1.3 Working with Function Notation

GOAL

Interpret and evaluate relationships expressed in function notation.

YOU WILL NEED

- graph paper

LEARN ABOUT the Math

A rocket is shot into the air from the top of a building. Its height, in metres, after t seconds is modelled by the function $h(t) = -4.9t^2 + 19.6t + 34.3$.

? What is the meaning of $h(t)$ and how does it change for different values of t?

EXAMPLE 1 Connecting the rocket's height to function notation

Use function notation and a table of values to show the height of the rocket above the ground during its flight.

Leila's Solution: Using a Table of Values

$$h(t) = -4.9t^2 + 19.6t + 34.3$$

$$\begin{aligned} h(0) &= -4.9(0)^2 + 19.6(0) + 34.3 \\ &= 0 + 0 + 34.3 \\ &= 34.3 \end{aligned}$$

I substituted 0 for t in the function. I started at $t = 0$ because time can never be negative.

$$h(0) = 34.3 \text{ m}$$

$h(0) = 34.3$ means that when the rocket was first shot, at 0 s, it was 34.3 m above the ground. This is the height of the building.

$$\begin{aligned} h(3) &= -4.9(3)^2 + 19.6(3) + 34.3 \\ &= -4.9(9) + 58.8 + 34.3 \\ &= -44.1 + 58.8 + 34.3 \\ &= 49 \end{aligned}$$

I evaluated the height of the rocket at $t = 3$.

$$h(3) = 49 \text{ m}$$

$h(3) = 49$ means that after 3 s, the rocket is 49 m above the ground.

t	0	1	2	3	4	5	6
$h(t)$	34.3	49.0	53.9	49.0	34.3	9.8	−24.5

I found the height of the rocket at different times and created a table of values. $h(6)$ gave a negative result. Since the height is always positive, the rocket hit the ground between 5 s and 6 s.

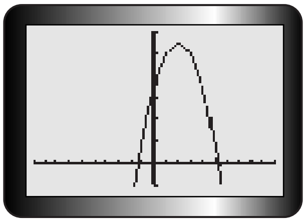
t	5.1	5.2	5.3	5.4
$h(t)$	6.81	3.72	0.54	−2.744

I evaluated the function for different times between 5 s and 6 s. The rocket hit the ground between 5.3 s and 5.4 s after it was shot into the air.

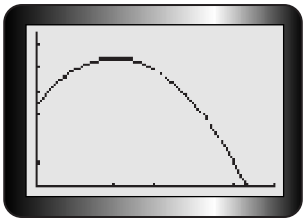
Tina's Solution: Using a Graph of the Height Function

Tech *Support*

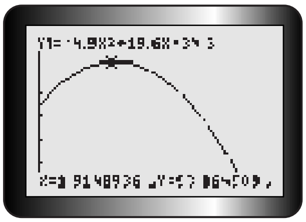
For help graphing and tracing along functions, see Technical Appendix, B-2.



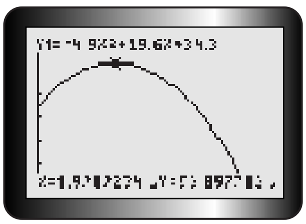
I used the equivalent equation $y = -4.9x^2 + 19.6x + 34.3$ and entered this for Y1 in the equation editor to plot the graph on a graphing calculator.



I adjusted the calculator window to show the part of the graph where the rocket is in the air. The height of the rocket is negative after about 5.3 s, so the domain is from 0 to 5.3 s.



By tracing along the graph, I can see that the maximum height, 53.89 m, occurs sometime between 1.91 s and 1.97 s.



Reflecting

A. How are the two methods similar? How are they different?

B. How did each student determine the domain? What is the domain of the function $h(t)$?

C. How did each student determine the valid values for the range? What is the range of the function $h(t)$?

D. Tina determined that the maximum height of the rocket would occur between 1.91 s and 1.97 s. Explain how Tina could find a more exact answer.

APPLY the Math

EXAMPLE 2 Using a substitution strategy to evaluate a function

Given $f(x) = 2x^2 + 3x - 1$, evaluate

a) $f(3)$ **b)** $f\left(\frac{1}{2}\right)$ **c)** $f(5 - 3)$ **d)** $f(5) - f(4)$

Richard's Solution

a)
$$\begin{aligned} f(x) &= 2x^2 + 3x - 1 \\ f(3) &= 2(3)^2 + 3(3) - 1 \\ &= 2(9) + 9 - 1 \\ f(3) &= 26 \end{aligned}$$

$f(3)$ means "find the value of the dependent variable or output when the independent variable or input is $x = 3$." I substituted 3 for x and then evaluated the expression.

b)
$$\begin{aligned} f(x) &= 2x^2 + 3x - 1 \\ f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1 \\ &= 2\left(\frac{1}{4}\right) + \frac{3}{2} - 1 \\ &= \frac{2}{4} + \frac{6}{4} - \frac{4}{4} \\ f\left(\frac{1}{2}\right) &= 1 \end{aligned}$$

I substituted $\frac{1}{2}$ for x and then evaluated the expression using a common denominator of 4.

c)
$$\begin{aligned} f(5 - 3) &= 2(5 - 3)^2 + 3(5 - 3) - 1 \\ &= 2(2)^2 + 3(2) - 1 \\ &= 2(4) + 6 - 1 \\ f(5 - 3) &= 13 \end{aligned}$$

$f(5 - 3)$ is the same as $f(2)$, so I evaluated $f(5 - 3)$ by evaluating $f(2)$.

d) $f(5) - f(4)$

$= [2(5)^2 + 3(5) - 1] - [2(4)^2 + 3(4) - 1]$

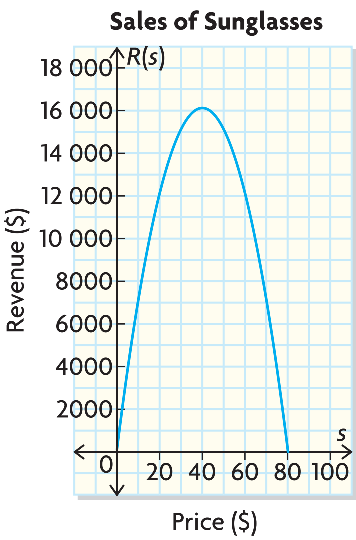
$= [2(25) + 15 - 1] - [2(16) + 12 - 1]$

$= 64 - 43$

$f(5) - f(4) = 21$

$f(5) - f(4)$ is the difference in the value of the function evaluated at $x = 5$ and at $x = 4$.

So, I subtracted the value for $f(4)$ from the value for $f(5)$.



EXAMPLE 3 Representing and comparing the value of a function

The relationship between the selling price of a new brand of sunglasses and revenue, $R(s)$, is represented by the function $R(s) = -10s^2 + 800s + 120$ and its graph at the left.

a) Determine the revenue when the selling price is \$5.

b) Explain what $R(20) = 12\,120$ means.

c) If $R(s) = 16\,120$, determine the selling price, s.

Kedar's Solution

a) $R(s) = -10s^2 + 800s + 120$

$R(5) = -10(5)^2 + 800(5) + 120$

$= -10(25) + 4000 + 120$

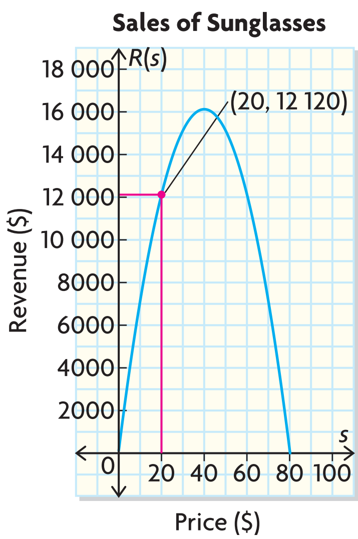
$= 3870$

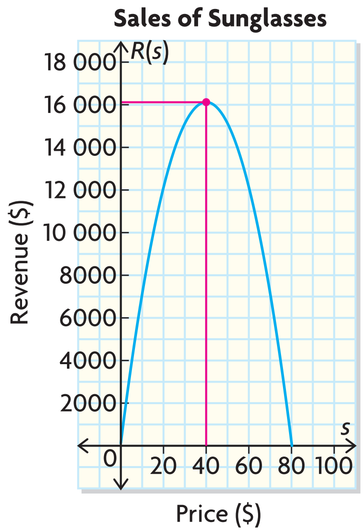
$R(5)$ is the revenue generated at a selling price of \$5. I substituted 5 for s into the equation and evaluated the expression.

When the selling price is \$5, the revenue is \$3870.

b) $R(20)$ represents the revenue generated from a selling price of \$20. In this case $R(20)$ is \$12 120. This corresponds to the point (20, 12 120) on the graph.

When the value of the independent variable, s, is 20, the dependent variable, $R(s)$, has a value of 12 120.



c) 

I used the graph to estimate the selling price that corresponds to a revenue of $16 120. I drew a horizontal line from 16 120 on the revenue axis until it touched the curve. From this point I drew a vertical line down to the selling price axis.

For a revenue of $16 120 to occur, the selling price must be $40.

EXAMPLE 4 Representing new functions

If $g(x) = 2x^2 - 3x + 5$, determine

a) $g(m)$ **b)** $g(3x)$

Sinead's Solution

a)
$$g(x) = 2x^2 - 3x + 5$$
$$g(m) = 2m^2 - 3m + 5$$

In this case, *x* is replaced by the variable *m*, not by a number.

b)
$$g(3x) = 2(3x)^2 - 3(3x) + 5$$
$$= 2(9x^2) - 9x + 5$$
$$g(3x) = 18x^2 - 9x + 5$$

I substituted $3x$ for x in the equation for $g(x)$ and then simplified.

In Summary

Key Idea

- $f(x)$ is called function notation and is used to represent the value of the dependent variable for a given value of the independent variable, x. For this reason, y and $f(x)$ are interchangeable in the equation or graph of a function, so $y = f(x)$.

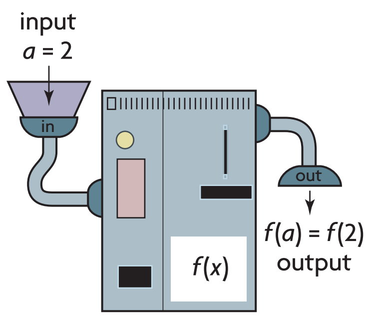
Need to Know

- When a function is defined by an equation, it is convenient to name the function to distinguish it from other equations of other functions.

  For example, the set of ordered pairs (x, y) that satisfies the equation $y = 3x + 2$ forms a function. By naming the function f, we can use function notation.

(x, y) Notation	Function Notation
(x, y) is a solution of $y = 3x + 2$.	$(x, f(x))$ is a solution of $f(x) = 3x + 2$.

- $f(a)$ represents the value or output of the function when the input is $x = a$. The output depends on the equation of the function. To evaluate $f(a)$, substitute a for x in the equation for $f(x)$.
- $f(a)$ is the y-coordinate of the point on the graph of f with x-coordinate a. For example, if $f(x)$ takes the value 3 at $x = 2$, then $f(2) = 3$ and the point $(2, 3)$ lies on the graph of f.

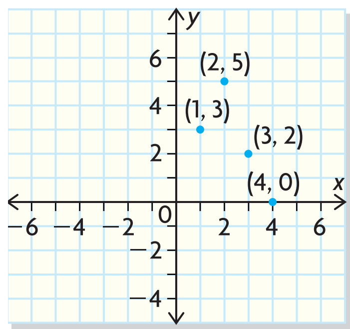


CHECK Your Understanding

1. Explain the meaning of $f(3) = \frac{1}{2}$.
2. Evaluate $f(3)$ for each of the following.
   a) $\{(1, 2), (2, 0), (3, 1), (4, 2)\}$
   b)

x	1	2	3	4
y	2	3	4	5

   c)



3. Determine $f(-2)$, $f(0)$, $f(2)$, and $f(2x)$ for each function.
a) $f(x) = -3x^2 + 5$ **b)** $f(x) = 4x^2 - 2x + 1$

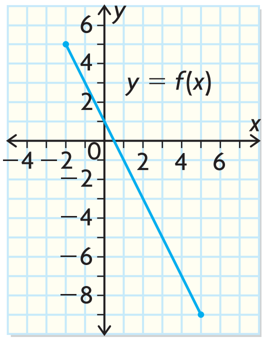
4. A stone is thrown from a bridge into a river. The height of the stone above the river at any time after it is released is modelled by the function $h(t) = 72 - 4.9t^2$. The height of the stone, $h(t)$, is measured in centimetres and time, t, is measured in seconds.
a) Evaluate $h(0)$. What does it represent?
b) Evaluate $h(2.5)$. What does it represent?
c) If $h(3) = 27.9$, explain what you know about the stone's position.

PRACTISING

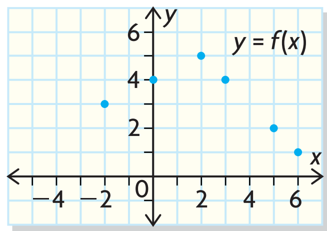
5. Evaluate $f(4)$ for each of the following.
a) $f = \{(1, 5), (3, 2), (4, 1), (6, 2)\}$
b)

x	2	4	6	8
f(x)	4	8	12	16

c) $f(x) = 3x^2 - 2x + 1$
d)



6. The graph of $y = f(x)$ is shown at the right.
a) State the domain and range of f.
b) Evaluate.
i) $f(3)$ **ii)** $f(5)$ **iii)** $f(5 - 3)$ **iv)** $f(5) - f(3)$
c) In part (b), why is the value in (iv) not the same as that in (iii)?
d) $f(2) = 5$. What is the corresponding ordered pair? What does 2 represent? What does $f(2)$ represent?



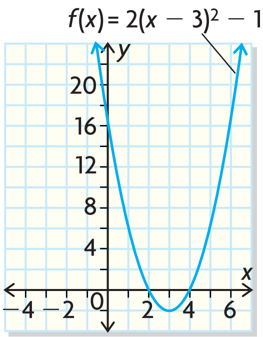
7. If the point (2, 6) is on the graph of $y = f(x)$, what is the value of $f(2)$? Explain.

8. If $f(-2) = 6$, what point must be on the graph of f? Explain.

9. Evaluate each function for the given x-values.
a) $f(x) = 9x + 1; x = 0, x = 2$
b) $f(x) = -2x - 3; x = -1, x = 3$
c) $f(x) = 2x^2 + 5; x = 2, x = 3$
d) $f(x) = 3x^2 - 4; x = 0, x = 4$

10. Consider the function $g(t) = 3t + 5$.
   a) Determine
      i) $g(0)$
      ii) $g(1)$
      iii) $g(2)$
      iv) $g(3)$
      v) $g(1) - g(0)$
      vi) $g(2) - g(1)$
   b) In part (a), what are the answers to (v) and (vi) commonly called?

11. Consider the function $f(x) = s^2 - 6s + 9$.
   a) Determine each value.
      i) $f(0)$
      ii) $f(1)$
      iii) $f(2)$
      iv) $f(3)$
      v) $[f(2) - f(1)] - [f(1) - f(0)]$
      vi) $[f(3) - f(2)] - [f(2) - f(1)]$
   b) In part (a), what are the answers to (v) and (vi) commonly called?



12. The graph shows $f(x) = 2(x - 3)^2 - 1$.
   a) Evaluate $f(0)$.
   b) What does $f(0)$ represent on the graph of f?
   c) If $f(x) = 6$, determine possible values of x.
   d) Does $f(3) = 4$ for this function? Explain.

13. **T** The sum of two whole numbers is 10. Their product can be modelled by the function $P(x) = x(10 - x)$.
   a) What does x represent? What does $(10 - x)$ represent? What does $P(x)$ represent?
   b) What is the domain of this function?
   c) Evaluate the function for all valid values of the domain. Show the results in a table of values.

x	0										
P(x)	0										

   d) What two numbers do you estimate give the largest product? What is the largest product? Explain.

14. **A** A farming cooperative has recorded information about the relationship between tonnes of carrots produced and the amount of fertilizer used. The function $f(x) = -0.53x^2 + 1.38x + 0.14$ models the effect of different amounts of fertilizer, x, in hundreds of kilograms per hectare (kg/ha), on the yield of carrots, in tonnes.
   a) Evaluate $f(x)$ for the given values of x and complete the table.

Fertilizer, x (kg/ha)	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Yield, y(x) (tonnes)									

   b) According to the table, how much fertilizer should the farmers use to produce the most tonnes of carrots?
   c) Check your answer with a graphing calculator or by evaluating $f(x)$ for values between 1.25 and 1.50. Why does the answer change? Explain.

15. Sam read the following paragraph in his textbook. He does not understand the explanation.

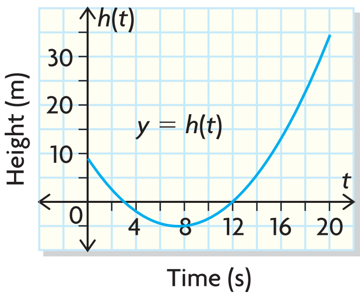
C

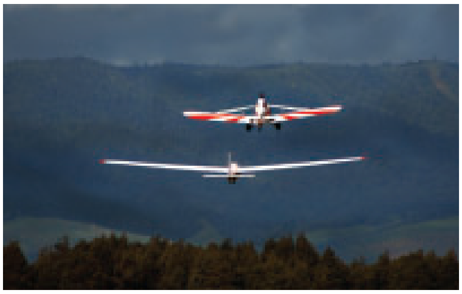
"$f(x)$ represents an expression for determining the value of the function f for any value of x. $f(3)$ represents the value of the function (the output) when x (the input) is 3. If $f(x)$ is graphed, then $f(3)$ is the y-coordinate of a point on the graph of f, and the x-coordinate of that point is 3."

Create an example (equation and graph) using a quadratic function to help him understand what he has read.

Extending

16. A glider is launched from a tower on a hilltop. The height, in metres, is negative whenever the glider is below the height of the hilltop. The equation representing the flight is $h(t) = \frac{1}{4}(t - 3)(t - 12)$, where time, t, is measured in seconds.

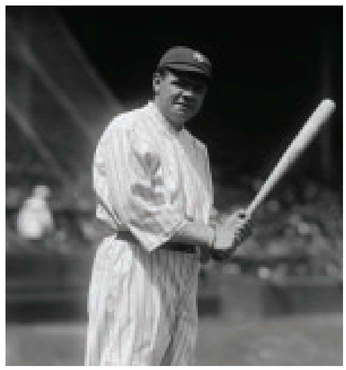




a) What does $h(0)$ represent?
b) What does $h(3)$ represent?
c) When is the glider at its lowest point? What is the vertical distance between the top of the tower and the glider at this time?

17. Babe Ruth, a baseball player, hits a "major league pop-up" so that the height of the ball, in metres, is modelled by the function $h(t) = 1 + 30t - 5t^2$, where t is time in seconds.

a) Evaluate the function for each of the given times to complete the table of values below.
b) Graph the function.
c) When does the ball reach its maximum height?
d) What is the ball's maximum height?
e) How long does it take for the ball to hit the ground?



Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
Height (m)																	