# 1.1 The Characteristics of a Function

**YOU WILL NEED**

- graph paper
- graphing calculator (*optional*)

**GOAL**

Identify the difference between a relation and a function.

## *INVESTIGATE the Math*

Nathan examines two temperature **relations** for the month of March. In relation *A*, the **dependent variable** *y* takes on the same or opposite value of the **independent variable** *x*. In relation *B*, the dependent variable *y* takes on the same value of the independent variable *x*. Each relation is represented by a table of values and a mapping diagram.

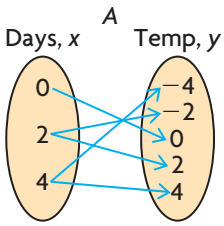
**mapping diagram**
a drawing with arrows to show the relationship between each value of *x* and the corresponding values of *y*

**Tech *Support***

For help using the graphing calculator to create a scatter plot, see Technical Appendix, B-10.

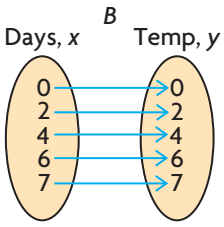
### Relation *A*

| **Number of Days in the Month, *x*** | 0 | 2 | 2 | 4 | 4 |
|---|---|---|---|---|---|
| **Minimum Daily Temperature (°C), *y*** | 0 | 2 | −2 | −4 | 4 |




### Relation *B*

| **Number of Days in the Month, *x*** | 0 | 2 | 4 | 6 | 7 |
|---|---|---|---|---|---|
| **Maximum Daily Temperature (°C), *y*** | 0 | 2 | 4 | 6 | 7 |




? Which relation is also a function?

**A.** The table of values of relation $A$ represents the set of ordered pairs $(x, y) =$ (days, minimum temp.). Create a scatter plot of the ordered pairs of $A$ on a coordinate grid.

**B.** The table of values of relation $B$ represents the set of ordered pairs $(x, y) =$ (days, maximum temp.) Create a scatter plot of the ordered pairs of $B$ on a different coordinate grid.

**C.** The domain of relation $A$ can be expressed using set notation, $D = \{0, 2, 4\}$. What do these numbers represent? Express the domain of relation $B$ in set notation.

**D.** The range of relation $A$ is $R = \{-4, -2, 0, 2, 4\}$. What do these numbers represent? Express the range of relation $B$ in set notation.

**E.** Examine the mapping diagram for relation $B$. Each $x$-value in the domain (number of days) is mapped to exactly one $y$-value (maximum temp.) in the range by an arrow. Examine the mapping diagram for relation $A$. How are the mappings in $A$ different from the mappings in $B$?

**F.** Explain how following the arrows of a mapping diagram helps you see which relation is a function.

**G.** Draw the vertical line $x = 2$ on your scatter plots of relation $A$ and relation $B$. What do you notice? Explain how drawing a vertical line helps you see which relation is a function.

**domain**
the set of all values for which the independent variable is defined

**set notation**
a way of writing a set of items or numbers within curly brackets, { }

**range**
the set of all values of the dependent variable. All such values are determined from the values in the domain

**function**
a relation in which there is only one value of the dependent variable for each value of the independent variable (i.e., for every $x$-value, there is only one $y$-value)

## Reflecting

**H.** Relations can be expressed by equations. Write the equations for relations $A$ and $B$ that define the relationship between the number of days in the month and the maximum or minimum temperature.

**I.** A relation can be represented as a table of values, a mapping diagram, a graph, and an equation. Which methods make it easier to help you decide whether a relation is a function? Explain.

**J.** Write a definition for the term *function* in terms of the domain and range.

## *APPLY* the Math

### EXAMPLE 1 Using reasoning to decide whether a relation is a function

For each of the following relations, determine

- the domain and the range
- whether or not it is a function

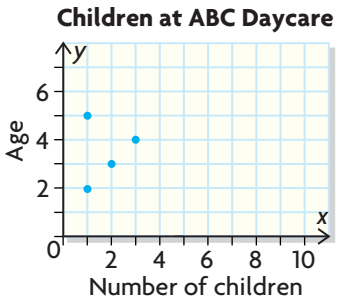
**a)** $G$: $(x, y)$ = (number of golfers, score below or above par)

$= \{(0, -2), (0, -1), (0, 0)\ (0, 5)\}$

**b)**

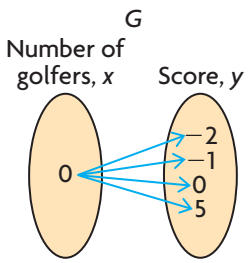
| $x$ | $y$ |
|---|---|
| $-1$ | $-3$ |
| 0 | 1 |
| 1 | 5 |
| 2 | 9 |

**c)**



## Anisha's Solution

**a)**




I drew a mapping diagram to represent the ordered pairs. The *x*-coordinate, 0, maps onto four different *y*-coordinates.

Domain $= \{0\}$
Range $= \{-2, -1, 0, 5\}$

This relation is not a function.

**b)**

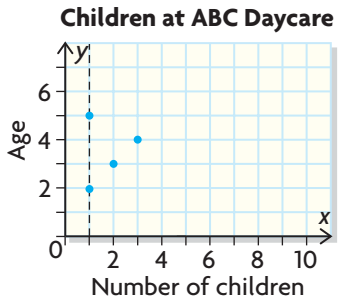
| *x* | *y* |
|---|---|
| −1 | −3 |
| 0 | 1 |
| 1 | 5 |
| 2 | 9 |

Each *x*-value in the table corresponds to one unique *y*-value.

Domain $= \{-1, 0, 1, 2\}$
Range $= \{-3, 1, 5, 9\}$

This relation is a function.

**c)**




The vertical line crosses two points, (1, 2) and (1, 5), when *x* equals 1. The *x*-coordinate, 1, corresponds to two different *y*-coordinates.

Domain $= \{1, 2, 3\}$
Range $= \{2, 3, 4, 5\}$

This relation is not a function.

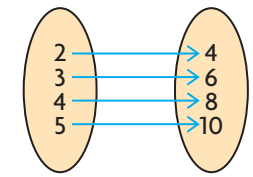
For any relation that is a function, if you know the value of the independent variable, you can predict the value of the dependent variable.

## EXAMPLE 2 Connecting mapping diagrams with functions

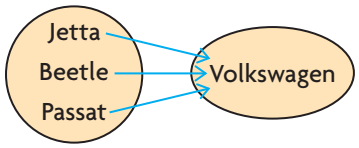
For each relation, determine

- the domain and the range
- whether or not it is a function

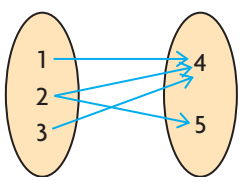
**a)**



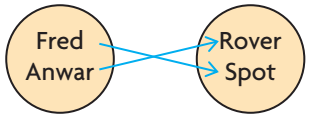

**b)**




**c)**




**d)**




### Kinson's Solution

**a)** The domain is {2, 3, 4, 5}.

The range is {4, 6, 8, 10}.

This relation is a function.

Each *x*-value corresponds to only one *y*-value. If I know the quantity of time, I can predict the number of e-mails received.

**b)** The domain is {Jetta, Beetle, Passat}.

The range is {Volkswagen}.

This relation is a function.

Each model of car in the domain corresponds with only one manufacturer in the range. If I know the model of car, I can predict the manufacturer.

**c)** The domain is {1, 2, 3}.

The range is {4, 5}.

This relation is not a function.

The 2 in the domain corresponds to two different values in the range. I can't predict the number of kittens born from knowing the age of the mother.

**d)** The domain is {Fred, Anwar}.

The range is {Rover, Spot}.

This relation is a function.

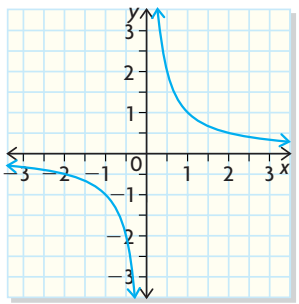
Each name in the domain corresponds to one name in the range. If I know the owner's name, I can predict the dog's name.

If you are given the graph of a relation, you can also identify if it is a function by comparing the values of the independent variable with the values of the dependent variable.

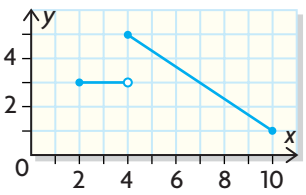
## EXAMPLE 3 Connecting functions with graphs

For each of the following relations, determine the domain and the range, using real numbers. State whether or not the relation is a function.

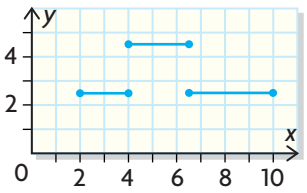
**a)**



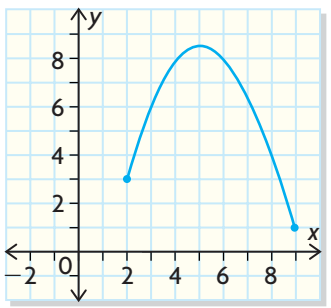

**b)**




**c)**




**d)**




### Dimitri's Solution

**a)** The domain is $D = \{x \in \mathbf{R} | x \neq 0\}$.

The range is $R = \{y \in \mathbf{R} | y \neq 0\}$.

This relation is a function.

This relation passes the vertical-line test. This means that each $x$-coordinate corresponds to exactly one $y$-coordinate.

**b)** The domain is $D = \{x \in \mathbf{R} | 2 \leq x \leq 10\}$.

The range is $R = \{y \in \mathbf{R} | 1 \leq y \leq 5\}$.

This relation is a function.

This relation passes the vertical-line test. The open dot at the point (4, 3) means that $y \neq 3$ when $x = 4$, and the closed dot at the point (4, 5) means that $y = 5$ when $x = 4$. So when $x = 4$, there is only one $y$-value.

**c)** The domain is $D = \{x \in \mathbf{R} | 2 \leq x \leq 10\}$.

The range is $R = \{y \in \mathbf{R} | y = 2.5, 5.5\}$.

This relation is a not function.

This relation fails the vertical-line test. At $x = 4$, $y = 2.5$ and 5.5; and at $x = 6.5$, $y = 2.5$ and 5.5.

**real numbers**

the set of real numbers is the set of all decimals—positive, negative, and 0, terminating and nonterminating. This statement is expressed mathematically with the set notation $\{x \in \mathbf{R}\}$

**Communication Tip**

$\{x \in \mathbf{R} | x \neq 0\}$ is the mathematical notation for saying "the set of $x$ such that $x$ can be any real number except 0."

**vertical-line test**

a test to determine whether the graph of a relation is a function. The relation is not a function if at least one vertical line drawn through the graph of the relation passes through two or more points

**d)** The domain is
$D = \{x \in \mathbf{R} | 2 \leq x \leq 9\}$.

The range is
$R = \{y \in \mathbf{R} | 1 \leq y \leq 8.5\}$.

This relation is a function.

The relation passes the vertical-line test; for each $x$-value, there is exactly one $y$-value.

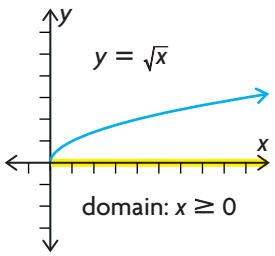
## In Summary

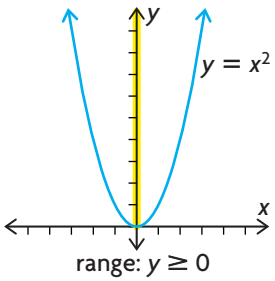
### Key Ideas

- A function is a relation where each value of the independent variable corresponds with only one value of the dependent variable. The dependent variable is then said to be a function of the independent variable.
- For any function, knowing the value of the independent variable enables you to predict the value of the dependent variable.
- Functions can be represented in words, by a table of values, by a set of ordered pairs, in set notation, by a mapping diagram, by a graph, or by an equation.

### Need to Know

- The set of all values of the independent variable is called the domain. On a Cartesian graph, the domain is the set of all possible values of the independent variable $x$.
  *Example*: The relation $y = \sqrt{x}$ exists only for positive values of $x$. The domain of this relation is $x = 0$ and all positive values of $x$.
- The set of all values of the dependent variable is called the range. On a Cartesian graph, the range is the set of all possible values of the dependent variable $y$.
  *Example*: $y = x^2$. The range is $y = 0$ and all positive values of $y$.
- A function can also be defined as a relation in which each element of the domain corresponds to only one element of the range.
- The vertical-line test can be used to check whether the graph of a relation represents a function. If two or more points lie on the same vertical line, then the relation is not a function.
- It is often necessary to define the domain and range using set notation. For example, the set of numbers ..., $-2$, $-1$, 0, 1, 2, ... is the set of integers and can be written in set notation as $\{x | x \in \mathbf{I}\}$, where the symbol "|" means "such that" and the symbol "$\in$" means "belongs to" or "is a member of." So set notation for the integers would be read as "The set of all $x$-values such that $x$ belongs to the integers."



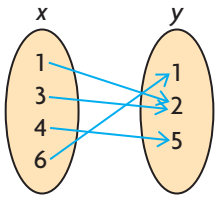
## *CHECK* Your Understanding

**1.** For each relation, state
- **i)** the domain and the range
- **ii)** whether or not it is a function, and justify your answer

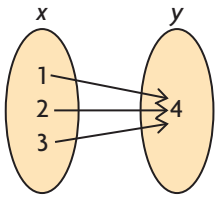
**a)** $g = \{(1, 2), (3, 1), (4, 2), (7, 2)\}$
**b)** $h = \{(1, 2), (1, 3), (4, 5), (6, 1)\}$
**c)** $f = \{(1, 0), (0, 1), (2, 3), (3, 2)\}$
**d)** $m = \{(1, 2), (1, 5), (1, 9), (1, 10)\}$

**2.** For each relation, state
- **i)** the domain and the range
- **ii)** whether or not it is a function, and justify your answer

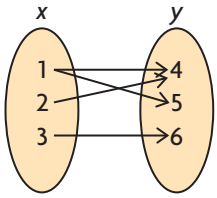
**a)**



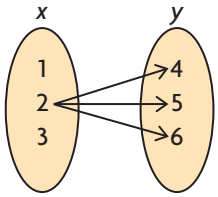

**c)**




**b)**

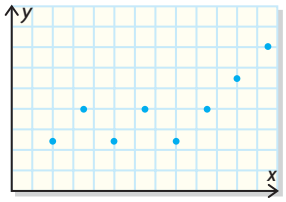



**d)**

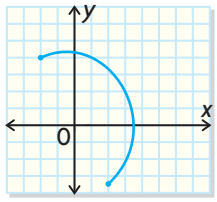



**3.** For each relation, state
- **i)** the domain and the range
- **ii)** whether or not it is a function, and justify your answer

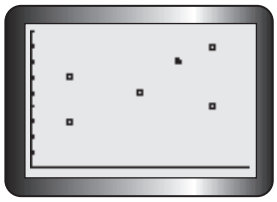
**a)**



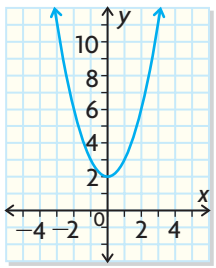

**c)**




**b)**



**d)**



## PRACTISING

**4.** Is the relation a function? Why or why not?

**a)** $h$: $(x, y)$ = (age of child, number of siblings)
$= \{(1, 4), (2, 4), (3, 3), (4, 2)\}$

**b)** $k$: $(x, y)$ = (number of households, number of TV's)
$= \{(1, 5), (1, 4), (2, 3), (3, 2)\}$

**c)**

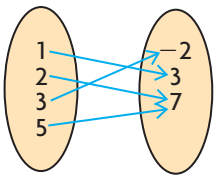
| $x$ | $y$ |
|---|---|
| 0 | −2 |
| 2 | 0 |
| 5 | −2 |

**d)**

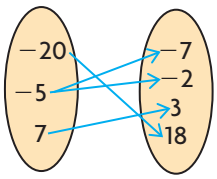
| Number of Drivers, $x$ | Number of Speeding Tickets, $y$ |
|---|---|
| 1 | 6 |
| 1 | 4 |
| 2 | 3 |
| 7 | 8 |

**5.** Is each relation a function? If not, which ordered pair should be removed to make the relation a function?

**a)**

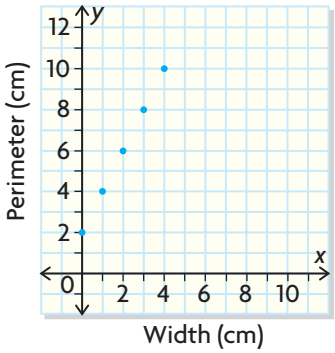



**b)**

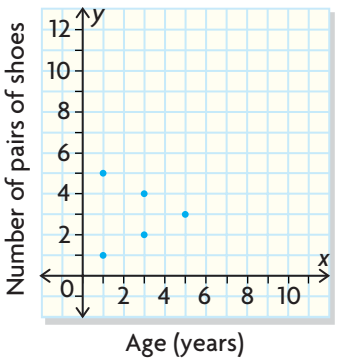



**6.** Which scatter plot represents a function? Explain. For each graph, state the domain and range.

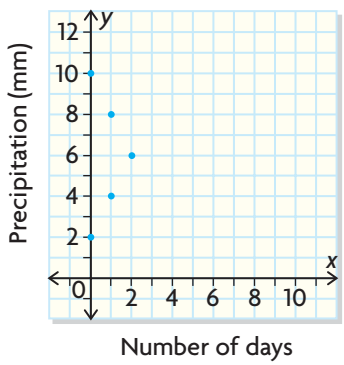
**a)**



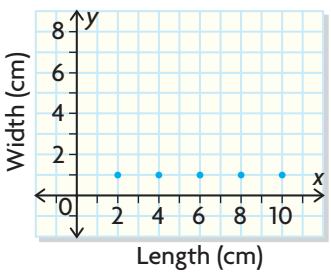

**c)**




**b)**

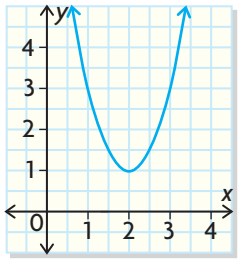



**d)**

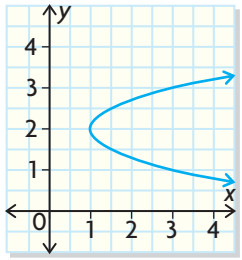


**7.** K Which relations are functions? Explain. For each graph, state the domain and range.

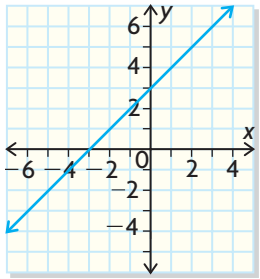
**a)**



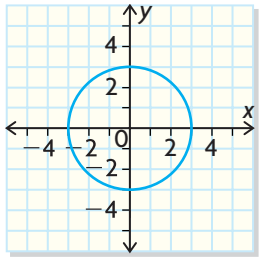

**b)**




**c)**

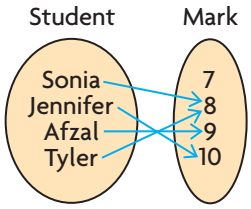



**d)**




**8.** **a)** Draw a line that is both a relation and a function.
**b)** Draw a line that is a relation but is not a function.
**c)** Which type of line cannot be the graph of a linear function? Explain.

**9.** The mapping diagram at the right shows the relation between students and their marks on a math quiz.
**a)** Write the relation as a set of ordered pairs.
**b)** State the domain and range.
**c)** Is the relation a function? Explain.




**10.** A The table gives typical resting pulse rates for six different mammals.

| **Mammal, *x*** | Cat | Elephant | Human | Mouse | Rabbit | Rat |
|---|---|---|---|---|---|---|
| **Pulse Rate, *y* (beats/minute)** | 176 | 27 | 67 | 667 | 260 | 315 |

**a)** Is the resting pulse rate a function of the type of mammal? Explain.
**b)** If more mammals and their resting pulse rates were included, would the extended table of values be a function? Explain.

**11.** Which variable would be associated with the domain for the following pairs of related quantities? Which variable would be associated with the range? Explain.
**a)** heating bill, outdoor temperature
**b)** report card mark, time spent doing homework
**c)** person, date of birth
**d)** number of slices of pizza, number of cuts

**12.** Bill called a garage to ask for a price quote on the size and type of tire he needed.

**a)** Explain why this scenario represents a function.

**b)** If Bill had given the clerk the tire price, would the clerk be able to tell Bill the tire size and type? Would this scenario represent a function?

**13.** T Dates and outdoor temperatures are related. The hottest temperature recorded in Canada was 45 °C at Midvale and Yellow Grass, Saskatchewan, on July 5, 1937. The coldest temperature recorded in Canada was −63 °C, in Snag, in the Yukon Territories, on February 3, 1947.

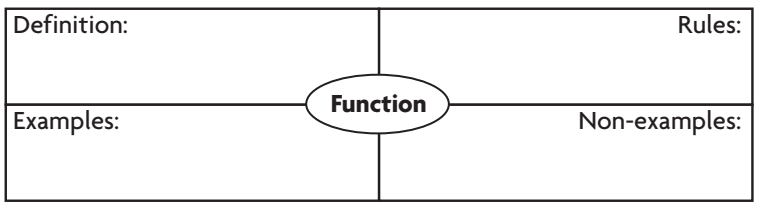
**a)** What is the independent variable in this relation? What is the dependent variable?

**b)** What is the domain?

**c)** What is the range?

**d)** Is one variable a function of the other? Explain.

**14.** C Summarize your understanding of functions in a chart similar to the one shown. Use at least three different types of representation for the examples and the non-examples.




## Extending

**15.** A rock rolls off a cliff 66 m high. Which set of values best represents the range in the relationship between the time elapsed, in seconds, and the resultant height, in metres? Explain.

**a)** 100 to 200

**b)** −100 to 0

**c)** −100 to 100

**d)** 0 to 66

**16.** If a company's profit or loss depends on the number of items sold, which is the most reasonable set of values for the domain in this relation? Explain.

**a)** negative integers

**b)** positive integers

**c)** integers between −1500 and 1500