1 Chapter Review

FREQUENTLY ASKED Questions

Q: What are some of the different kinds of transformations that can be applied to the function $f(x) = x^2$?

A: The transformations include:

1. horizontal translations: The graph of the function is shifted to the left or right.
2. vertical translations: The graph of the function is shifted up or down.
3. vertical stretches: The graph is stretched vertically. Each y-value on the graph is multiplied by a factor that is greater than 1.
4. vertical compressions: The graph is compressed vertically. Each y-value on the graph is multiplied by a factor that is between 0 and 1.
5. reflections: The graph is reflected in the x-axis when each y-value is multiplied by a negative factor.

Study *Aid*

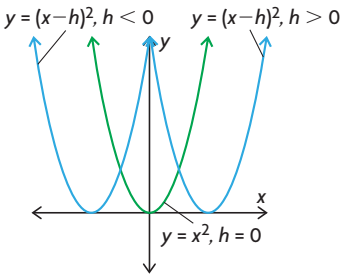
- See Lesson 1.5, Examples 1 to 4.
- Try Chapter Review Question 6.

Q: How is the form $f(x) = a(x - h)^2 + k$ of a quadratic function useful?

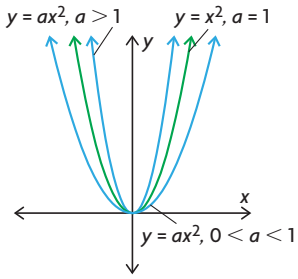
A: Every quadratic function can be written in the form $f(x) = a(x - h)^2 + k$.

The constants a, h, and k each change the location and/or shape of the graph of $f(x) = x^2$.

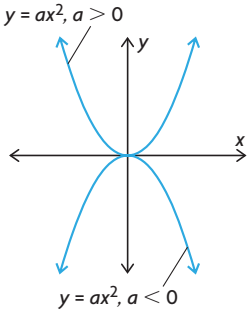
- Horizontal translations: The graph moves to the right when $h > 0$ and to the left when $h < 0$.
- Vertical stretches: The graph is stretched vertically when $a > 1$.
- Vertical compressions: The graph is compressed vertically when $0 < a < 1$.
- Vertical reflections: The graph is reflected in the x-axis when $a < 0$.
- Vertical translations: The graph moves up when $k > 0$ and down when $k < 0$.



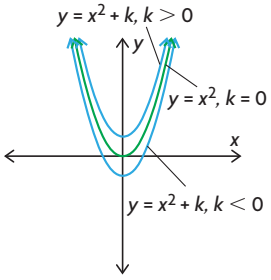
horizontal translations



vertical stretch/compressions



vertical reflections



vertical translations

Q: Why is the order in which I apply the transformations when graphing a quadratic function important?

A: Horizontal translations are applied first, vertical stretches and compressions second, reflections third, and then vertical translations. This order of transformations ensures that the y-values of $f(x) = x^2$ are transformed correctly by following the order of operations for every function of the form $f(x) = a(x - h)^2 + k$.

Study Aid

- See Lesson 1.6, Examples 1 to 4.
- Try Chapter Review Questions 7 to 10.

Q: How do you determine the domain and range of a linear and a quadratic function?

A: The domain of a function is the set of values for which the function is defined. As a result, the range of each function depends on the defining equation of the function.

Most linear functions have the set of real numbers as their domain and range. The exceptions are horizontal and vertical lines. When the line is horizontal, the range is the value of y. When the line is vertical, the domain is the value of x.

Quadratic functions have the set of real numbers for their domain. The range depends on the location of the vertex and whether the parabola opens up or down.

When linear and quadratic functions are used to model real-world situations, the domain and range are often restricted to values that make sense for the situation. Values that don't make sense are excluded.

Study Aid

- See Lesson 1.7, Examples 1 to 4.
- Try Chapter Review Questions 11 to 13.

PRACTICE Questions

Lesson 1.1

1. The data in the table show the average mass of a boy as he grows between the ages of 1 and 12. State the following:

a) domain

b) range

c) whether the relation is a function

Age (years)	1	2	3	4	5	6
Mass (kg)	11.5	13.7	16.0	20.5	23.0	23.0

Age (years)	7	8	9	10	11	12
Mass (kg)	30.0	33.0	39.0	38.5	41.0	49.5

Lesson 1.2

2. Determine, without graphing, which type of relationship (linear, quadratic, or neither) best models this table of values. Explain.

x	-1	0	1	2	3
y	1	2	-3	-14	-31

3. State the degree of each function and whether each is linear, or quadratic, or neither.

a) $f(x) = -8 + 3x$

b) $g(x) = 4x^2 - 3x + 5$

c) $y = (x - 4)(4x^2 - 3)$

Lesson 1.3

4. Evaluate the function $f(x) = 3x^2 - 3x + 1$ at the given values.

a) $f(-1)$ **b)** $f(3)$ **c)** $f(0.5)$

5. For each of the following, determine $f(3)$.

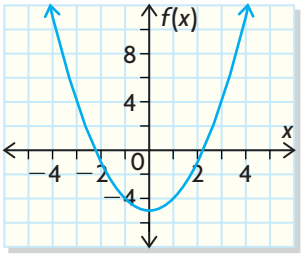
a) $f = \{(1, 2), (2, 3), (3, 5), (4, 5)\}$

b)

x	1	3	5	7
$f(x)$	2	4	6	8

c) $f(x) = 4x^2 - 2x + 1$

d)



Lesson 1.5

6. Use transformations to determine the vertex, axis of symmetry, and direction of opening of each parabola. Sketch the graph.

a) $y = x^2 - 7$

b) $y = -(x + 1)^2 + 10$

c) $y = -\frac{1}{2}(x + 2)^2 - 3$

d) $y = 2(x - 5)^2$

Lesson 1.6

7. Describe how the graph of $y = x^2$ can be transformed to the graphs of the relations from question 6.

8. **a)** Describe how the graph of $y = x^2$ can be transformed into the graph of the given quadratic function.

i) $y = 5x^2 - 4$

ii) $y = \frac{1}{4}(x - 5)^2$

iii) $y = -3(x + 5)^2 - 7$

b) List the domain and range of each function. Compare these with the original graph of $y = x^2$.

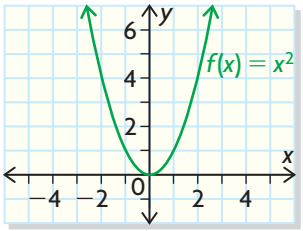
9. **a)** Describe the transformations to the graph of $y = x^2$ to obtain $y = -2(x + 5)^2 - 3$.

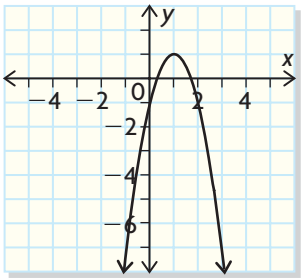
b) Graph $y = x^2$. Then apply the transformations in part (a) to graph $y = -2(x + 5)^2 - 3$.

10. The graphs of $f(x) = x^2$ (in green) and another parabola (in black) are shown.

a) Draw a combination of transformations that would produce the second parabola from the first.

b) Determine a possible equation for the second parabola.





11. On a calculator like the TI-83 Plus, enter these equations: $Y_1 = X^2$, $Y_2 = Y_1 - 4$, and $Y_3 = 2 * Y_2$.

a) Use an appropriate WINDOW setting to graph Y_1, Y_2, and Y_3. Write an equation in vertex form for Y_3.

b) Change Y_2 and Y_3 to $Y_2 = 2 * Y_1$ and $Y_3 = Y_2 - 4$. Find the coordinates of the vertex for Y_3. Write an equation in vertex form for Y_3.

c) Explain why the graph of Y_3 in part (a) is different from the one in part (b).

d) Describe the sequence of transformations needed to transform the graph of $y = x^2$ into the graph of $y = 2x^2 - 4$.

Lesson 1.7

12. A football is thrown into the air. The height, $h(t)$, of the ball, in metres, after t seconds is modelled by $h(t) = -4.9(t - 1.25)^2 + 9$.

a) How high off the ground was the ball when it was thrown?

b) What was the maximum height of the football?

c) How high was the ball at 2.5 s?

d) Is the football in the air after 6 s?

e) When does the ball hit the ground?

13. Clay shooting disks are launched from the ground into the air from a machine 12 m above the ground. The height of each disk, $h(t)$, in metres, is modelled by $h(t) = -5t^2 + 30t + 12$, where t is the time in seconds since it was launched.

a) What is the maximum height the disks reach?

b) At what time do the disks hit the ground?

c) Determine the domain and range of this model.

14. The height, h, in metres, of an object t seconds after it is dropped is $h = -0.5gt^2 + k$, where g is the acceleration due to gravity and k is the height from which the object is released. If an object is released from a height of 400 m, how much longer does it take to fall to a height of 75 m on the Moon compared with falling to the same height on Earth? The acceleration due to gravity is 9.8 m/s^2 on Earth and 1.6 m/s^2 on the Moon.

1 Chapter Self-Test

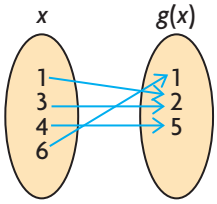
1. For each of the following relations, state

i) the domain and range

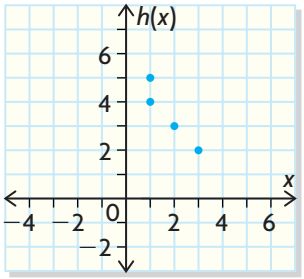
ii) whether or not it is a function, and justify your answer

a) $f = \{(1, 2), (3, 1), (4, 2), (7, 2)\}$

b)



c)



2. Define the term *function* and give an example and a non-example.

Time (s)	Height (m)
0	0
1	30
2	40
3	40
4	30
5	0

3. Use a difference table to determine whether the data in the table at the left represent a linear or quadratic relationship. Justify your decision.

4. If $f(x) = 3x^2 - 2x + 6$, determine

a) $f(2)$ **b)** $f(x - 1)$

5. $f(x) = 3(x - 2)^2 + 1$

a) Evaluate $f(-1)$.

b) What does $f(1)$ represent on the graph of f?

c) State the domain and range of the relation.

d) How do you know if f is a function from its graph?

e) How do you know if f is a function from its equation?

6. A function is defined by the equation $d(x) = 5(x - 3)^2 + 1$.

a) List the transformations to the graph of $f(x) = x^2$ to get $d(x)$.

b) What is the maximum or minimum value of the transformed function $d(x)$?

c) State the domain and range of $d(x)$.

d) Graph the function $d(x)$.

7. A football is kicked from a height of 0.5 m. The height of the football is modelled by the function $h(t) = -5t^2 + 18t + 0.5$, where t is time in seconds and $h(t)$ is height in metres.

a) Graph the function for reasonable values of t.

b) Explain why the values you chose for t in part (a) are reasonable.

c) What is the maximum height of the football?

d) At what time does the football reach the maximum height?

e) For how many seconds is the football in the air?

f) Express the domain and range in set notation.

1 Chapter Task

Using Transformations and Quadratic Function Models

Quadratic functions can be used as mathematical models of many real-life situations. Since the graphs of these functions are parabolas, many familiar examples can be found that appear to utilize this shape. Here are some of them:




? How can you use transformations to determine the equation of a quadratic function that models a parabola seen in a picture?

A. Use the Internet to find a picture that appears to have a parabola (or part of one) in it.

B. Copy the picture and paste it into *Geometer's Sketchpad*. Place a grid over top of your picture.

C. Create a quadratic function of the form $f(x) = ax^2$ by estimating an appropriate value for a. Adjust this value and re-graph your function until you are satisfied with the width of your parabola.

D. Create a quadratic function of the form $f(x) = a(x - h)^2 + k$ by estimating appropriate values for h and k. Adjust these values and re-graph your function until you are satisfied that the graph of your function closely matches that of the parabola in the photo. Print your final results.

E. List the transformations you used to create your graphical model as well as the final equation of your quadratic function.

F. State any restrictions on the domain and range that must be applied to ensure that your graph matches only the parabola shown in the photo.

G. Repeat parts A to F using a picture that contains a parabola that opens in the opposite direction to the one you originally found.

Tech Support

For help using *Geometer's Sketchpad* to graph functions, see Technical Appendix, B-19.

Task Checklist

- ✔ Did you show all your steps?
- ✔ Did you include a printout of your graph and photo?
- ✔ Did you support your choice of transformations used?
- ✔ Did you explain your thinking clearly?
- ✔ Did you restrict the domain and range?