2.1 Working with Quadratic Expressions

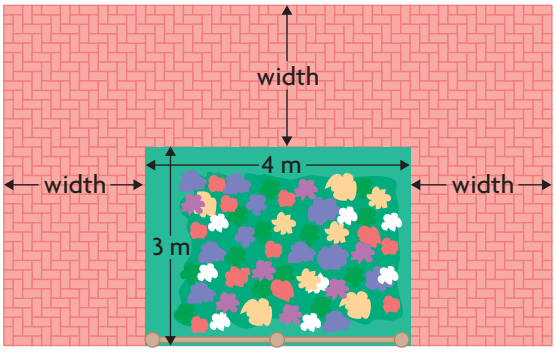
YOU WILL NEED

- algebra tiles

GOAL

Expand and simplify quadratic expressions.

INVESTIGATE the Math



Ali wants to know the area of brick he needs to build a path around his garden.

? What is a simplified expression for the area of the path?

A. Use w to represent the width of the path.

B. Think about the large rectangle that includes the garden and the path. What algebraic expressions represent the length and width of this rectangle?

C. What algebraic expression represents the area of the large rectangle in part B? How do you know?

D. What is the area of the garden without the path?

E. Use your results from parts C and D to write an expression for the area of the path.

F. What simplified expression represents the area of the path?

Reflecting

G. Why was a variable introduced in part A?

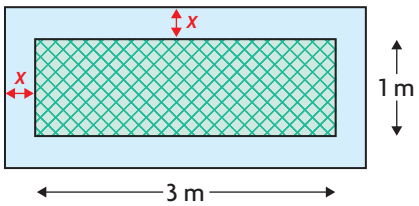
H. Explain how you expanded and simplified the expression you wrote in part E.

I. Is the algebraic expression representing the area of the path linear or quadratic? Explain how you know.

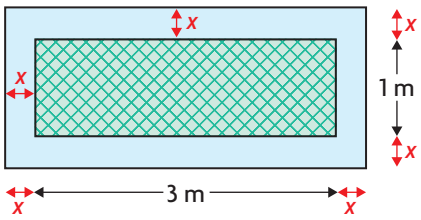
APPLY the Math

EXAMPLE 1 Multiplying two binomials by using algebra tiles

Evelyn is sewing a quilt as shown. If the width of the border is x, state the area of the quilt as a function of x.



Devin's Solution

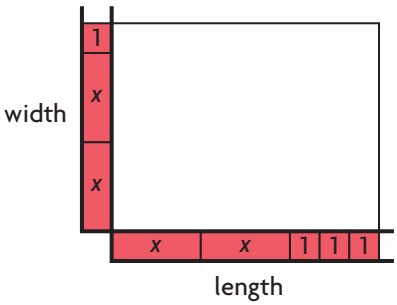


If the width of the border is x, then I know the length and width of the quilt in terms of x.

length: $x + 3 + x = 2x + 3$

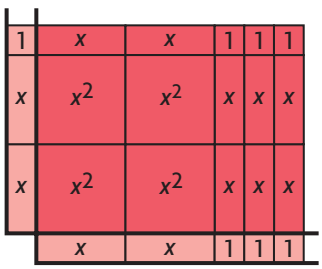
width: $x + 1 + x = 2x + 1$

The dimensions are $2x + 3$ and $2x + 1$.

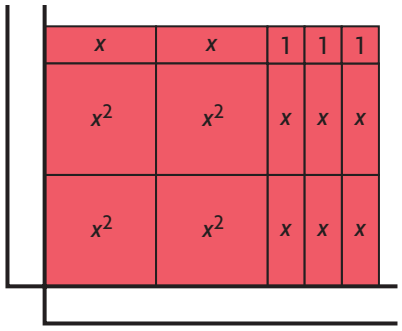


I modelled the area by using algebra tiles.

I represented the length of $2x + 3$ and width of $2x + 1$.



I formed a rectangle with tiles to model the area of the quilt. I used the length and width tiles to make sure I made the rectangle the right size.



I removed the tiles I used to represent the length and width, since they are not part of the area.

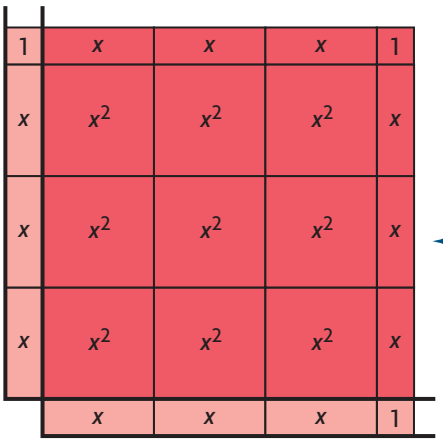
$$A(x) = 4x^2 + 2x + 6x + 3$$
$$= 4x^2 + 8x + 3$$

I counted the tiles in the rectangle to get an expression for its area $A(x)$, then simplified by adding like terms.

EXAMPLE 2 Squaring a binomial by using algebra tiles

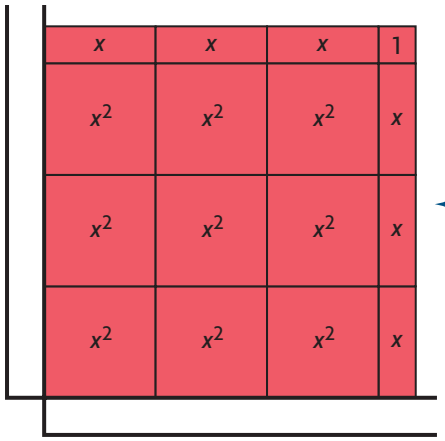
Use algebra tiles to simplify $(3x + 1)^2$.

Lisa's Solution



I used tiles to create a model with a length and width of $3x + 1$.

$(3x + 1)^2$ is the area of a square with side length $3x + 1$.



I removed the tiles that represented the length and width. They were not part of the area of the square.

$$(3x + 1)^2 = 9x^2 + 3x + 3x + 1$$
$$= 9x^2 + 6x + 1$$

To determine the area, I counted the tiles, then simplified the expression by adding like terms.

EXAMPLE 3 Determining the product of a sum and difference of two terms using algebra tiles

Dave claims that

$$(2x + 3)(2x - 3) = (2x)^2 - 3^2$$
$$= 4x^2 - 9$$

a) Confirm the relationship by evaluating each expression when $x = 2$ and $x = 3$.
b) Show that Dave is correct no matter what the value of x.

Tracy's Solution

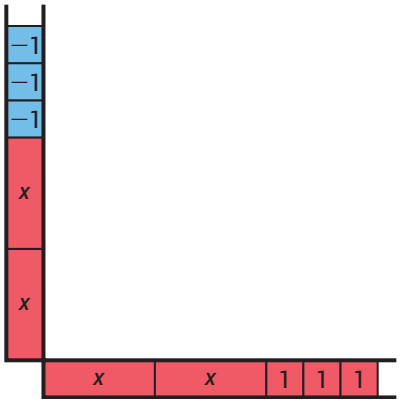
a)

	Left Side	Right Side	
	$(2x + 3)(2x - 3)$	$4x^2 - 9$	
$x = 2$	$(2 \times 2 + 3)(2 \times 2 - 3)$ $= 7 \times 1$ $= 7$	$4 \times 2^2 - 9$ $= 7$	Values are equal.
$x = 3$	$(2 \times 3 + 3)(2 \times 3 - 3)$ $= 9 \times 3$ $= 27$	$4 \times 3^2 - 9$ $= 27$	Values are equal.

I substituted the values 2 and 3 into both sides of the equation.

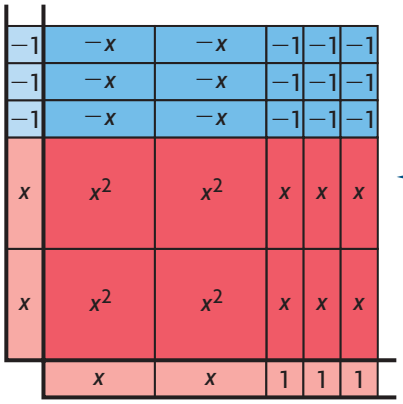
It appears Dave's claim is true, since both expressions give the same result when $x = 2$ and $x = 3$. However, I can't be certain since I only showed that this worked for two cases.

b)

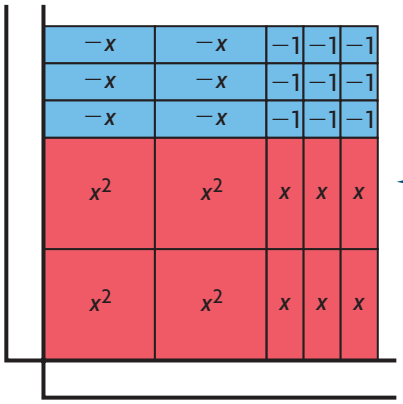


The area of a rectangle with dimensions $2x + 3$ and $2x - 3$ represents the product $(2x + 3)(2x - 3)$.

I used algebra tiles to represent the lengths and widths, red tiles for positives and blue tiles for negatives.



I built the rectangle I needed. I used blue negative tiles for the part that was multiplied by -3.



I removed the tiles representing the length and width.

$$(2x + 3)(2x - 3) = 4x^2 - 6x + 6x - 9$$
$$= 4x^2 - 9$$

To determine the area, I counted the tiles in the square, then simplified. I also noticed that since the 6 blue x tiles are negative and the 6 red x tiles are positive, they cancel out.

You can multiply algebraic expressions without using algebra tiles if you use the distributive property.

EXAMPLE 4 Expanding and simplifying quadratic expressions symbolically

Expand and simplify.

a) $x(2x + 1)$

b) $5(2x - 3)^2$

c) $-2(a - 5)(3a - 2)$

d) $4n(n - 3) + (5n + 1)(3n + 2)$

Link's Solutions

a) $x(2x + 1)$

$= x \times 2x + x \times 1$

$= 2x^2 + x$

I used the distributive property to multiply each term in the brackets by x.

b) $5(2x - 3)^2$

$= 5(2x - 3)(2x - 3)$

$= 5[(2x)(2x - 3) + (-3)(2x - 3)]$

$= 5[4x^2 - 6x - 6x + 9]$

$= 5[4x^2 - 12x - 9]$

$= 20x^2 - 60x + 45$

$(2x - 3)^2$ means $(2x - 3)(2x - 3)$.

I used the distributive property to expand the two binomials. I multiplied $2x$ by both $2x$ and -3, then multiplied -3 by both $2x$ and -3. I simplified by collecting like terms.

I multiplied the product by 5, again using the distributive property.

c) $-2(a - 5)(3a - 2)$

$= -2[a(3a - 2) - 5(3a - 2)]$

$= -2(3a^2 - 2a - 15a + 10)$

$= -2(3a^2 - 17a + 10)$

$= -6a^2 + 34a - 20$

I used the distributive property to find the product of the two binomials. I multiplied a by both $3a$ and -2, and -5 by both $3a$ and -2.

Then I collected like terms and multiplied by -2, using the distributive property again.

d) $4n(n-3) + (5n+1)(3n+2)$

$= (4n \times n - 4n \times 3) + (5n \times 3n + 5n \times 2 + 1 \times 3n + 1 \times 2)$

$= (4n^2 - 12n) + (15n^2 + 10n + 3n + 2)$

$= 4n^2 - 12n + 15n^2 + 13n + 2$

$= 19n^2 + 1n + 2$

$= 19n^2 + n + 2$

I used the distributive property again. I multiplied the first product and then the second one.

Then I collected like terms. Since I was adding $13n$ and $-12n$, only $1n$ was left.

I wrote $1n$ as n.

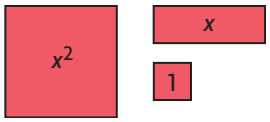
In Summary

Key Idea

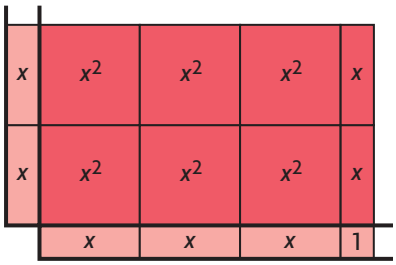
- Quadratic expressions can be expanded by using the distributive property and then simplified by collecting like terms.

Need to Know

- One way to multiply two linear expressions is to use an area model with algebra tiles. If you multiply two expressions, the expressions describe the length and width of a rectangle. The area of the rectangle is the product.
- In these models, x^2 can be represented as the area of a square with side length x.
- x can be represented as the area of a rectangle with side lengths of 1 and x.
- 1 can be represented as a square with side length 1.



- Red is used to represent positive quantities, blue to represent negative quantities.
- For example, to multiply $2x(3x + 1)$, build a rectangle with a width of $2x$ and a length of $3x + 1$ and count the tiles in the area as the product.



- For the product of a monomial and a binomial, the distributive property states that

$$a(b + c) = ab + ac$$

- For the product of a binomial and a binomial, apply the distributive property twice:

$$(a + b)(c + d)$$
$$= a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

- Like terms have the same variables with the same exponents.
- Three special multiplication patterns are

$$(a + b)^2 = (a + b)(a + b) = (a^2 + ab + ba + b^2) = a^2 + 2ab + b^2$$

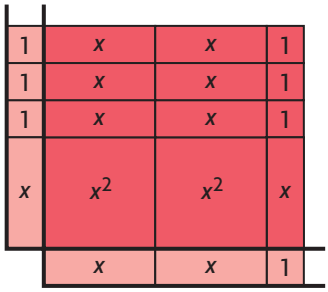
$$(a - b)^2 = (a - b)(a - b) = (a^2 - ab - ba + b^2) = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = (a^2 - ab + ba - b^2) = a^2 - b^2$$

CHECK Your Understanding

1. For each diagram, state the terms representing the length and the width of the rectangle. Then determine the product represented by the area.

a)



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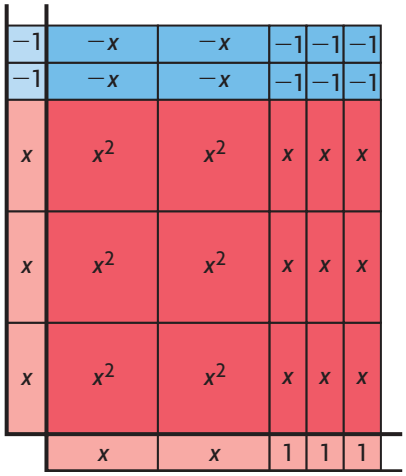


2. Expand and simplify.

a) $(x + 7)(x - 3)$

b) $(a + 6)(a + 6)$

c) $(2x - 5)^2$

d) $(m - 9)(m + 9)$

3. Expand and simplify

a) $3(x - 6)(x + 5)$

b) $3a(a - 5) - (2a + 1)(a - 7)$

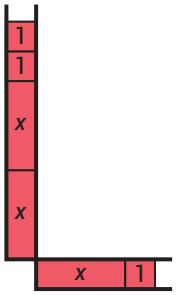
c) $-2n(2n + 1) + (n + 2)^2$

d) $3(2x + 1)^2 - 2(3x - 1)^2$

PRACTISING

4. For each diagram, describe the terms representing the length and the width of the rectangle and the product represented by the area.

a)



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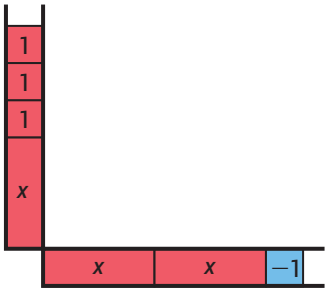


5. Expand and simplify.

a) $(3x - 2)(4x + 5)$

b) $5(3x + 2)^2$

c) $2(x - 3)^2 - (4x + 1)(4x - 1)$

d) $-a(2a + 3) + 2(a + 4)(3a + 4)$

e) $2n(n + 3) - 3n(5 - 2n) + 7n(1 - 4n)$

f) $(2x + 5)^2 + (2x - 5)^2 - (2x - 5)(2x + 5)$

6. **a)** Evaluate $3 - 2x$ for $x = -4$.

b) Evaluate $x + 2$ for $x = -4$.

c) Expand and simplify $(3 - 2x)(x + 2)$.

d) Evaluate your answer from part (c) for $x = -4$.

e) How are your answers to parts (a) and (b) related to the answer in part (d)?

7. Expand and simplify.

K **a)** $2x(x - 5)$

b) $(a + 7)(a - 9)$

c) $(3x + 7)(6x - 5)$

d) $2m(3m + 1) + (m - 4)(5m + 3)$

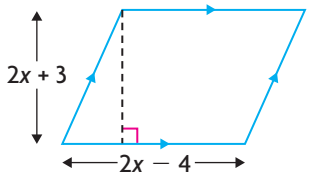
8. How do you know that the product will be quadratic when you expand an expression such as $(2x + 3)(3x - 2)$?

9. **a)** Sketch two different rectangles with an area of $4x^2 + 8x$.

b) List the dimensions of each rectangle.

10. Write an expression for the area of each shape. Expand and simplify.

A **a)**



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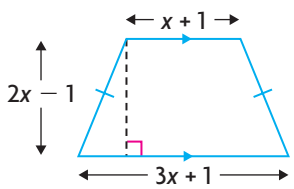


Communication Tip

The formula for the area of

- a parallelogram is $A = bh$
- a trapezoid is $A = h(b_1 + b_2) \div 2$

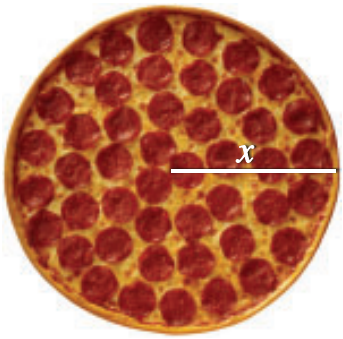
11. A circular pizza has a radius of x cm.

T

a) Write an expression for the area of the pizza.

b) Write an expression for the area of a pizza with a radius that is 5 cm greater.

c) How much greater is the second area? Write the difference as a simplified expression.



Communication Tip

The formula for the area of a circle is $A = \pi r^2$.

12. When you add $(6x^2 - 8x)$ and $(-15x^2 - 18)$ and collect like terms, you end up with three terms.

a) Give an example to show that the sum of two quadratic binomials could have only two terms.

b) Give an example to show that the sum of two quadratic binomials could have only one term.

13. C

a) Use an example to show that you can multiply two binomials and end up with three terms after you simplify.

b) Use other binomials to show that you can end up with two terms after you multiply and simplify.

Extending

14. Expand and simplify.

a) $(2x - y)(3x + y)$

b) $(3a - 5b)^2$

c) $(5m - 7n)(5m + 7n)$

d) $-2(x + 3y)(2x - y)$

15. A Pythagorean triple is a triple of natural numbers a, b, c that satisfies the Pythagorean theorem $a^2 + b^2 = c^2$, or equivalently, $a^2 = c^2 - b^2$. For example, 3, 4, 5 is a Pythagorean triple, since $3^2 + 4^2 = 5^2$, or $3^2 = 5^2 - 4^2$.

a) Show that $(n + 1)^2 - n^2 = 2n + 1$.

b) Determine n if $5^2 = 2n + 1$.

c) Use your answer from part (b) to determine a Pythagorean triple.

d) Determine three more Pythagorean triples.