1.7 The Domain and Range of a Quadratic Function

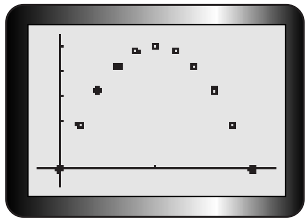
GOAL

Determine the domain and range of quadratic functions that model situations.

LEARN ABOUT the Math

A flare is shot vertically upward. A motion sensor records its height above ground every 0.2 s. The results are shown in the table.

Time (s)	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Height (m)	0.0	1.8	3.2	4.2	4.8	5.0	4.8	4.2	3.2	1.8	0.0

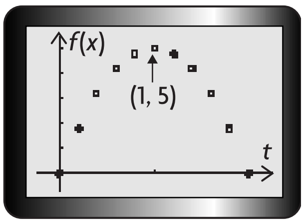


The data are plotted. The function $h(t) = -5t^2 + 10t$ models the height of the flare, in metres, as a function of time from the time the flare is first shot into the air to the time that it returns to the ground.

? How does the motion of the flare affect the domain and range of this quadratic function?

EXAMPLE 1 Reasoning about restricting the domain and range of a function

Wanda's Solution



The flare is initially shot at $t = 0$ s. The height of the flare when it is first shot is 0 m.

The flare reaches a maximum height of 5 m at $t = 1.0$ s.

After reaching 5 m, the flare starts to fall back to the ground.

At $t = 2$ s, the flare is on the ground.

The domain of this function is $\{t \in \mathbf{R} \mid 0 \leq t \leq 2\}$.

In many situations, the domain of a function is the set of all **real numbers**. However, in this situation, where time is the independent variable, we are interested only in the time from when the flare was first shot into the air until the time it hits the ground—the first 2 s of its flight.

The range is $\{h(t) \in \mathbf{R} \mid 0 \leq h(t) \leq 5\}$.

Since the height depends on the time the flare is in the air, the range of the function $h(t)$ is the set of all possible heights for the function. The maximum height is 5 m; the minimum is 0 m.

Reflecting

A. Use your graphing calculator to graph the quadratic function defined by $f(x) = -5x^2 + 10x$. Then identify the domain and range. Explain why these differ from the domain and range of the function used to model the height of the flare.

B. For each of the following descriptions, identify the independent variable and the dependent variable. Then describe and justify reasonable values for the domain.

i) The height of a stone that is thrown upward and falls to the ground, as a function of time

ii) The height of a stone that is thrown upward and falls over a cliff, as a function of time

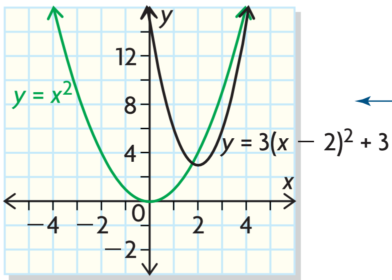
iii) The percent of sale prices that a supermarket cashier can remember, as a function of time

APPLY the Math

EXAMPLE 2 Connecting the domain and range of a function to its graph

Find the domain and range of $y = 3(x - 2)^2 + 3$.

Indira's Solution



I graphed the function by translating $y = x^2$ to the right 2 units, vertically stretching by a factor of 3, and finally translating 3 units up.

The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all values of x. The domain is the set of all real numbers.

The range is $\{y \in \mathbf{R} \mid y \geq 3\}$.

The vertex is (2, 3).

Since the function is a parabola that opens upward, the y-coordinate of the vertex is a minimum value.

The range is all values of y greater than or equal to 3.

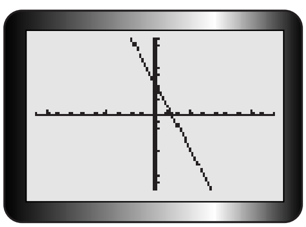
EXAMPLE 3 Connecting the domain and range to linear functions

Find the domain and range of each linear function.

a) $f(x) = -3x + 4$ **b)** $y = 5$

Paul's Solution

a)



I used a graphing calculator to graph the function.

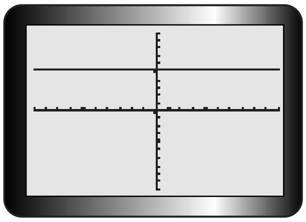
The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all real numbers.

The range is $\{y \in \mathbf{R}\}$.

Every real number corresponds to a value in the domain.

b)



I graphed the function on a graphing calculator.

The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all real numbers.

The range is $\{y \in \mathbf{R} | \, y = 5\}$.

Every real number in the domain corresponds to the number 5 in the range.

Any time you work with a function that models a real-world situation, it is necessary to restrict the domain and range.

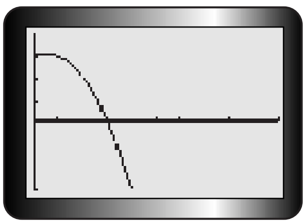
EXAMPLE 4 Placing restrictions on the domain and range

A baseball thrown from the top of a building falls to the ground below. The path of the ball is modelled by the function $h(t) = -5t^2 + 5t + 30$, where $h(t)$ is the height of the ball above ground, in metres, and t is the elapsed time in seconds. What are the domain and range of this function?

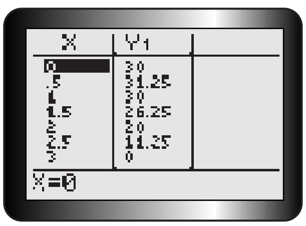
Kendall's Solution

Tech Support

For help on using the table of values, see Technical Appendix, B-6.



Time is the independent variable, so the domain is all of the times the ball is in the air. I graphed the function on a graphing calculator, using only positive numbers for time.



I examined the resulting table of values.

The domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 3\}$.

The baseball is thrown at $t = 0$ s and is in the air until it hits the ground at $t = 3$ s.

The range is $\{h(t) \in \mathbf{R} \mid 0 \leq h(t) \leq 31.25\}$.

Height is the dependent variable, so the range is all of the heights the ball reaches during its flight.

The ball reaches its maximum height at $t = 0.5$, so

$$\begin{aligned} h(0.5) &= 5(0.5)^2 + 5(0.5) + 30 \\ &= 31.25 \text{ m} \end{aligned}$$

It reaches its minimum height of 0 m when it hits the ground at $t = 3$, so $h(3) = 0$.

Since the height cannot be negative, I disregarded the y-values below 0.

In Summary

Key Ideas

- The domain of a function is the set of values for which the function is defined. As a result, the range of a function depends on the defining equation of the function.
- When quadratic functions model real-world situations, the domain and range are often restricted to values that make sense for the situation. Values that don't make sense are excluded.

Need to Know

- Most linear functions have the set of real numbers as their domain and range. The exceptions are horizontal and vertical lines.
- The range of a horizontal line is the value of y.
- The domain of a vertical line is the value of x.
- Quadratic functions have the set of real numbers as their domain. The range depends on the location of the vertex and whether the parabola opens up or down.

CHECK Your Understanding

1. a) If $f(x)$ is a linear function with a positive or negative slope, what will the domain and range of this function be?

b) Consider linear relations that represent either horizontal lines, such as $y = 4$, or vertical lines, such as $x = 6$. What are the domain and range of these relations?

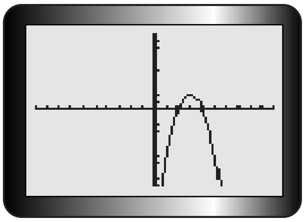
2. Find the domain and range of each function. Explain your answers.

a) $f(x) = -2(x + 3)^2 + 5$
b) $f(x) = 2x^2 + 4x + 7$
c) $f(x) = 3x - 4$
d) $x = 5$

3. The height of a flare is a function of the elapsed time since it was fired. An expression for its height is $f(t) = -5t^2 + 100t$. Express the domain and range of this function in set notation. Explain your answers.

PRACTISING

4. For each graph, state the domain and range in set notation. The scale on both axes is 1 unit per tick mark in each calculator screen.

a)


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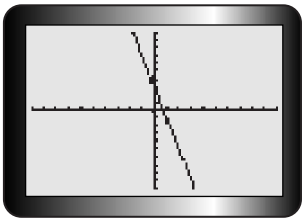

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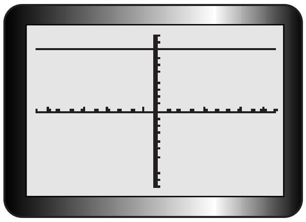

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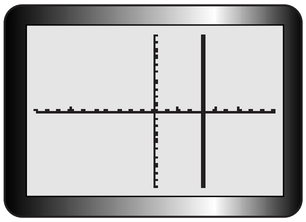

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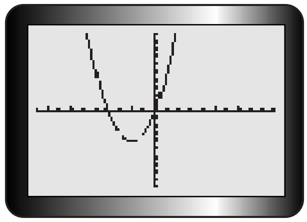

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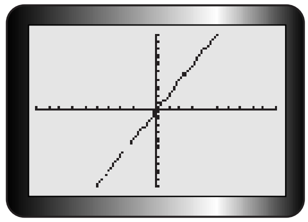


5. State the domain and range of the quadratic function represented by the following table of values.

x	1	2	3	4	5	6	7	8	9	10	11
y	12	20	27	32	35	36	35	32	27	20	12

6. **K** Use a graphing calculator or graphing software to graph each function. State its domain and range.

a) $f(x) = x^2 - 6$
b) $g(t) = 2 + t - 10t^2$
c) $f(x) = -2x^2 + 5$, where $x \geq 0$
d) $h(x) = 2(x - 1)^2 + 3$
e) $g(x) = x^2 - 6x + 2$
f) $h(x) = 3x^2 - 14x - 5$, where $x \geq 0$

7. **A** A pebble is dropped from a bridge into a river. The height of the pebble above the water after it has been released is modelled by the function $h(t) = 80 - 5t^2$, where $h(t)$ is the height in metres and t is time in seconds.

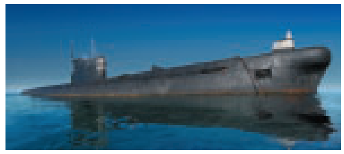
a) Graph the function for reasonable values of t.
b) Explain why the values you chose for t in part (a) are reasonable.
c) How high is the bridge? Explain.
d) How long does it take the pebble to hit the water? Explain.
e) Express the domain and range in set notation.

8. The cost of a banquet is \$550 for the room rental, plus \$18 for each person served.
   a) Create a table of values and construct a scatter plot for this function.
   b) Determine the equation that models the function.
   c) State the domain and range of the function in this situation.

Tech Support

For help with scatter plots, see Technical Appendix, B-10.

9. A submarine at sea level descends 50 m every 5 min.
   T a) Determine the function that models this situation.
   b) If the ocean floor is 3 km beneath the surface of the water, how long will it take the submarine to reach the ocean floor.
   c) State the domain and range of this function in this situation.



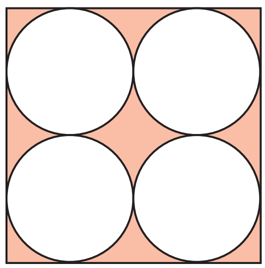
10. A baseball is hit from a height of 1 m. The height of the ball is modelled by the function $h(t) = -5t^2 + 10t + 1$, where t is time in seconds.
    a) Graph the function for reasonable values of t.
    b) Explain why the values you chose for t in part (a) are reasonable.
    c) What is the maximum height of the ball?
    d) At what time does the ball reach the maximum height?
    e) For how many seconds is the ball in the air?
    f) For how many seconds is the ball higher than 10 m?
    g) Express the domain and range in set notation.

11. In the problem about selling raffle tickets in Getting Started on page 5, the student council wants to determine the price of the tickets. The council surveyed students to find out how many tickets would be bought at different prices. The council found that
    - if they charge \$0.50, they will be able to sell 200 tickets; and
    - if they raise the price to \$1.00, they will sell only 50 tickets.

    a) The formula for revenue, $R(x)$, as a function of ticket price, x, is $R(x) = -300x^2 + 350x$. Use a graphing calculator to graph the function.
    b) What ticket price should they charge to generate the maximum amount of revenue?
    c) State the range and domain of the function.

12. Sometimes when functions are used to model real-world situations, the **C** domain and range of the function must be restricted.
    a) Explain what this means.
    b) Explain why restrictions are necessary. Use an example in your explanation.

Extending



13. The picture shows four circles inside a square. Each small circle has a radius r. The area of the shaded region as a function is $A(r) = (16 - 4\pi)r^2$.
    a) What is the domain of the function?
    b) What is the range of the function?