

Exploring Transformations of Quadratic Functions

YOU WILL NEED

- graphing calculator or graphing software
- graph paper

transformations

transformations are operations performed on functions to change the position or shape of the associated curves or lines

Tech Support

For help on graphing functions, see Technical Appendix, B-2.

GOAL

Understand how the parameters a , h , and k in $f(x) = a(x - h)^2 + k$ affect and change the graph of $f(x) = x^2$.

EXPLORE the Math

Transformations of curves, lines, and shapes form the basis for many patterns, like those you see on fabrics, wallpapers, and gift-wrapping paper. They are also used in designing tessellations such as those used in the graphic art of Maurits Cornelis Escher (1898–1972).



? How do the parameters a , h , and k in a quadratic function of the form $f(x) = a(x - h)^2 + k$ change the graph of $f(x) = x^2$?

- A.** Use a graphing calculator or graphing software to graph the quadratic functions shown in the table on the same set of axes. Sketch and label the graph of each function on the same set of axes. Then copy and complete the table.

	Function	Value of k in $f(x) = x^2 + k$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = x^2 + 2$	2				
c)	$f(x) = x^2 + 4$					
d)	$f(x) = x^2 - 1$					
e)	$f(x) = x^2 - 3$					

- B.** Use the graphs and the table from part A to answer the following:
- What information about the graph does the value of k provide in functions of the form $f(x) = x^2 + k$?
 - What happens to the graph of the function $f(x) = x^2 + k$ when the value of k is changed? Consider both positive and negative values of k .
 - What happens to the x -coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = x^2 + k$? What happens to the y -coordinates?

- C. Clear all previous equations from your calculator or graphing program. Repeat part A for the quadratic functions shown in the table.

	Function	Value of h in $f(x) = (x - h)^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = (x - 2)^2$	2				
c)	$f(x) = (x - 4)^2$					
d)	$f(x) = (x + 2)^2$					
e)	$f(x) = (x + 4)^2$					

- D. Use the graphs and the table from part C to answer the following:
- What information about the graph does the value of h provide in functions of the form $f(x) = (x - h)^2$?
 - What happens to the graph of the function $f(x) = (x - h)^2$ when the value of h is changed? Consider both positive and negative values of h .
 - What happens to the x -coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = (x - h)^2$? What happens to the y -coordinates?
- E. Clear all previous equations from your calculator or graphing program. Repeat part A for the quadratic functions shown in the table.

	Function	Value of a in $f(x) = ax^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	1	up	(0, 0)	$x = 0$	yes
b)	$f(x) = 2x^2$					
c)	$f(x) = 0.5x^2$					
d)	$f(x) = -2x^2$					
e)	$f(x) = -0.5x^2$					

- F. Use the graphs and the table from part E to answer the following:
- What information about the graph does the value of a provide in functions of the form $f(x) = ax^2$? Consider values of a that are greater than 1 and values of a between 0 and 1.
 - What happens to the graph of the function $f(x) = ax^2$ when the value of a is changed? Consider both positive and negative values of a .
 - What happens to the x -coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = ax^2$? What happens to the y -coordinates?

Reflecting

- G. Use examples and labelled graphs that compare the transformed functions with the function $f(x) = x^2$. Summarize the effect that a , h , and k have on the parabola's
- direction of opening
 - vertex
 - axis of symmetry
 - congruency to $f(x) = x^2$
 - x -coordinates and y -coordinates

In Summary

Key Ideas

- Quadratic functions can be written in the form $g(x) = a(x - h)^2 + k$.
- Each of the constants a , h , and k changes the position and/or shape of the graph of $f(x) = x^2$.

Need to Know

- Changing the values of h and k changes the position of the parabola and, as a result, the locations of the vertex and the axis of symmetry. The new parabola is congruent to the parabola $f(x) = x^2$.
- Changing the value of a can change the shape of the parabola, as well as the direction in which the parabola opens. The new parabola is not congruent to the parabola $f(x) = x^2$ when $a \neq 1$ or -1 .

FURTHER Your Understanding

- The parabola $f(x) = x^2$ is transformed as described. The equation of the transformed graph has the form $f(x) = a(x - h)^2 + k$. Determine the values of a , h , and k for each of the following transformations.
 - The parabola moves 4 units to the right.
 - The parabola moves 5 units up.
 - The parabola moves 2 units to the left
 - The parabola moves 3 unit down.
 - The parabola is congruent to $f(x) = x^2$ and opens down.
 - The parabola is narrower and opens upward. The y -coordinates have been multiplied by a factor of 2.
 - The parabola is wider and opens downward. The y -coordinates have been multiplied by a factor of $\frac{1}{2}$.