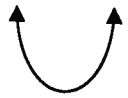
Quadratic Relations

- A quadratic relation is a 'u' shaped curve, though it may be upside down.

i.e.  or 

Linear, Quadratic or Neither? How can we tell?

1) Table of Values
   - Calculate the finite differences (differences in consecutive y-values).
     - If the 1st differences are the constant: it is a linear function
     - If the 2nd differences are the constant: it is a quadratic function
     - If neither are constant, we say the function is neither.

Ex/ Determine if the functions are linear, quadratic or neither.

x	5	10	15	20
y	10	20	30	40

↓10 ↓10 ↓10

All same value

∴ Linear

x	1	2	3	4
y	0	3	8	15

↓3 ↓5 ↓7

↓2 ↓2

2nd differences the same → quadratic

- The value of the 2nd differences actually determines which way the parabola opens. A positive 2nd difference: parabola opens upwards; a negative: downwards.

2) The Equation
   - Look at the degree (highest exponent) of the equation after brackets have been removed.
     - If the degree is 1: it is a linear function
     - If the degree is 2: it is a quadratic function

Ex/ Determine if the following functions are linear, quadratic or neither.

a) $y = x^2 + 4$

↑

Degree 2

∴ Quadratic

b) $y = 3x - 7$

↑

Degree 1

∴ Linear

c) $y = 5x(x-3)$ ↓ simplify 1st

$= 5x^2 - 15x$

Degree 2

∴ Quadratic

d) $y = x^3$

Degree 3

∴ Neither

Properties of Quadratics

a) Vertex: The point where the curve changes direction.

b) X-intercepts/zeros: The point(s) where the curve crosses the x-axis.

c) Y-intercept: The point where the curve crosses the y-axis.

d) Maximum/minimum: Does the curve have a highest or lowest value.

e) Axis of symmetry: The imaginary line that divides the curve in half vertically. It has the equation x = *x-coordinate of vertex*

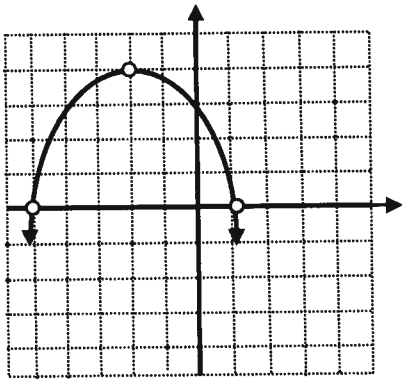
f) Optimal value: The highest or lowest point that the curve reaches. It is the y-coordinate of vertex.

g) Direction of opening: The direction in which the curve get wider.

h) 2nd differences: opens up - positive differences, opens down - negative differences

Ex/ Label or determine the properties of the following quadratics.

a)



Vertex: (-2,4)

Zeros: (1,0), (-5,0)

or $x=1$, $x=-5$

y-int: (0,3)

max (reaches a highest value)

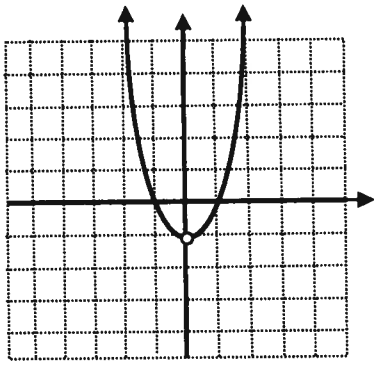
Axis: $x=-2$

Optimal: $y=4$

opens down

Negative 2nd differences

b)



Vertex: (0,-1)

Zeros: (-1,0), (1,0)

y-int: (0,-1)

min

Axis: $x=0$

optimal: $y=-1$

opens up

Positive 2nd differences

Ex/ The x-intercepts of a quadratic relation are -2 and 6. The second differences are positive.

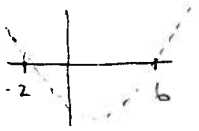
a) Which direction will the parabola open?

b) Is the optimum value positive or negative?

c) What is the axis of symmetry?

a) Opens up - positive 2nd dif

b) Negative



c) $x = 2$ * axis of symmetry is always midway between zeros $x = \frac{-2+6}{2} = \frac{4}{2} = 2$

Ex/ The zeros of a parabola are 3 and 5. The maximum value is 7.

a) Which way does the quadratic open?

b) Determine the vertex.

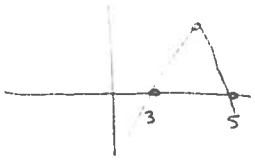
c) Will the y-intercept be a positive or negative value?

a) Down - it has a maximum

b) $x = \frac{3+5}{2} = \frac{8}{2} = 4$

Vertex: $(4, 7)$

c)



Negative y-int

Homework: Handout - Properties of Quadratics