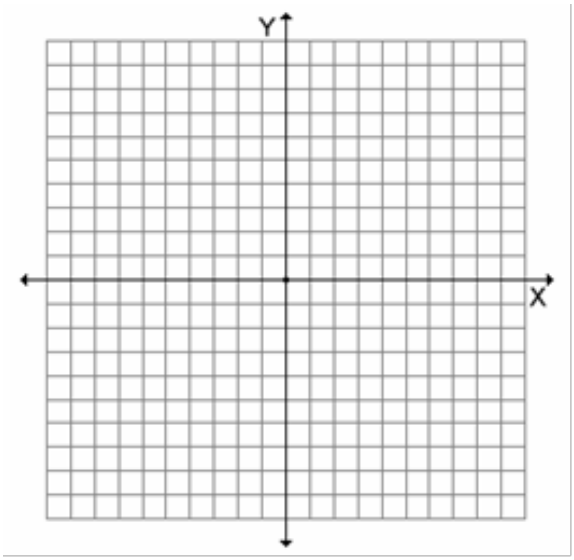
PART E & F SOLUTIONS

Function
$f(x) = x^2$
$f(x) = 2x^2$
$f(x) = 0.5x^2$
$f(x) = -2x^2$
$f(x) = -0.5x^2$



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SOLUTIONS TO THE INVESTIGATION

E.

	Function	Value of a in $f(x)=ax^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x)=x^2$?
a)	$f(x) = x^2$	1	up	(0, 0)	$x = 0$	yes
b)	$f(x) = 2x^2$	2	up	(0, 0)	$x = 0$	no
c)	$f(x) = 0.5x^2$	0.5	up	(0, 0)	$x = 0$	no
d)	$f(x) = -2x^2$	−2	down	(0, 0)	$x = 0$	no
e)	$f(x) = -0.5x^2$	−0.5	down	(0, 0)	$x = 0$	no

F. **i)** and **ii)** The value of a determines whether the graph opens up ($a > 0$) or down ($a < 0$) and determines how wide the opening of the graph is. For $a > 1$ or $a < -1$, the graph is narrower than $f(x) = x^2$. For $-1 < a < 1$ and $a \neq 0$, the graph is wider than $f(x) = x^2$.

iii) For the same x-coordinate, the y-coordinate of $f(x) = ax^2$ is greater than the y-coordinate of $f(x) = x^2$ if $a > 1$ or $a < -1$ and is less than the y-coordinate of $f(x) = x^2$ if $-1 < a < 1$ and $a \neq 0$.

Mathematical Process Focus
- Reflecting
- Selecting Tools and Computational Strategies
- Connecting
- Representing

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Summary

$f(x) = x^2$ (base curve - original function)

$g(x) = -a(x-h)^2 + k$ (transformed curve - new function)

reflection about x axis

vertical stretch (narrow-taller) a > 1

vertical compression (shorter-wider) 0<a<1

horizontal shift left / right

vertical shift up / down

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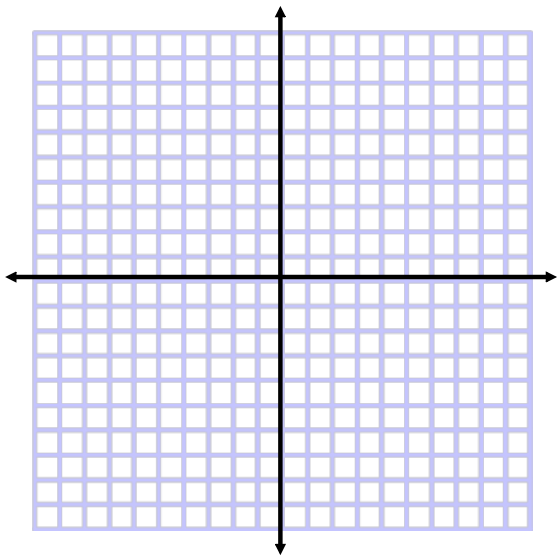
$g(x) = (x-2)^2 - 5$



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Consolidation Questions

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