Section 1.2 - Linear and Quadratic Functions

Learning Goal:

By the end of today I will be able to recognize the degree of a function from its finite differences and from its equation.

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Determine if the following are linear or quadratic relations using finite difference (first and second differences).

X	Y
1	-4
2	-1
3	2
4	5
5	8

X	Y
1	-8
2	-5
3	0
4	7
5	16

Summary

A linear relation has a first difference that is constant.

A quadratic relation has a second difference that is constant.

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Degree of a Function

The degree of a function is the largest exponent value found on the independent variable.

BE CAREFUL, to determine the largest exponent value you may have to expand or simplify the function before making your decision.

The degree of the function also tells you which finite difference should be constant.

Example

$y = 3x + 1$	first degree	first differences are constant
$y = 2x^2 + 4x - 7$	second degree	second differences are constant
$y = 5x^3 + 7x - 9$	third degree	third differences are constant
$y = 13x^5 + 6$	fifth degree	fifth differences are constant

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State the degree of the following:

(a) $y = 4x + 3x^2$

(b) $y = -2x + 5$

(c) $y = (x-4)(x-5)$

(d) $y = (x-1)^2 + 8$

(e) $y = (x-2)^2(x-4)$

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Use finite differences to determine the degree of the following:

X	Y
-2	2
-1	0
0	0
1	2
2	6

X	Y
-1	-5
0	0
1	-3
2	-8
3	-9
4	0
5	25

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PROBLEM 1

Water is poured into a tank at a constant rate. The volume of water in the tank is measured every minute until the tank is full. The measurements are recorded in the table.

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0.0	1.3	2.6	3.9	5.2	6.5	7.8	9.1

a) Use difference tables to determine whether the volume of water poured into the tank, $V(t)$, is a linear or quadratic function of time. Explain.

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0.0	1.3	2.6	3.9	5.2	6.5	7.8	9.1
First Differences		1.3	1.3	1.3	1.3	1.3	1.3	1.3

b) State the domain and range using set notation.

$$\text{Domain} = \{t \in \mathbf{R} \mid 0 \leq t \leq 7\}$$

$$\text{Range} = \{V(t) \in \mathbf{R} \mid 0 \leq V(t) \leq 9.1\}$$

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PROBLEM 2

A migrating butterfly travels about 128 km each day. The distance it travels, $g(t)$, in kilometres, is a function of time, t, in days.

a) Write an equation using function notation to represent the distance a butterfly travels in t days.

b) State the degree of this function and whether it is linear or quadratic.

c) Use the function to calculate the distance the butterfly travels in 20 days.

d) What are the domain and range of this function, assuming that a butterfly lives 40 days, on average? Express your answers in set notation.

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Solution

a) $g(t) = 128t$

b) The degree is 1.

The function is linear.

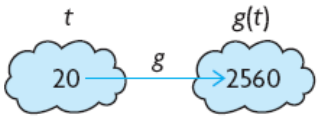
c) $g(t) = 128t$

$g(20) = 128(20)$

$= 2560$

The butterfly travels 2560 km in 20 days.

Substitute $t = 20$ into the function $g(t)$.



d) Domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 40\}$

Range $= \{g(t) \in \mathbf{R} \mid 0 \leq g(t) \leq 5120\}$

The maximum number of days is 40.

The maximum distance is $g(40) = 5120$ km.

In this case, neither distance nor time can be negative.

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Practice / Consolidation Questions

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