2.4 Factoring Quadratic Expressions: $ax^2 + bx + c$

GOAL

Factor quadratic expressions of the form $ax^2 + bx + c$, where $a \neq 1$.

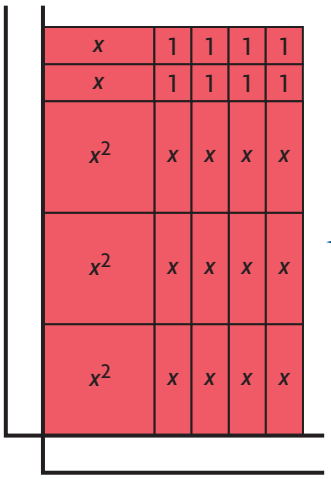
Martina is asked to factor the expression $3x^2 + 14x + 8$. She is unsure of what to do because the first term of the expression has the coefficient 3, and she hasn't worked with these kinds of polynomials before.

? How do you factor $3x^2 + 14x + 8$?

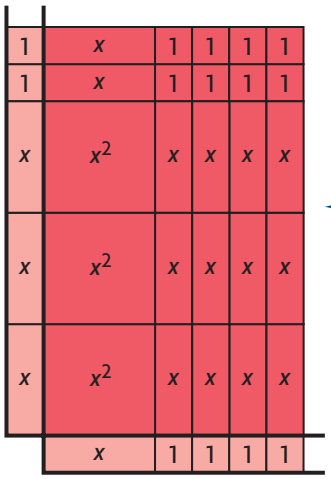
LEARN ABOUT the Math

EXAMPLE 1 Selecting a strategy to factor a trinomial where $a \neq 1$

Llewelyn's Solution: Using Algebra Tiles



I used 3 x^2 tiles, 14 x tiles, and 8 unit tiles to create a rectangle area model of the trinomial.



I placed the tiles along the length and width of the rectangle to read off the factors. The length was $3x + 2$ and the width was $x + 4$.

$3x^2 + 14x + 8 = (3x + 2)(x + 4)$

Albert's Solution: Using Guess and Check

$3x^2 + 14x + 8 = (x + ?)(3x + ?)$

To get $3x^2$, I had to multiply x by $3x$, so I set up the equation to show the factors.

$(x + 8)(3x + 1) = 3x^2 + 25x + 8$

WRONG FACTORS

$(x + 1)(3x + 8) = 3x^2 + 11x + 8$

WRONG FACTORS

$(x + 4)(3x + 2) = 3x^2 + 14x + 8$

WORKED!

$3x^2 + 14x + 8 = (x + 4)(3x + 2)$

I had to multiply two numbers together to get 8.
The ways to get a product of 8 are 8×1, 4×2, $-8 \times (-1)$, and $-4 \times (-2)$. Since the middle term was positive, I tried the positive values only.
I used guess and check to see which pair of numbers worked.

decomposing
breaking a number or expression into parts that make it up

Murray's Solution: Using Decomposition

$3x^2 + 14x + 8 = 3x^2 + 12x + 2x + 8$ ← I rewrote the original expression by **decomposing** $14x$ into $12x + 2x$.

$3x^2 + 12x + 2x + 8$
$= (3x^2 + 12x) + (2x + 8)$ ← I divided out the greatest common factors from the first two terms and then from the last two terms.
$= 3x(x + 4) + 2(x + 4)$
$= (x + 4)(3x + 2)$ ← I factored out the binomial common factor.

Reflecting

A. Why did Llewlyn try to form a rectangle with the algebra tiles?

B. What was Murray's goal when he "decomposed" the coefficient of the x-term?

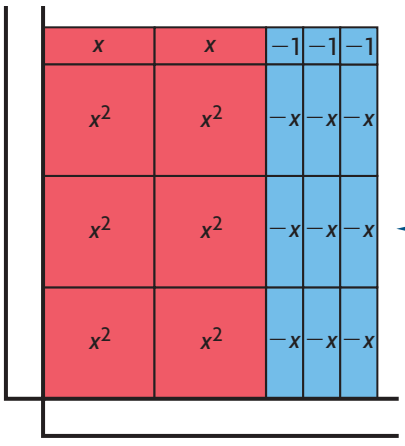
C. How was Albert's factoring method similar to the sum-and-product method?

D. What are the advantages and disadvantages of each of the three factoring methods?

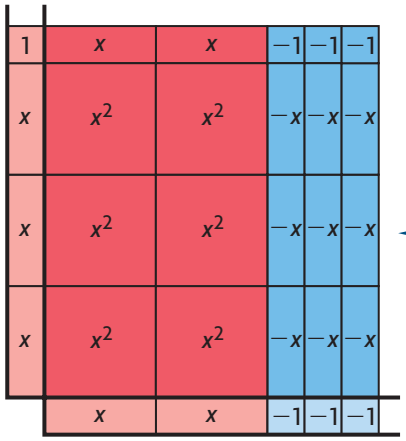
EXAMPLE 2 Factoring a trinomial by using algebra tiles

Use algebra tiles to factor $6x^2 - 7x - 3$.

Ariel's Solution



I used 6 x^2 tiles, 9 $-x$ tiles, 2 x tiles, and 3 negative unit tiles to build a rectangle area model of the trinomial.



I placed tiles along the length and width of the rectangle to read off the factors.

$6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

EXAMPLE 3 Factoring a trinomial by decomposition

Factor $4x^2 - 3x - 1$.

Florian's Solution

$(4)x^2 \boxed{-3}\, x\,(-1)$

$= 4x^2 - 4x + x - 1$

I found two numbers whose sum is -3 and whose product is $(4)(-1) = -4$.

The numbers are -4 and 1. I used them to "decompose" the middle term.

$= 4x(x - 1) + 1(x - 1)$

$= (x - 1)(4x + 1)$

I factored the first two terms and then the last two terms. Next, I divided out the common factor $x - 1$.

Check:

$(x - 1)(4x + 1) = 4x^2 + 1x - 4x - 1$

$= 4x^2 - 3x - 1$

I checked by multiplying.

EXAMPLE 4 Factoring a trinomial by guess and check

Factor $3x^2 - 5x + 2$.

Nadia's Solution

$3x^2 - 5x + 2$

$(3x - 1)(x - 2) = 3x^2 - 7x + 2$

WRONG FACTORS

$(3x - 2)(x - 1) = 3x^2 - 5x + 2$

WORKED

$3x^2 - 5x + 2 = (3x - 2)(x - 1)$

The factors of $3x^2$ are $3x$ and x.

The factors of 2 are 2 and 1 or -2 and -1.

I tried the negative values, since the middle term was negative.

I used guess and check to place the values in each set of brackets.

In Summary

Key Idea

- If the quadratic expression $ax^2 + bx + c$, where $a \neq 1$ can be factored, then the factors are of the form $(px + r)(qx + s)$, where $pq = a$, $rs = c$, and $ps + rq = b$.

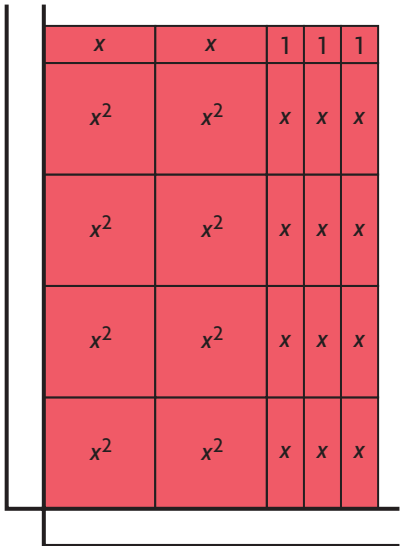
Need to Know

- If the quadratic expression $ax^2 + bx + c$, where $a \neq 1$ can be factored, then the factors can be found by a variety of strategies, such as
  - forming a rectangle with algebra tiles
  - decomposition
  - guess and check
- A trinomial of the form $ax^2 + bx + c$ can be factored if two integers can be found whose product is ac and whose sum is b.
- The decomposition method involves decomposing b into a sum of two numbers whose product is ac.

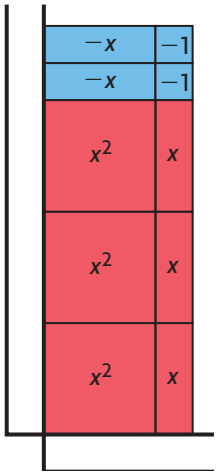
CHECK Your Understanding

1. Each diagram represents a polynomial. Identify the polynomial and its factors.

a)



b)



2. State the missing factor.

a) $6a^2 + 5a - 4 = (3a + 4)(?)$

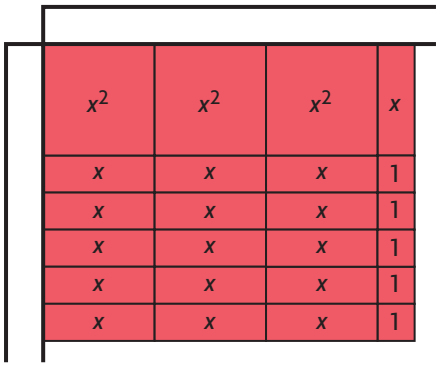
b) $5x^2 - 22x + 8 = (?)(x - 4)$

c) $3x^2 + 7x + 2 = (3x + 1)(?)$

d) $4n^2 + 8n - 60 = (?)(n + 5)$

PRACTISING

3. The diagram below represents a polynomial. Identify the polynomial and its factors.



4. Factor. You may first need to determine a common factor.

a) $2x^2 - 7x - 4$
b) $3x^2 + 18x + 15$
c) $5x^2 + 17x + 6$
d) $2x^2 + 10x + 8$
e) $3x^2 + 12x - 63$
f) $2x^2 - 15x + 7$

5. Factor.

a) $8x^2 + 10x + 3$
b) $6m^2 - 3m - 3$
c) $2a^2 - 11a + 12$
d) $15x^2 - 4x - 4$
e) $6n^2 + 26n - 20$
f) $16x^2 + 4x - 6$

6. Write three different quadratic trinomials that have $(2x - 5)$ as a factor.

7. For each expression, name an integer, k, such that the quadratic trinomial can be factored.

a) $kx^2 + 4x + 1$ **b)** $4x^2 + kx - 10$ **c)** $8x^2 - 14x + k$

8. Can the guess and check method for factoring trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, be applied when $a = 1$? Explain.

9. K Factor.

a) $6x^2 - x - 12$
b) $8k^2 + 43k + 15$
c) $30r^2 - 85r - 70$
d) $12n^3 - 75n^2 + 108n$
e) $3k^2 - 6k - 24$
f) $24y^2 - 10y - 25$

10. Factor.

a) $x^2 + 5x + 6$
b) $x^2 - 36$
c) $5a^2 - 13a - 6$
d) $a^2 - a - 12$
e) $4x^2 + 16x - 48$
f) $6x^2 + 7x - 3$

11. T Is there an integer, n, such that $6n^2 + 10n + 4$ is divisible by 50? Explain.

12. C How does knowing that factoring is the opposite of expanding help you factor a polynomial such as $-4x^2 + 38x - 48$?

Extending

13. Factor.

a) $6x^2 + 11xy + 3y^2$
b) $5a^2 - 7ab - 6b^2$
c) $8x^2 - 14xy + 3y^2$
d) $12a^2 + 52a - 40$

14. Can $ax^2 + bx + c$ be factored if a, b, and c are odd? Explain.